Young Trotter Edinburgh

Octor. 28 : 1782

1782 The

Young

Young Trotter

Edinburgh

October 1782

Euclides

THE ELEMENTS

Young OF Trotter

Edin

EUCLID,

In which the Propositions are demonstrated in a new and shorter Manner than in former Translations, and the Arrangement of many of them altered,

To which are annexed

Plain and Spherical Trigonometry, Tables of Logarithms from 1 to 10,000, and Tables of Sines, Tangents, and Secants, Natural and Artificial.

BY

GEORGE DOUGLAS,

Teacher of Mathematics in the Academy at Ayr.

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M,DCC,LXXVI.

TO

MATTHEW STUART, D.D.

THE

FOLLOWING EDITION

OF THE

ELEMENTS OF EUCLID

ARE INSCRIBED

BY

HIS MOST OBEDIENT

AND MOST HUMBLE SERVANT

GEORGE DOUGLAS.

P R E F A C E.

MOST authors, from a natural anxiety to render their subjects as compleat as poſſible, are in danger of being betrayed into prolixity: An attention to minute circumſtances may be neceſſary in ſome kinds of compoſition, but prolixity is altogether inexcuſable in a ſcientific writer. His object is to explain the principles of ſcience in the moſt ſimple and perſpicuous manner. To accompliſh this end, every ſuperfluity of language and reaſoning ought to be ſtrictly guarded againſt. Whoever has attended to books of ſcience will readily allow, that moſt of them are capable of abridgement; and that this abridgement, inſtead of obſcuring, or rendering the ſubject more difficult, will make it more clear and intelligible to the generality of ſtudents.

Simplicity and conciſeneſs are peculiarly neceſſary in communicating the Elements of ſcience, which are always leſs intereſting to the ſtudent than the practical parts. If the author be tedious in this article, the mind, being entirely unacquainted with the utility or application of elementary truths, is apt to revolt and abandon the ſtudy. But ſimplicity and conciſeneſs are more indiſpenſible in the elements of mathematics than any other ſcience. Unfortunately, however, too little attention has hitherto been given to this circumſtance.

Euclid, an author long and juſtly admired for the excellency of his general method, has often gone ſo minutely to work in his demonſtrations, as to render many plain propoſitions not only tedious, but difficult. His manner of demonſtrating is unqueſtionably the beſt that has yet appeared, and therefore ought to be followed: But it is by no means impoſſible to make his demonſtrations as plain in much fewer words, and even to arrange many of them in a different manner, without doing the leaſt injury to his principles.

This taſk I have undertaken in the following ſheets. If I have ſucceeded, one capital objection to the ſtudy of mathematics is happily removed, as the Elements of Euclid may now be learned in one half of the uſual time, and with greater eaſe to the ſtudent.

That the reader may be the better prepared for the alterations he may meet with, I have here mentioned a few, with the reaſons which induced me to make them.

Book

Book I. ax. 10. "Two right lines do not bound a figure;" instead of "include a space," the boundaries of space, being disputed by metaphysical writers, become unfit for a mathematical axiom. Prop. 5. which is rather too tedious, I have proved from prop. 4. in very few words, and have not used more freedom than is done in the demonstration as it now stands. The second part, *viz.* the angles below the base, I have left out till the 13th is proved, from which it easily follows; and likewise in proving the bases equal in the 4th, I have changed the indirect proof, and given a direct one, by which it is both shorter and easier comprehended. The manner in which I have enunced the 7th prop. renders the second part of the 5th unnecessary; yet have supposed no more given than what must be supposed before a proof can be begun. But, those who think it ought to be in more general terms, I have indulged in the 21st, from which it naturally follows. As some have thought axiom 12. not self-evident, and therefore ought not to be an axiom, I have added a cor. to prop. 17. that convincingly proves it. The 35th and 37th are joined in one, as nothing can follow more naturally than, if the wholes are equal, their halfs are likewise so. The same may be said of the 36th and 38th; nor is it less natural to prove it from half the parallelogram than to double the triangle, and then take its half. I cannot agree with Mr Simpson in leaving out the corollaries from prop. 32. nor can I find any reason for his so doing.

Book II. I have varied the enunciation of several of the propositions, and expressed them in clearer terms. In the 8th proposition, the equality of the squares is proved in a shorter but clearer manner than that presently used. The 13th is retained much in the same manner as in Commandine's Euclid; for, though it be true of every side of a triangle subtending an acute angle; yet, as the demonstration is general, and the perpendicular falling within or without the triangle, makes no real alteration, proving it in different figures becomes unnecessary.

Book III. The first definition is challenged by Mr Simpson, which, he says, ought to be proved; for this I can see no reason, or any necessity of a proof, as the equality of coincident figures is admitted, ax. 8. Book I. I have taken another demonstration in place of that used in the 2d proposition, which I thought as mathematical as that used either by Commandine or Simpson, and much shorter. To the 8th prop. I have added, "that only two "equal lines can fall either upon the convex or concave part of "the circumference;" but the demonstration of the whole is shorter than that presently used. In the 16th, "the angle of a "semicircle" is omitted, because it follows more naturally as a corollary. The 18th and 19th are joined in one, for the reasons already given. I have put a short and natural demonstration

tion in place of the 2d part of prop. 21. and changed the figure. The 25th is ſhortened, and the 28th and 29th joined in one. In the 31ſt, "the angle of a ſegment" is left out, but reſumed in the cor. as it follows naturally from the propoſition. I have added a cor. to prop 37. which is found neceſſary in practice.

Book IV. is much ſhortened, the 12th, 13th, and 15th, are demonſtrated in a different manner.

Book V. is ſhortened almoſt in every propoſition.

In Book VI. I have added a few words to the 5th def. which renders it compleat; the lemma added to prop. 22. is therefore unneceſſary; as alſo def. A. inſerted after def. 11. book V. by Mr Simpſon. The 5th and 6th propoſitions are joined in one, as alſo the 14th and 15th; the demonſtrations are in general ſhorter.

Book XI. Def. 10. is retained, as univerſally true, for the reaſons given in the note at the end of the preface. Prop. 7. As this propoſition has no dependence on any of the preceding propoſitions of this book, I have put it in place of the 6th, and joined the 6th and 8th in one, by which the propoſition is made both ſhorter and plainer than when ſeparate. The greateſt part of the propoſitions of this book are conſiderably ſhortened.

Book XII. Prop. 5. and 6. are joined in one, and much ſhortened, and the demonſtrations in part new. The 8th and 9th are demonſtrated in a much ſhorter and more familiar manner; the greateſt part of the 10th and 11th being only a repetition of the 2d, that Prop. is only referred to, as it is not neceſſary to demonſtrate a prop. twice over, nor has Euclid done ſo any where at ſo great length as in this book.

In Plain Trigonometry I have not inſerted any thing that depends for illuſtration on infinite ſeries, that being a ſubject more proper for the higher parts of mathematics; but have rendered the elements ſhort and comprehenſive, ſo as fully to contain the principles of trigonometry, as well as to explain the nature and uſe of the logarithmic canon.

In Spherical Trigonometry, the propoſitions are demonſtrated in a ſhort and eaſy method, from the principles of plain trigonometry. The obſervations made on them by Mr Cunn are left out, being wholly contained in the propoſitions, and what he intends by them eaſily diſcovered in practice.

I have added a ſhort explanation, of the nature and uſe of Sines, Tangents, Secants, and verſed Sines, both natural and artificial; and how to change Briggs's Logarithms to the Hyperbolic, and *vice verſa*, with examples of the above. To which are annexed Tables of the Logarithms of Numbers, of Sines, Tangents, and Secants, both natural and artificial, which will work to the ſame exactneſs, of any extant, even to ſecond and third minutes, or farther, if thought neceſſary.

Upon

Upon the whole, although the above alterations are intended to render the elements easier and sooner acquired, yet are not intended to indulge the indolence of either master or student. The Elements of Geometry being of such extensive use, that a thorough knowledge of them is absolutely necessary, whether in the literary or mechanic profession; the conciseness of the reasoning, and conclusiveness of the arguments, render that knowledge a necessary qualification for the pulpit or bar; and in prosecuting the sciences, this knowledge becomes absolutely necessary: but the sooner it can be acquired, a thorough knowledge of it may more easily be attained: and what is reserved of that time, which even an experienced Teacher would formerly have taken up in barely demonstrating the propositions, may be employed in pointing out their particular beauties, the accuracy of the reasoning, their use in the affairs of life, and their application to the sciences, which will be of great advantage to the student, as he is hereby let into the beauties of the science by the time he formerly could have had but even a tolerable knowledge of the method of demonstration.

The author does not hereby mean to insinuate, that this work is without exception; that notwithstanding the pains he has taken to render it as correct as possible, yet several inaccuracies, both in the language and demonstrations, may have escaped his notice, which he hopes the learned will excuse, and lend their assistance to render it more useful, if they shall think it worthy of another impression.

That Mr Simpson has fallen into a mistake, in the demonstration he has given to prove the falsity of def. 10. Book XI. will appear from the following observations:

He has proved that the triangles EAB, EBC, ECA, containing the one solid, are equal and similar to the three triangles FAB, FBC, FCA, containing the other solid, and having the common base ABC; he does not deny the equality of these solids, but compares them with another solid contained by three triangles GAB, GBC, GCA, and common base ABC, which three triangles he neither proves equal nor similar; but concludes, that the solid contained by the three triangles GAB, GBC, GCA, is not equal to the solid contained by the three triangles EAB, EBC, ECA, and common base ABC, because the one contains the other. If he had proved, that the triangles GAB, GBC, GCA, were equal and similar to the other three triangles EAB, EBC, ECA, and common base ABC, and then proved the solids not equal, he would then have gained his point; but as he has not even so much as attempted this, def. 10. must be held as universally true; at least till some better argument is produced against it.

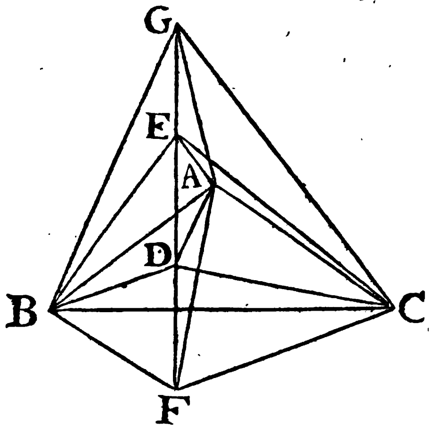
But

But as he ſuppoſes it proved not univerſally true, he preſents us with prop. A, B, C, after prop. 23. Book XI. to ſupply its defect. prop. C. "Solid figures contained by the ſame number of equal and ſimilar planes alike ſituated, and having none of their ſolid angles contained by more than three plane angles, are equal and ſimilar to one another." But this prop. C. will evidently appear inſufficient to ſupply this ſuppoſed defect, on account of the limited ſenſe in which it is taken; for, if ſolid figures, bounded by an equal number of equal and ſimilar planes, are not equal and ſimilar, but under this limitation, then prop. 15. Book V. muſt not be univerſally true, which I ſuppoſe will not eaſily be admitted; and, if not admitted, then prop. C muſt be a very inſufficient foundation for proof of the following propoſitions depending on it, *viz.* Prop. 25. 26. and 28. and conſequently eight others, *viz.* 27th, 31ſt, 32d, 33d, 34th, 36th, 37th, and 40th. Book XI. all which are by this author toſſed off their baſe, which is univerſally true, and placed upon this limited one.

Mr Simpſon farther objects, that though this definition be true, yet ought not to be a definition, but a propoſition, and the truth of it proved.

The ſame objection might be made with equal propriety to ſeveral others; for example, why not prove the equality of theſe angles which determine the equal inclination of planes, Def. 7. Book XI. and the equality of right lines equally diſtant from the center, both which we may conclude to be Euclid's, as Mr Simpſon does not object to them; for he would make us believe none are Euclid's that he does not affirm to be ſo, and that frequently without any other reaſon given for it, but his own *ipſe dixit*. If we conſider the nature of a definition, it is, if I miſtake not, diſtinguiſhing bodies from one another, by ſuch properties as cannot be applied to any other bodies, but thoſe it is intended to diſtinguiſh. In which ſenſe, if the properties given in this definition are ſuch as diſtinguiſh ſimilar and equal bodies from others that are not ſo in every inſtance, then it is certainly a proper definition; but Euclid has ſometimes thought proper to prove his definitions; for example, def. 4. Book III. which he has proved, prop. 14. of that book. This, it would appear, he has not thought neceſſary to prove, probably, if we may be allowed to aſſign a reaſon in his name, that he has thought it ſo ſelf-evident, that none would ever call the truth of it in queſtion; but as the truth of it has been called in queſtion, the definition may be proved in the following manner from Mr Simpſon's demonſtration to prove the contrary; for which obſerve his own figure and demonſtration. He has proved the three triangles EAB, EBC, ECA, containing the one ſolid, equal and ſimilar to the three triangles FAB, FBC, FCA, containing the other ſolid

lid, having the common baſe, ABC ; then, if the ſolid EABC is not equal to the ſolid FABC, let it be equal to ſome ſolid as GA BC, either greater or leſs than EABC, which cannot be; for the one would contain the other ; and if the ſolid angle is contained by more than three plane angles, equal and ſimilar to one another, then it can be divided into angles which are contained by three equal and ſimilar plane angles, by Prop. 20. Book VI. and parts have the ſame proportion as their like multiples, by Prop. 15. Book V. wherefore univerſally, figures bounded by an equal number of equal and ſimilar planes are equal and ſimilar.



N. B. In the references, when the propoſition referred to is in the ſame book with the propoſition to be proved, the book is not named, but only the number of the propoſition, but, if in any other book, both are named.

THE ELEMENTS OF EUCLID.

BOOK I.

DEFINITIONS.

I.

A Point is that which hath no parts or magnitude.

II.

A line is length without breadth.

III.

The bounds of a line are points.

IV.

A right line is that which lieth evenly between its points.

V.

A ſuperficies is that which hath only length and breadth.

VI.

The bounds of a ſuperficies are lines.

VII.

A plain ſuperficies is that which lieth evenly between its lines.

VIII.

A plain angle is the inclination of two lines to one another in the ſame plain, which touch each other, but do not lie in the ſame right line.

IX.

If the lines containing the angle be right ones, then the angle is called a right-lined angle.

X.

When one right line ſtanding on another right line makes the angles on each ſide thereof equal to one another, each of theſe angles is a right one, and that line which ſtands upon the other is called a perpendicular to that whereon it ſtands.

Book I.

XI.

An obtuſe angle is that which is greater than a right one.

XII.

An acute angle is that which is leſs than a right one.

XIII.

A term, or bound, is the extreme of any thing.

XIV.

A figure is that which is contained under one or more terms.

XV.

A circle is a plain figure bounded by one line, called the circumference, to which all right lines drawn from a certain point within the ſame are equal.

XVI.

That point is called the center of the circle.

XVII.

The diameter of a circle is a right line drawn through the center, and terminated on both ends by the circumference, and divides the circle into two equal parts.

XVIII.

A ſemicircle is a figure contained under any diameter, and the circumference cut off by that diameter.

XIX.

A ſegment of a circle is a figure contained under a right line, and circumference cut off by that right line.

XX.

Right-lined figures are ſuch as are contained by right lines.

XXI.

Three ſided figures are ſuch as are contained by three lines.

XXII.

Four ſided figures are ſuch as are contained by four lines.

XXIII.

Many ſided figures are ſuch as are contained by more than four lines.

XXIV.

An equilateral triangle is that which hath three equal ſides.

XXV.

An iſoſceles triangle, that which hath two ſides equal.

XXVI.

A ſcalene triangle, that which hath all the three ſides unequal.

XXVII.

A right angled triangle is that which hath one right angle in it.

XXVIII.

An obtuſe angled one, that which hath one obtuſe angle in it.

XXIX.

An acute angled triangle is that which hath all the angles leſs than right ones.

XXX. A

XXX.

A ſquare is that which hath four equal ſides, and its angles all right ones.

XXXI.

An oblong, or rectangle, is longer than broad, its oppoſite ſides are equal, and its angles all right ones.

XXXII.

A rhombus, that which hath four equal ſides, but not right angles.

XXXIII.

A rhomboides, whoſe oppoſite ſides and angles are equal.

XXXIV.

All quadrilateral figures beſide theſe are called trapezia.

XXXV.

Parallel right lines are ſuch as, being produced both ways in the ſame plain, never meet.

XXXVI.

A parallelogram is a figure whoſe oppoſite ſides are parallel.

POSTULATES.

I.

GRANT that a right line may be drawn from any one point to another:

II.

That a finite right line may be continued directly forwards: And,

III.

That a circle may be deſcribed about any center, with any diſtance.

AXIOMS.

I.

THINGS equal to one and the ſame thing are equal to one another.

II.

If equal things are added to equal things, the wholes will be equal.

III.

If from equal things equal things be taken, the remainders will be equal.

IV.

If to unequal things equal things are added, the whole will be unequal.

Book I.

V.

If from unequal things equal parts are taken, the remainders will be unequal.

VI.

Things which are double one and the ſame thing are equal between themſelves.

VII.

Things which are half one and the ſame thing are equal between themſelves.

VIII.

Things which mutually agree together are equal to one another.

IX.

Any whole is greater than its part.

X.

Two right lines do not bound a figure.

XI.

All right angles are equal to one another.

XII.

If a right line fall upon two right lines, making the inward angles on the ſame ſide leſs than two right angles, theſe right lines continually produced will at laſt meet one another on that ſide where the angles are leſs than right ones.

N. B. Any angle is expreſſed by three letters, of which that at the vertex is named betwixt the other two.

PRO-

PROPOSITION I. PROBLEM.

To describe an equilateral triangle upon a given right line.

Let AB be the given right line, upon which it is required to describe an equilateral triangle.

About the center A, with the distance AB, describe the circle BCD[a]; and about the center B, with the distance BA, describe the circle ACE[b]; from the point C, where the two circles intersect each other, draw the lines CA, CB[b]. a Postulate 3. b Post. 1.

Then, because A is the center of the circle DBC, AC is equal to AB[c]; and because B is the center of the circle ACE; BC is equal to BA[c], but CA is proved equal to AB, and BC to AB; therefore BC is equal to AC[d]: Therefore the three sides AB, BC, CA, are equal to one another: Therefore, upon the given right line AB, there is described an equilateral triangle ABC: Which was required. c Definition 15. d Axiom 1.

PROP. II. PROB.

At a given point to put a right line equal to a given right line.

Let A be the given point, and BC the given right line, it is required to put a right line at the point A, equal to the given right line BC.

With the center C and distance BC, describe the circle BGH[a]; join AC[b], upon which describe an equilateral triangle DAC[c]; produce DC that passes through the center to G, in the circumference; and DA to any distance E[d]; with the center D, and distance DG, describe the circle KGL. a Post. 3. b Post. 1. c 1. d Post. 2.

Then, because C is the center of the circle BGH, BC is equal to CG[e]; and because D is the center of the circle KGL, DG is equal to DL[e], but DC is equal to DA[e]: Therefore the remainders AL, and CG, are equal[f]; but BC, AL, are each proved equal to CG; and therefore equal to one another[g]: Therefore, from the point A, there is drawn the right line AL, equal to the given right line BC: Which was required. e Def. 15. f Ax. 3. g Ax. 1.

PROP.

Book I.

PROP. III. PROB.

TWO unequal right lines being given, to cut off from the greater a part equal to the lesser.

Required to cut off from the greater AB a part AE, equal to the lesser C.

a 2. From the point A draw a right line AD equal to C [a], about
b Post. 3. the center A, with the distance AD, describe a circle DEF [b]; then, because A is the center of the circle DEF, AE is equal to
c Def. 15. AD [c], but AD is equal to C: Therefore AE is likewise equal
d Ax. 1. to C [d]: Which was required.

PROP. IV. THEOREM.

IF there be two triangles having two sides of the one equal to two sides of the other, each to each; and the angle contained by the two sides of the one, equal to the angle contained by the correspondent sides of the other; then the base of the one triangle will be equal to the base of the other; the two triangles will be equal, and the remaining angles of the one equal to the remaining angles of the other, each to each, which the equal sides subtend.

Let the two triangles be ABC, DEF, having the two sides AB, AC, equal to the two sides DE, DF, each to each; that is, AB equal to DE, and AC equal to DF, and the angle BAC equal to EDF; then the bases BC and EF will be equal, the triangle ABC equal to the triangle DEF, the angle ABC equal to the angle DEF; and ACB equal to DFE, each to each, which the equal sides subtend.

For, let the triangle ABC be applied to the triangle DEF, so as the point A may coincide with the point D, the right line AB with DE, then the point B will coincide with the point E; for AB and DE are equal; and, because the angles BAC EDF are equal, the right line AC will coincide with DF, and the point C with F; for AC and DF are equal: Then, because the point B coincides with the point E, and C with F, the right
a Def. 4. lines BC and EF, will coincide [a]; and therefore equal [b]: There-
b Ax. 8. fore the triangles ABC, DEF, are equal, the angle ABC equal to the angle DEF, and ACB to DFE, each to each, which the equal sides subtend. Wherefore, &c.

PROP.

PROP. V. THEOR.

THE angles above the base of every isosceles triangle are equal to one another.

Let the triangle ABC be the isosceles triangle, having the side AB equal to the side AC, then the angle ABC will be equal to the angle ACB.

For, let a triangle DEF be likewise given, having the sides DE, DF, equal to the sides AB, AC, each to each; and the angle BAC equal to the angle EDF, then the bases EF, BC, are equal; and the angle ABC equal to the angle DEF, and ACB to DFE, each to each; which the equal sides subtend[a]; but the sides AB, DF, are equal[b]; therefore the angles ACB, DEF, are likewise equal; but the angles DEF, ABC, are equal[b]; therefore the angles ABC, ACB, are likewise equal[b]. Wherefore, &c.

a 4.
b Ax. 1.

CORROLLARY. If any triangle is equilateral, it will likewise be equiangular.

PROP. VI. THEOR.

IF any triangle have two angles in it equal to one another, the sides subtending these angles will likewise be equal.

Let the triangle ABC have the angles ABC, ACB, equal; then the sides AB, AC, will likewise be equal to one another.

For, if AB is not equal to AC, let one of them, as AB, be the greater; from which cut off DB equal to AC[a], and join DC; then, because DB is equal to AC, the two sides DB, BC, are equal to the two sides AC, BC, and the angles ACB, DBC, equal; therefore the base AB is equal to the base DC, and the triangles ABC, DBC, equal[b]; that is, a part equal to the whole; which is absurd: Wherefore the sides AB, AC, are equal; that is, the triangle is isosceles. Wherefore, &c.

a 3.
b 4.

COR. Hence every equiangular triangle is also equilateral.

PROP. VII. THEOR.

IF from the extremity of any right line, to two different points on the same side, there be drawn two right lines equal to one another, the lines drawn from the other extremity, to the same points, cannot be equal to one another.

If from the extremity A, of the right line AB, to the points C, D, on the same side, the two lines AD, AC, are drawn equal

to

BOOK. I. to one another, from the extremity B, to the same points C, D, the lines BC, BD, are not equal to one another.

Join DC; for, because AC, AD, are equal, the angles ADC, ACD are equal [a]; but the angle BDC is greater than ADC [b], or ACD, and much greater than BCD; but, if BD is equal to BC, the angles BCD, BDC, are equal, and likewise greater; which is impossible: Therefore, BD is not equal to BC. If BD is made equal to BC, it is proved in the same manner that AC is not equal to AD. Wherefore, &c.

a 5.
b Ax. 9.

PROP. VIII. THEOR.

If two triangles have two sides of the one, equal to two sides of the other, and the base of the one equal to the base of the other, then the angles that these equal bases subtend will be equal to one another.

Let the two triangles be ABC, DEF, having the two sides AB, AC, equal to the two sides DE, DF, each to each; and the bases BC, EF, equal; then the angles BAC, EDF, will be equal.

For, let the triangle ABC be applied to the triangle DEF, so that the right line BC may coincide with EF; then the point B will coincide with the point E, and C with F; for the sides BC, EF, are equal: And the sides BA, AC, will coincide with ED, DF, and the point A with D. If not, let the point A fall in G; then EG, ED, are equal; for each are equal to BA [a]; and FG, FD, will likewise be equal, which is impossible [b]. Wherefore the point A cannot fall in G, and, for the same reason, in no point but D; therefore the angle BAC is equal to the angle EDF. Wherefore, &c.

a Ax. 1.
b 7.

PROP. IX. PROB.

To cut a given right lined angle into two equal parts.

Let BAC be the given right lined angle required to be cut into two equal parts.

Assume any point D, in the right line AB, and cut off AE equal to AD [a]; join DE, upon which describe the equilateral triangle DEF [b], and join AF; then the right line AF will bisect the given angle BAC; for, because DA is equal to EA, and AF is common, and the base DF equal to EF [b]; therefore the angle DAF is equal to the angle EAF [c]: Therefore the angle BAC is bisected by the right line AF. Which was required.

a 3.
b 1.
c 8.

PROP.

PROP. X. PROB.

To cut a given finite right line into two equal parts.

Let AB be a given right line, required to be cut into two equal parts; upon it describe an equilateral triangle ABC; bisect the angle ACB by the right line CD [a]; then is the right line AB bisected in D. a 9.

For, because AC is equal to CB, and CD common, and the angles ACD, BCD, equal [a], the bases AD, BD, are equal [b]: Therefore the right line AB is bisected in D. Which was required. b 4.

PROP. XI. PROB.

To draw a line at right angles to a given right line from a point given in the same.

Let AB be the given right line, and C the given point in it, from which it is required to draw a right line, at right angles to the given right line AB.

Assume any point D in AC, and make CE equal to CD [a]; a 3.
upon DE describe an equilateral triangle DEF [b], and join FC, b 1.
which will be at right angles to AB. For, because DC, CE, are equal [c], and CF common, the two sides DC, CF, are equal c. const.
to the two sides EC, CF, and the bases FD, FE, equal [b], the angles FCD, FCE, are likewise equal [d]: Therefore each of them d 8.
is a right angle, and FC perpendicular to AB [e]. Which was e Def. 10.
required.

PROP. XII. PROB.

To draw a right line perpendicular to a given indefinite right line from a given point out of it.

Let AB be the given right line, and C the point given out of it, from which its required to let fall a perpendicular upon the indefinite given right line AB.

Assume any point D, on the opposite side of the right line AB; about the center C, with the distance CD, describe a circle EDG; bisect EG in H, join CG, CH, CE; then CH is the perpendicular required.

Book I. For GH, HC, are equal to EH, HC; and the baſes GC, EC [a], equal: Therefore the angles GHC, EHC, are equal [b]. a def. 15. Each of them are right angles, and HC perpendicular to AB [c]. b 8. Which was required.
c def. 10.

PROP. XIII. THEOR.

WHEN a right line ſtands upon a right line, making angles with it, theſe angles are either two right angles, or, together, equal to two right angles.

For, let a right line, AB, ſtand upon the right line CD, making angles CBA, ABD; theſe angles ſhall either be two right angles, or, together, equal to two right angles.

a def. 10. For, if CBA, ABD, be equal, they are right angles [a]; if not, b 11. from the point B draw BE, at right angles, to DC [b]: Therefore the two right angles CBE, EBD, are equal to the three angles ABC, ABE, EBD; but the two angles ABC, ABD, are equal to the ſame three angles: Therefore the two angles ABC, ABD, are equal to the two angles CBE, EBD; that is, equal to two c ax. 1. right angles [c]. Wherefore, &c.

Cor. If the two ſides of an iſoſceles triangle ADE, be produced to B, C, the angles below the baſe will be equal to one a- * Fig. to prop. 9. nother; for the angles ADE, EDB*, are equal to two right d 13. angles [d], and AED, DEC, are equal to two right angles [d]; but e 5. the angles ADE, AED, above the baſe, are proved equal [e]: f ax. 3. Therefore the remaining angles BDE, DEC, are equal [f]: Therefore, in every iſoſceles triangle, the angles above the baſe are equal to one another; and, if the ſides be produced, the angles below the baſe are likewiſe equal to one another.

PROP. XIV. THEOR.

IF to any right line, and point therein, two right lines be drawn from oppoſite points, making the adjacent angles together equal to two right angles, theſe two right lines will make one continued right line.

For, if to any right line AB, and point B therein, be drawn two right lines, CB, DB, from the oppoſite points, C, D, making the angles ABC, ABD, equal to two right angles; then DBC will be one right line. If not, let CBE be one right line; then a. 13. the angles ABC, ABE, will be equal to two right angles [a]; but b. hyp. ABC, ABD, are equal to two right angles [b]; Therefore the

two

two angles ABC, ABD, are equal to the two angles ABC, ABE. Take ABC from both; then the angle ABE will be equal to the angle ABD, a part to the whole; which cannot be. Wherefore, &c.

Cor. Hence two right lines CBD, CBE, cannot have a common segment as CB; or BD, BE, cannot both be in a right line with CB.

PROP. XV. THEOR.

If two right lines mutually cut each other, the opposite angles are equal.

Let the right lines AB, CD, mutually cut each other in the point E, the angles AEC, DEB, will be equal; and likewise the angles CEB, AED, equal to one another. For, because the right line CE falls upon the right line AB, the angles AEC, CEB, are equal to two right angles [a]. For the same reason the angles AED, AEC, are equal to two right angles [a]: Therefore the two angles AEC, CEB, are equal to the two angles AEC, AED [b]. Take the common angle AEC from both, the remaining angles CEB, AED, are equal [c]. Again, because AEC, AED, are equal to two right angles [a], and AED, DEB, equal to two right angles, take the common angle AED from both, the remaining angles AEC, DEB, will be equal [c]. Wherefore, &c.

a 13.
b Ax. 1.
c Ax. 3.

Cor. 1. Hence, two right lines cutting each other, the angles at the section are equal to four right angles.

2. All the angles constitute about any point are equal to four right angles.

PROP. XVI. THEOR.

If one side of a triangle be produced, the outward angle will be greater than either of the inward opposite angles.

Let ABC be a triangle, and one of its sides BC be produced to D, the outward angle ACD will be greater than the angle CBA, or BAC.

For, bisect AC in E [a]; join BE, which produce to F; make EF equal to EB, and join FC; then the two sides AE, EB, are equal to the two sides FE, EC, and the angles AEB, FEC, equal [b]: Therefore the bases FC, AB, are equal; and the angles ECF,

a 10.
b 15.

Book. I. ECF, EAB, likewise equal[c]: Therefore the angle ACD is greater than the angle BAC. In like manner, if the side
c 4. BC is bisected in E, EF made equal to AE, and FC join-
d 15. ed, the angle BCG, or ACD[d], is greater than ABC; but ACD is likewise proved greater than BAC. Wherefore, &c.

PROP. XVII. THEOR.

Two angles of any triangle, however taken, are, together, less than two right angles.

Let ABC be the triangle, any two angles in it are less than two right angles.

For, produce BC both ways to D, E; then, because the out-
a 16. ward angle ACD is greater than ABC[a], add ACB to both; then the angles ACD, ACB, are greater than ABC, ACB, or,
b 13. BAC, ACB; but ACD, ACB, are equal to two right angles[b]: Therefore ABC, ACB, or ACB, BAC, are less than two right angles. For the same reason, ABE, ABC, are greater than BAC, ABC. Wherefore, &c.

Cor. Hence, if a right line fall upon two right lines, making the inward angles on the same side less than two right angles, these lines will meet one another on that side where the angles are less than right ones.

PROP. XVIII. THEOR.

The greater side of every triangle subtends the greater angle.

Let ABC be a triangle, and the side AC greater than AB; then the angle ABC will be greater than the angle ACB.

a 3. For, from the greater AC cut off AD, equal to AB[a], join
b 16. DB; then, because ADB is greater than ACB[b], ABD is likewise greater than ACB, and ABC much greater. Wherefore, &c.

PROP. XIX. THEOR.

The greater angle of every triangle is subtended by the greater side.

In

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In the triangle ABC let the angle ABC be greater than the angle BCA; the side AC will be greater than AB. If not, let AC be either equal or less than AB. If equal, then the angle ABC is equal to ACB [a]; but it is not [b]: Therefore AC is not equal to AB. If AC is less than AB, the angle ABC is less than ACB [c]; but it is not: Therefore AC is not less than AB. It is therefore greater, since it has been proved neither equal nor less. Wherefore, &c.

a 5.
b Hyp.
c 18.

PROP. XX. THEOR.

TWO sides of any triangle, however taken, are greater than the third.

In any triangle, ABC, two sides of it, however taken, are greater than the third, viz. AB, AC, greater than BC; AC, BC, greater than AB; or BC, AB, greater than AC. For, produce any side, as BA, to D; make AD equal to AC [a]; and join DC: Then, because AD is equal to AC, the angles ADC, ACD, are equal [b]; but the angle BCD is greater than ACD, that is, than ADC: Therefore the side BD is greater than BC [c]; but BD is equal to BA, AC: Therefore the sides BA, AC, are greater than BC. Wherefore, &c.

a 3.
b 5.
c 19.

PROP. XXI. THEOR.

IF two right lines be drawn from the extreme points of one side of a triangle, to a point within the same, these two right lines will be less than the sides of the triangle, but contain a greater angle.

From the extreme points of the right line BC, let the two right lines BD, CD, be drawn to the point D, within the same; these lines shall be less than the sides BA, AC; but the angle BDC will be greater than BAC.

For, produce BD to E; then the two sides BA, AE, are greater than the third side BE [a]; add EC to both; then BA, AC, are greater than BE, EC [b]. For the same reason, BE, EC, are greater than BD, DC; but BA, AC, are greater than BE, EC; therefore much greater than BD, DC. But the angle BDC is greater than BAC; for the angle BDC is greater than BEC [c]; and BEC is greater than BAC [c]: Therefore BDC is much greater than BAC. Wherefore, &c.

a 20.
b Ax. 4.
c 16.

Cor.

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Cor. Hence BD, DC, are not equal to BA, AC, each to each. Wherefore, if in any case it is thought necessary to prove that part of Prop. VII. when the one point falls within the triangle, it is evident from this.

PROP. XXII. PROB.

To make a triangle, whose sides are equal to three given right lines, if any two of them, however taken, are greater than the third.

Let A, B, C, be the three given right lines, any two of which are greater than the third. Take any right line bounded at D, but not bounded at E, from which cut off DF equal to A, FG equal to B, and make GH equal to C; then, with the center F, and distance DF, describe the circle DKL; with the center G, and distance GH, describe the circle KLH; from the point K, where the circles cut each other, draw the right lines FK, KG; then FD is equal to FK [a]; but FD is equal to A; therefore FK is equal to A. For the same reason GK is equal to C, and FG is equal to B: Therefore the three sides FK, FG, GK, of the triangle FKG, are equal to the three given right lines, A, B, C. Wherefore there is constitute, &c.

a Def. 15.

PROP. XXIII. PROB.

At a given point, in any right line, to make an angle equal to a given right lined angle.

Let A be the given point in the right line AB; it is required to make an angle equal to the right lined angle DCE.

Assume any points D, E, in the right lines CD, CE, and join DE. At the point A, in the line AB, make a triangle AFG, whose sides are equal to the three right lines CD, CE, DE [a]; then, because the two sides GA, AF, are equal to the two sides CE, CD, each to each, and the bases GF, ED, equal, the angles GAF, ECD, are equal [b]. Wherefore there is constitute, &c.

a 22.

b 8.

PROP. XXIV. THEOR.

If two triangles have two sides of the one equal to two sides of the other, each to each, and the angle contained by the two sides of the one greater than the angle contained by the correspond-

ent

ent sides of the other; then the base that subtends the greater angle of the one triangle shall be greater than the base of the other. Book I.

Let ABC, DEF, be the two triangles, having the two sides BA, AC, equal to the two sides ED, DF, each to each, but the angle BAC greater than EDF; then the base BC will be greater than EF.

For, make the angle EDG equal to BAC, and DG to AC; join EG; then the bases BC, EG, will be equal [a]. Now, 1st, if the right line EF fall upon EG, then EG will be greater than EF [b]; and therefore, BC greater than EF. a 4. b Ax. 9.

2. If EF fall above EG, then F is a point within the triangle; therefore the sides DF, FE, are less than DG, GE [c]; but DG, DF, are equal [d]; therefore EG, or BC, is greater than EF [e]. c 21. d Ax. 1. e Ax. 5.

3. If EF fall below EG, join FG; then DF, DG, are equal [d]: Therefore the angles DGF, DFG, are equal [f]; and the whole angle EFG greater than DFG, or DGF, and much greater than EGF [b]; but the greater angle is subtended by the greater side [g]: Therefore EG or BC is greater than EF. Wherefore, &c. f 5. g 19.

PROP. XXV. THEOR.

If two triangles have two sides of the one equal to two sides of the other, each to each, and the base of the one greater than the base of the other, the angle that the greater base subtends shall be greater than the other.

Let the two triangles be ABC, DEF, having the sides AB, AC, equal to the two sides DE, DF, each to each, and the base BC greater than the base EF; then the angle BAC will be greater than the angle EDF. If not, it will be equal or less. If equal, the bases BC, EF, will be equal [a]; but they are not [b]. If less, the base BC will be less than EF [c]; but it is not [b]: Therefore, since the angle BAC is neither equal nor less than EDF, it must be greater. Wherefore, &c. a 4. b Hyp. c 24.

PROP. XXVI. THEOR.

If two triangles have two angles of the one equal to two angles of the other, each to each, and a side of the one equal to a side of the other, either the side lying between the equal angles, or subtending

ing

Book I. *ing one of them, the remaining ſides of the one triangle will be equal to the remaining ſides of the other, each to each, and the remaining angle of the one equal to the remaining angle of the other.*

Let the two triangles be ABC, DEF, having the two angles ABC, ACB, of the one, equal to DEF, DFE, of the other, each to each.

1. Let the ſide BC be equal to EF, viz. the ſides lying between the equal angles; then the ſides BA, AC, will be equal to the ſides ED, DF, each to each; and the angles BAC, EDF, equal. For, if the ſide AB be not equal to DE, let one of them,
a 3. as AB, be the greater; from which cut off GB equal to DE [a], join GC; then, ſince GB, BC, are equal to DE, EF, and the angles GBC, DEF, equal, the baſes GC, DF, are equal; and
b 4. the angles GCB, DFE, equal [b]; but the angle DFE is equal
c Hyp. to ACB [c]: Therefore GCB is equal to ACB, a part to the whole; which is impoſſible: Therefore GB is not equal to DE, nor is any ſide but AB equal to DE: Therefore AB, BC, are equal to DE, EF; the angle ABC, to DEF; and the baſe AC
d 4. to DF [d].

2. Let the ſides AB, DE, which ſubtend the equal angles, be equal; if any of the ſides, as BC, be not equal to EF, let BC be the greater; cut off BH equal to EF [a]; join AH; then, becauſe AB, BH, are equal to DE, EF, and the angle ABH to DEF, the baſe AH equal to DF, and the angle AHB to DFE [b]; but the angle ACB is equal to DFE [c]: Therefore the
e Ax. 1. angle AHB is equal to ACB [e], and likewiſe greater [f]; which is
f 16. impoſſible. Wherefore, &c.

PROP. XXVII. THEOR.

IF a right line fall upon two right lines, making the alternate angles equal, theſe right lines will be parallel.

Let the right line EF fall upon the two right lines AB, CD, making the angles AEF, EFD, equal, the right lines AB, CD, will not meet one another, whether produced towards B, D, or A, C. Let them be produced; and, if poſſible, meet in the point G; then EGF is a triangle; the outward angle AEF is
a 16. greater than EGF, or EFG [a]: but AEF, EFG, are equal [b],
b Hyp. and likewiſe greater; which is impoſſible: Therefore AB, CD, will not meet, if produced toward B, D. For the ſame reaſon they will not meet, if produced toward A, C: Wherefore AB, CD, are parallel.

PROP.

PROP. XXVIII. THEOR.

If a right line fall upon two right lines, making the outward angle equal to the inward and opposite, on the same side, or the inward angles on the same side equal to two right angles; these two right lines shall be parallel.

Let the right line EF fall upon the two right lines AB, CD, making the outward angle EGB equal to the inward and opposite GHD; or the inward angles BGH, GHD, together, equal to two right angles; then AB, CD, will be parallel.

For, because the angles EGB, GHD, are equal[a], AGH is equal to EGB[b]; and therefore equal to GHD[c]: therefore AB is parallel to CD[d]. Again, because the angles BGH, GHD, are equal to two right ones[a]; but AGH, BGH, are equal to two right angles[e]; therefore AGH, BGH, are equal to BGH, GHD[c]. Take the common angle BGH from both, the remainders AGH, GHD, are equal[f]; but these are alternate angles: Therefore AB is parallel to CD[d]. Wherefore, &c.

a hyp.
b 15.
c Ax. 1.
d 27.
e 13.
f Ax. 3.

PROP. XXIX. THEOR.

If a right line fall upon two parallel lines, the alternate angles will be equal; the outward angle equal to the inward and opposite, on the same side; and the two inward angles on the same side equal to two right angles.

For, let EF fall upon the two parallel lines AB, CD, the alternate angles AGH, GHD, will be equal; the outward angle EGB equal to the inward GHD; and the two inward angles BGH, GHD, equal to two right angles.

For, if the angle AGH is not equal to GHD, let one of them be greater, as AGH; then the right lines AB, CD, produced toward B, D, will meet one another in some point[a]; but they are parallel; therefore cannot meet[b]: Therefore AGH is not greater than GHD. For the same reason it is not less; therefore it is equal. But EGB is equal to AGH[c]; therefore EGB is equal to GHD[d]. Add BGH to both; then EGB, BGH, are equal to BGH, GHD[e]; but EGB, BGH, are equal to two right angles[f]: Therefore BGH, GHD, are equal to two right angles. Wherefore, &c.

a Cor. 17.
b Def. 35.
c 15.
d Ax. 1.
e Ax. 2.
f 13.

PROP. XXX. THEOR.

Right lines parallel to one and the same right line, are parallel to one another.

Let AB, CD, be two right lines, each parallel to EF; AB will be parallel to CD.

a 29. Let GK fall upon them; then, because GK falls upon the parallels AB, EF, the angles AGH, GHF, are equal[a]. Again, because GK falls upon the parallels EF, CD, the outward angle GHF is equal to the inward and opposite GKD; therefore the angles AGK, GKD, are each equal to GHF; therefore

b Ax. 1. equal to one another[b]: Therefore AB is parallel to CD[c].

c 27. Wherefore, &c.

PROP. XXXI. PROB.

To draw a right line through a given point parallel to a given right line.

It is required, through the point A, to draw a right line parallel to the right line BC.

a 23. Assume any point D, in BC; join AD; and make the angle DAE equal to the angle ADC[a]; join EA, and produce it to F; then the alternate angles EAD, ADC, are equal to one a-

b Const. nother[b]: Therefore EF, BC, are parallel[c]. Wherefore, &c.

c 27.

PROP. XXXII. THEOR.

If one side of a triangle be produced, the outward angle is equal to both the inward opposite angles; and the three inward angles are equal to two right angles.

Let ABC be a triangle, CD a side produced; the outward angle ACD is equal to the inward and opposite angles ABC, BAC; and the three angles ACB, ABC, and BAC, are together equal to two right angles. Through C draw CE parallel to

a 29. AB; then the angle BAC is equal to ACE[a]; and the angle ECD, to ABC[a]; therefore the whole angle ACD is equal to the two angles ABC, BAC. Add the angle ACB to both; then the two angles ACD, ACB, are equal to the three angles ABC, ACB,

ACB, BAC [b]; that is, equal to two right angles [c]. Wherefore, &c.

b Ax. 1.
c 13.

Cor. 1. Hence all the three angles of any one triangle are equal to all the three angles of any other triangle, either separately or taken together.

2. If two angles of one triangle be equal to two angles of another triangle, either separately or together, the remaining angle of the one is equal to the remaining angle of the other.

3. If one angle of a triangle be a right one, the other two angles are together equal to a right angle.

4. If the angle included by the equal sides of an isosceles triangle be a right one, each of the other angles will be half a right one.

5. Any angle in an equilateral triangle is one third of two right angles, or two thirds of one right angle.

6. If one angle of a triangle be equal to the other two, that angle is a right one; for, if the side is produced, the adjacent angle is equal to the other two; therefore each of them are right angles.

7. All the inward angles of any right lined figure make twice as many right angles, abating four, as the figure has sides. For any right lined figure can be divided into triangles, the inward angles of each equal to two right angles, and all the triangles together equal to the number of sides of the figure, abating two: Therefore all the inward angles will be equal to twice the number of sides, abating four.

8. All the outward angles of any right lined figure are equal to four right angles. For all the outward and inward angles together are equal to double the number of sides; but the inward angles are equal to double the number of sides, abating four: Therefore the outward are equal to four right angles.

PROP. XXXIII. THEOR.

If two right lines join two equal and parallel right lines toward the same part, these lines will be equal and parallel.

Let AB, CD, be two equal and parallel right lines; join AC, BD; then will the right lines AC, BD, be equal and parallel.

For, because AB, CD, are parallel, and BC falls upon them, the angle ABC is equal to BCD [a]; but, because AB is equal to CD, and BC common; and the angle ABC equal to BCD, the base AC is equal to BD, and the angle ACB equal to CBD [b];

a 29.
b 4.

but

Book I. but these are alternate angles: Therefore AC is parallel to BD [a], and likewise equal. Wherefore, &c.

a. 29.

PROP. XXXIV. THEOR.

The opposite sides and opposite angles of every parallelogram are equal; and the diameter divides it into two equal parts.

Let ABCD be a parallelogram, the opposite sides AB, CD; AC, BD, are equal; the angle CAB equal to BDC, and ACD to ABD; and the diameter BC bisects it.

a 29. For, because AB is parallel to CD, and BC falls upon them, the angle ABC is equal to BCD [a]. For the same reason ACB is equal to CBD; therefore the two angles ABC, ACB, in the triangle ABC, are equal to the two angles CBD, BCD, in the triangle BCD; and the side BC common to both: Therefore the two sides AC, AB, of the one triangle, are equal to the two sides BD, DC, of the other, each to each; and the angle

b 26. BAC equal to BDC, and ACD to ABD [b]. Again, because the two sides AC, AB, are equal to the two sides BD, DC, each to each, and the angle CAB equal to BDC, the base BC

c 4. common: Therefore the triangles are equal [c]; and BC bisects the parallelogram. Wherefore, &c.

PROP. XXXV. and XXXVII. THEOR.

Parallelograms and triangles, constitute upon the same base, and between the same parallels, are equal between themselves, viz. parallelogram to parallelogram, and triangle to triangle.

Let ABCD, EBCF, be two parallelograms (Fig. 2.) constitute upon the same base BC, and between the same parallels BC, AF; the parallelograms ABCD, EBCF, are equal.

a 34. For, because AD, EF, are each equal to BC [a], they are equal

b Ax. 1. to one another [b]. If the point E coincide with D. (Fig. 1.) each of the parallelograms are double the triangle DBC; therefore e-

c Ax. 6. qual to one another [c]. If AD is less than AE, add DE to

d Ax. 2. both; then the whole AE is equal to DF [d], DC to AB [e], and the

e 34. angle FDC to EAB [f]: Therefore the triangles FDC, EAB,

f 29. are equal [g]. Take DGE from both; the trapeziums, ADGB,

g 4.

h Ax. 3. FEGC, are equal [h]. Add the triangle GBC to both; then the whole parallelogram ABCD is equal to the parallelogram EBCF.

EBCF[d]. If AD is greater than AE, take DE from both; then the remainder AE will be equal to DF[h], the triangle AEB to DFC. Add EBCD to both, then the parallelograms ABCD, EBCF, are equal[d]: So, likewise, if the diameters AC, BF, be drawn, then the triangle ABC will be equal to FBC[i]. Wherefore, &c.

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d Ax. 2.
h Ax. 3.
i 34. and Ax. 7.

Cor. Hence every parallelogram is equal to a right angled parallelogram, constitute upon the same base, and between the same parallels; and every triangle constitute upon the same base, and betwixt the same parallels, is half the rectangle.

PROP. XXXVI. and XXXVIII. THEOR.

Parallelograms and triangles, constitute upon equal bases, and between the same parallels, are equal to one another, viz. parallelogram to parallelogram, and triangle to triangle.

Let the parallelograms ABCD, EFGH, be constitute upon the equal bases BC, FG, and between the same parallels AH, BG; the parallelogram ABCD will be equal to EFGH.

For, join EB, CH, the parallelograms AC, EG *, are each equal to the parallelogram EC[a]; therefore equal to one another[b]. Join AC, FH; then the triangles ABC, HGF, are equal[c]. Wherefore, &c.

a 35.
b Ax. 1.
c 34. and Ax. 7.

PROP. XXXIX. THEOR.

EQUAL triangles, constitute upon the same base, on the same side, are between the same parallels.

Let the equal triangles ABC, DBC, be constitute upon the same base, BC, on the same side; the right line AD, that joins their vertex, will be parallel to BC.

If not, draw AE, parallel to BC; join EC; then the triangles ABC, EBC, are equal[a]; but DBC is equal to ABC[b]; therefore DBC, EBC, are equal, a part to the whole; which is impossible. Therefore no line but AD is parallel to BC. Wherefore, &c.

a 35.
b Hyp.

PROP.

* Parallelograms are expressed by the letters at the opposite angles.

PROP. XL. THEOR.

EQUAL triangles, constitute upon equal bases, on the same side, are between the same parallels.

Let ABC, DGE, be equal triangles, constitute upon the equal bases BC, GE, on the same side; then AD is parallel to BE. If not, draw AF parallel to BE; join FE; then the
a 36. triangle ABC is equal to FGE [a]; but DGE is equal to ABC [b];
b hyp. therefore DGE is equal to FGE, a part to the whole; which is impossible: Therefore AD is parallel to BE. Wherefore, &c.

PROP. XLI. THEOR.

IF a parallelogram and triangle be constitute upon the same base, and between the same parallels, the parallelogram will be double the triangle.

Let the parallelogram be ABCD, and triangle EBC, having the same base BC, and be between the same parallels AE, BC, the parallelogram ABCD is double the triangle EBC; join AC.

a 35. Then the triangle ABC is equal to EBC [a]; but the paralle-
b 34. logram ABCD is double the triangle ABC [b]; and therefore double EBC. Wherefore, &c.

PROP. XLII. PROB.

TO constitute a parallelogram equal to a given triangle, having an angle in it equal to a given right lined angle.

Let the given triangle be ABC, and right lined angle D, it is required to constitute a parallelogram equal to the given triangle ABC, having an angle in it equal to D.

a 10. Bisect BC in E [a]; make an angle CEF equal to D [b]; through
b 23. A draw AG parallel to CE [c]; and through C draw CG parallel
c 31. to EF [c]; join AE; then the triangles ABE, AEC, are equal [d];
d 36. and ABC is double AEC; but the parallelogram EG is double
e 41. the triangle EAC [e]: Therefore the parallelogram EG is e-
f Ax. 6. qual to the triangle ABC [f], and the angle FEC equal to D. Wherefore, &c.

PROP. XLIII. THEOR.

IN every parallelogram, the complements that stand about the diameter are equal to one another.

Let ABCD be a parallelogram; BD its diameter; the parts of which BK, KD, the diameters of the parallelograms HKFD, EBGK; the remaining parallelograms AEKH, KGCF, its complements, are equal to one another.

For, because DB is the diameter of the parallelogram ABCD, the triangles ADB, DBC, are equal [a]. For the same reason, the triangle HKD is equal to DFK, and EBK to BKG; wherefore the triangles HKD and EKB are equal to DFK, BKG, [b]. Take HKD, EKB, from ADB, and DFK, BKG, from DBC, there remains AEKH equal to KGCF [c]. Wherefore, &c.

a 34.
b Ax. 2.
c Ax 3.

PROP. XLIV. PROB.

TO apply a parallelogram to a given right line equal to a given triangle, having an angle in it equal to a given right lined angle.

It is required, upon the given right line AB, to make a parallelogram equal to a given triangle C, having an angle in it equal to a given angle D.

Make the parallelogram FGBE equal to the triangle C, having the angle EBG equal to D [a]; put BE in a right line with AB; and produce FG to H; through A draw AH parallel to GB, or FE; join HB: Now, because the angles EFH, FHA, are equal to two right angles [b], the angles EFH, FHB, are less than two right angles; then FE, HB, being produced, will meet in some point [c]; which let be K; through which draw KL parallel to FH; and produce GB, HA, to M, L; wherefore FHLK is a parallelogram, whose diameter is HK; and whose complements FGBE, BALM, are equal [d]; but FGBE, was made equal to C; and the angle EBG equal to D; therefore BALM is equal to C, and the angle ABM [e] equal to D. Wherefore, &c.

a 42.
b 29.
c 17. Cor.
d 43.
e 15.

PROP. XLV. PROB.

TO make a parallelogram equal to a given right lined figure, having an angle in it equal to a given right lined angle.

Book I. It is required to make a parallolegram equal to a given right lined figure ABCD, having an angle in it equal to the right lined angle E: Join DB, and make the parallelogram FH e-
a 42. qual to the triangle ABD[a]; the angle FKH equal to E. Upon the right line GH make GM equal to DCB, and the angle
b 44. GHM equal to E[b]; then FM is the parallelogram equal to ABCD.

For, becaufe FH is a parallelogram, the angles FKH, GHK,
c 29. are equal to two right angles[c]; but the angles GHM, FKH, are each equal to the angle E; therefore equal to one another. Add GHK to both, then the angles GHM, GHK, are equal to FKH, GHK, that is, equal to two right angles; therefore
d 14. KHM is a right line[d]. For the fame reafon FGL is a right line[d]; but FK, LM, are each parallel to GH; therefore pa-
e 30. and conftruct. rallel to one another[e]. Wherefore FM is a parallelogram equal to the right lined figure ABCD, and an angle FKM equal to E. Wherefore, &c.

Cor. Hence a parallelogram may be made equal to a given right lined figure of any number of fides; for a parallelogram can be made equal to any triangle upon any given right line.

PROP. XLVI. PROB.

To defcribe a fquare upon a given right line.

It is required to defcribe a fquare upon the given right line AB.

From the point A, in the given right line AB, draw the per-
a 11. pendicular AC[a]; cut off AD equal to AB[b]; through D draw
b 3. c 31. DE parallel to AB[c], and BE parallel to AD; then ADEB is a
d 34. parallelogram, the oppofite fides of which are equal[d]; that is,
e by conftr. DE equal to AB, and BE to AD; but AD is equal to AB[e]; therefore the four fides are equal to one another. But the
f 29. angles ADE, BAD, are equal to two right angles[c]; and BAD is a right angle; therefore ADE is likewife a right angle; but the oppofite angles of every parallelogram are equal[d]:
g Def. 30. Therefore ADEB is a fquare[g]. Wherefore, &c.

PROP. XLVII. THEOR.

In every right angled triangle the fquare defcribed upon the fide fubtending the right angle is equal to the fquares of the fides containing the right angle.

Let

Let ABC be a right angled triangle, the ſquare of the ſide BC ſubtending the right angle is equal to the ſquares of the ſides BA, AC, containing the right angle. Upon BC deſcribe the ſquare BDEC [a]; upon BA, AC, the ſquares BG, AK; through A draw AL parallel to BD, or EC [b]; join AD, FC, BK, AE.

a 46.
b 31.

Then, becauſe BAC, BAG, are each right angles [a], GAC is a right line [c]. For the ſame reaſon BAH is a right line; likewiſe the angles DBC, ABF, are right angles [a]; add ABC to both, then the whole angle FBC is equal to ABD [d]; and AB, BD, are equal to FB, BC; and the angle FBC to ABD; therefore the triangles ABD, FBC, are equal [e]; but the parallelogram BL is double the triangle ABD [f]; and BG is double FBC [f], or ABD; therefore the parallelograms GB, BL, are equal [g]. For the ſame reaſon LC is equal to CH; but BL, LC, are equal to the ſquare of BC; therefore the ſquares of BA, AC, are equal to the ſquare of BC. Wherefore, &c.

c 14.
d Ax. 2.
e 4.
f 41.
g Ax. 6.

PROP. XLVIII. THEOR.

If the ſquare deſcribed upon one of the ſides of a triangle be equal to the ſquares of the other two ſides, the angle contained by theſe two ſides is a right angle.

Let the ſquare of the ſide BC of the triangle ABC be equal to the ſquares of the ſides BA, AC, the angle BAC is a right angle.

For, let AD be drawn from the point A, at right angles [a], to AC, and equal to AB; join DC. Then, becauſe the angle DAC is a right one, the ſquare of DC is equal to the ſquares of DA, AC [b]. But DA is equal to AB, and AC is common; therefore the ſquares of DA, AC, are equal to the ſquares of BA, AC; but the ſquare of BC is equal to the ſquares of BA, AC [c], or of DA, AC: Therefore the ſquare of BC is equal to the ſquare of DC [d]; therefore BC is equal to DC; but BA is equal to AD, and AC common; therefore BA, AC, are equal to DA, AC, and the baſes BC, DC, equal; therefore the angle BAC is equal to the angle DAC [e]. But DAC is a right angle; therefore BAC is a right angle. Wherefore, &c.

a 11.
b 47.
c Conſt.
d Ax. 1.
e 8.

THE

ELEMENTS

OF

EUCLID.

BOOK II.

DEFINITIONS.

I.

Book II. EVERY right angled parallelogram is said to be contained by two right lines containing the right angle.

II.

In every parallelogram, either of the two parallelograms that are about the diameter, together with the complement, is called a gnomon.

PROP. I. THEOR.

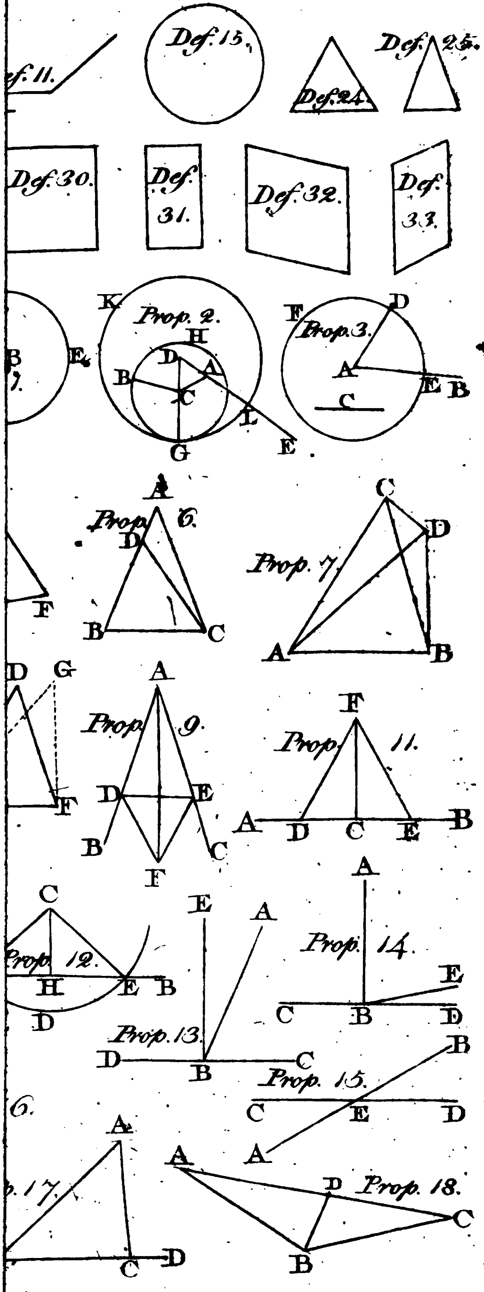
If there be two right lines, and one of them divided into any number of parts, the rectangle contained by the whole, and divided line, is equal to all the rectangles contained by the whole line, and the several parts of the divided line.

Let A, BC, be the two right lines, one of which, viz. BC, is divided into any number of parts, as D, E; the rectangle contained by A, BC, is equal to the rectangles contained by A, BD; A, DE; A, EC.

a 11. 1. For, from the point B draw BF, at right angles, to BC[a];
b 3. 1. make BG equal to A[b]; through G draw GH parallel to BC; and through the points D, E, C, draw DK, EL, CH, each pa-
c 31. 1. rallel to BG[c].
d Const. The rectangle BK is that contained by BD, BG; for BG[d] is equal to A; the rectangle DL is contained by A, DE; and

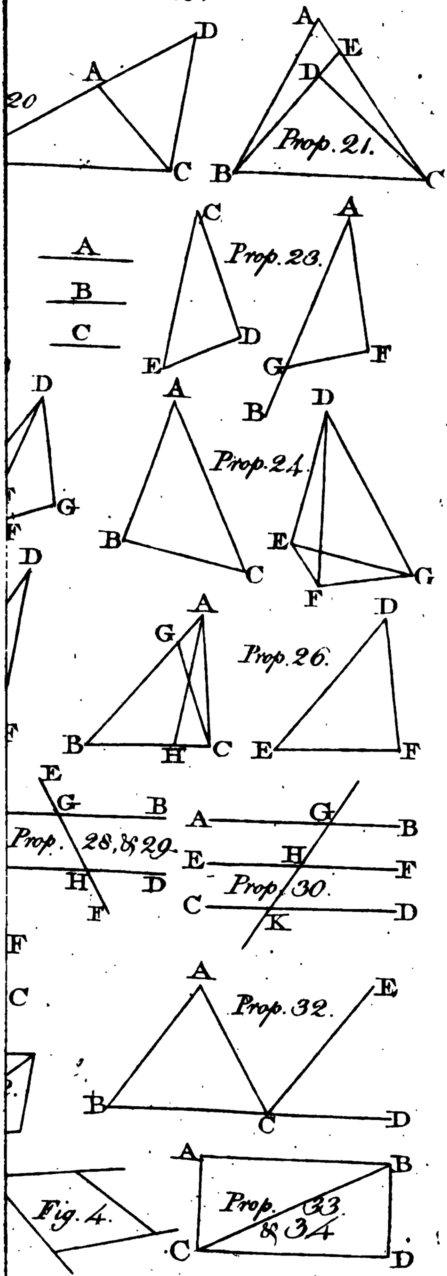
EH

ook I. *Plate 1.*



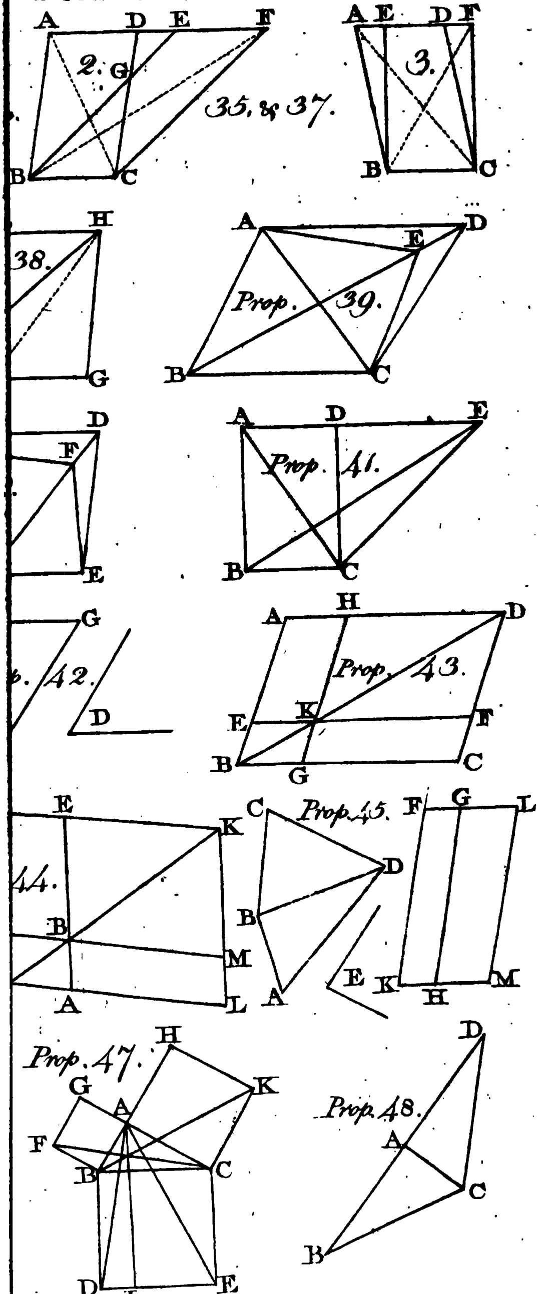
ef. 11.
Def. 15.
Def. 24.
Def. 25.
Def. 30.
Def. 31.
Def. 32.
Def. 33.
Prop. 2.
Prop. 3.
Prop. 6.
Prop. 7.
Prop. 9.
Prop. 11.
Prop. 12.
Prop. 13.
Prop. 14.
Prop. 15.
6.
. 17.
Prop. 18.

ok I. *Plate* 2.



20
Prop. 21.
Prop. 23.
Prop. 24.
Prop. 26.
Prop. 28, & 29.
Prop. 30.
Prop. 32.
Fig. 4.
Prop. 33 & 34

Book I. *Plate* 3.



EH by A, EC; for DK, EL, CH, are each equal to BG[c], that is, equal to A; but the rectangle BH is equal to the rectangles BK, DL, EH; and BH is contained by A, BC; therefore the rectangle by A, and BC, is equal to the rectangles by A, BD; A, DE, and A, EC. Wherefore, &c.

c 34. 1.

PROP. II. THEOR.

IF a right line be any how cut, the rectangles contained by the whole line, and each of the segments, are equal to the square of the whole line.

Let the right line AB be any how cut in C, the rectangles contained by AB, BC, and AB, AC, together, are equal to the square of AB. Upon AB describe the square ADEB[a]; thro' C draw CF parallel to AD[b], or BE; then, because AD is equal to AB[c], the rectangle under AD, AC, is equal to the rectangle under AB, AC; and the rectangle under EB, BC, is equal to the rectangle under AB, BC; but the rectangle under AD, AC, that is, the rectangle AF, together with the rectangle under EB, BC, that is, CE, are equal to the square of AB; that is, the square AE. Wherefore, &c.

a 46. 1.
b 31. 1.

PROP. III. THEOR.

IF a right line be any how cut, the rectangle under the whole line, and one of the parts, is equal to the rectangle under the two parts, together with the square of the first mentioned part.

Let the right line AB be any how cut in C, the rectangle under AB, BC, is equal to the rectangle under AC, CB, together with the square of BC. Upon BC describe the square BCDE[a]; produce ED to F, and draw AF parallel to CD or BE; the rectangle under AB, BC, that is, AE, is equal to the rectangles AC, CD; that is, the rectangle under AC, CB, and the square of CB. Wherefore, &c.

a 46. 1.

PROP. IV. THEOR.

IF a right line be any how cut, the square of the whole line is equal to the squares of the two parts, with twice the rectangle under these parts.

Let

Book II. Let the right line AB be any how cut in C, the square of AB is equal to the squares of AC, CB, and twice the rectangle under AC, CB.

For, upon AB describe the square ADEB; through C draw CF parallel to AD, or BE; draw DB, cutting CF in G; through which point draw HGK parallel to AB, or DE. (Then the figure is said to be constructed.) Now, because AB is equal
a 5. 1. to AD the angle ADB is equal to ABD[a]; and the angle CGB
b 29 1. to ADB[b]; therefore the angle CBG is equal to CGB[c]; there-
c Ax. 1. 1. fore CG is equal to CB[d]; therefore CGKB is equilateral[e];
d 6. 1. but the angles BCG, CBK, are equal to two right angles[b], and
e 34. 1.
f 46. 1. CBK is a right angle[f]; therefore BCG is likewise a right angle; therefore all the angles are right ones[e]; and CGKB is
g Def. 30. 1. a square[g]. For the same reason HF is a square. But the rec-
h 43. 1. tangles AG, GE, are equal[h]; and AG is the rectangle under AC, CB; for CG is equal to CB; but the squares HF, CK, with the rectangles AG, GE, make up the square of AB: Therefore the square of AB is equal to the squares of AC, CB, and twice the rectangle under AC, CB. Wherefore, &c.

Cor. Hence every parallelogram about the diameter of a square is a square.

PROP. V. THEOR.

If a right line be cut into two equal parts, and into two unequal parts, the rectangle under the two unequal parts, together with the square of the intermediate part, are equal to the square of half the line.

Let the right line AB be cut equally in C, and unequally in D, the rectangle under AD, DB, together with the square of CD, are equal to the square of CB.

For, upon CB describe the square CEFB; construct the figure, and produce OL to K; and through A draw AK parallel to EC.

a 36. 1. The parallelograms AL, CO, are equal[a], and CH is equal
b 43. 1. to HF[b]; add DO to both; then CO is equal to DF[c]; therefore
c Ax. 2. 1.
d Ax. 1. 1. AL is likewise equal to DF[d]; add CH to both; then the rectangle AH is equal to the gnomon LDF; add LG, that is, the
Cor. 4. square of CD[e], to both; then the rectangle AH, that is, the rectangle under AD, DB, with LG, that is, the square of CD, are equal to the gnomon LDF, and LG; that is, equal the square of CB. Wherefore, &c.

PROP.

PROP. VI. THEOR.

If a right line be divided into equal parts, and another line added to it, the rectangle contained by the whole and added line as one side of the rectangle, and the added line for the other side, together with the square of half the line, are equal to the square of the half and added line, as one side of the square.

Let the right line AB be bisected in C, and BD added to it, the rectangle under AD, DB, together with the square of BC, are equal to the square of CD.

Describe the square CEFD; construct the figure; and compleat the parallelogram under AC, CL.

Then the parallelograms AL, CH, are equal [a]; but CH is equal to HF [b]; add BM to both; then CM is equal to BF; add AL to both; then AM is equal to the gnomon CMG. To each add LG, that is, the square of CB [c]; then AM, LG, are equal to the gnomon CMG, and LG; that is, the rectangle under AD, DB, for DM is equal to DB, together with the square CB, are equal to the square of CD. Wherefore, &c.

a 36. 1.
b 43. 1.
c Cor. 4.

PROP. VII. THEOR.

If a right line be any how cut, the square of the whole line, and one of the parts, is equal to twice the rectangle contained by the whole line, and said part, together with the square of the other part.

Let the right line AB be any how cut in C, the squares of AB, BC, are equal to twice the rectangle under AB, BC, and the square of AC.

Upon AB describe the square ADEB [a], and construct the figure; then, because the rectangle AF is equal to CE [b], and AF, CE, together, are equal to twice AF, that is, equal to the gnomon AFK, together with the square of CB, that is, CF [c]; add HK to both; then twice AF, and HK, are equal to the gnomon AFK, and the squares of AC, BC; that is, to the squares of AB, BC. Wherefore, &c.

a 46. 1.
b 43. 1. and Ax. 2. 1.
c Cor. 4.

PROP.

Book II.

PROP. VIII. THEOR.

If a right line be cut into two parts, four times the rectangle under the whole line, and one of the parts, together with the square of the other part, are equal to the square of the whole line, and the first part taken as the side of the square.

Let the right line AB be any how cut in C, four times the rectangle under AB, BC, together with the square of AC, are equal to the square of AD; that is, AB produced to D, so that BD equal BC. Upon AD describe the square AEFD, and construct the double figure. Then, because BN, GR, are squares[a], and CK, BN, are equal parallelograms[b]; but the sides CB, BK, are equal, and CBK is a right angle, for it is equal to BDN[c]; therefore CK is a square[d]. For the same reason, KO is a square; therefore CK, BN, GR, KO, are each squares; but they are constitute upon equal right lines; therefore equal to one another, and, together, quadruple KC. But the rectangle AG is equal to MP[b], and PL to RF; but MP is equal to PL[f]; therefore the four rectangles are quadruple AG; and the four squares and four rectangles quadruple the rectangle AK, that is, the rectangle under AB, BC; add the square XH, that is, the square of AC; then four times AK, that is, four times the rectangle under AB, BC, together with the square of AC, are equal to the square of AD. Wherefore, &c.

a Cor. 4. b 36. 1. c 29. 1. d 34. 1. and def. 30. 1. f 43. 1.

PROP. IX. THEOR.

If a right line be cut into two equal parts, and into two unequal parts, the squares of the two unequal parts are double the square of the half line, and double the square of the intermediate part.

Let the right line AB be cut equally in C, and unequally in D, the squares of AD, BD, are double the squares of AC, CD.

For, through C draw CE, at right angles, to AB, and equal to AC, or CB; join EA, EB; through D draw DF parallel to to CE, and FG through F, parallel to AB; join AF.

Then, because AC is equal to CE, and the angle ACE a right angle, the angles AEC, EAC are each half right angles[a], and the squares of AC, CE double the square of AC; but the square of AE is equal to the squares of AC, CE[b]; therefore, double

a Cor. 32. 1. b 47. 1.

double the square of AC. For the same reason, the angles CEB, Book II.
EBC are each half right angles; but the angle EGF is a right angle[c]; therefore GFE is half a right angle[a]; therefore the sides c 29. 1.
EG, GF are equal[d]; but the square of EF is equal to the squares d 6. 1.
of EG, GF, or double the square of GF or CD[e]; but the squares e 34. 1.
of AE, EF are equal to the square of AF, for the angle AEF is a right angle; but the squares of AE, EF are double the squares of AC, CD; therefore the square of AF is double the squares of AC, CD; but the angle DFB is half a right angle; for it is equal to CEB[c]; therefore, DFB, DBF are each half right angles; therefore FD, DB are equal; but the square of AF is equal to the squares of AD, DF[b], or DB; therefore, the squares of AD, DB are double the squares of AC, CD. Wherefore, &c.

PROP. X. THEOR.

If a right line be cut into two equal parts, and another right line added to it, the square of the whole and added line taken as one line, and the square of the added line, are double the square of the half line, and double the square of the half and added line, taken as one line.

Let the right line AB be bisected in C, and BD added to it, the squares of AD, DB are double the squares of AC, CD.

For, from the point C, draw CE perpendicular to AB, and equal to AC or CB; join AE, EB; through E, draw EF parallel to AD; and through D, draw DF parallel to CE.

Because AC is equal to CE, and the angle ACE a right angle, each of the angles AEC, EAC are half right angles; and therefore the square of AE is equal to the squares of AC, CE,
or double the square of AC[a]. For the same reason, CEB, CBE a 47. 1.
are each half right angles; therefore AEB is a right angle; but
the angles FEC, ECD are equal to two right angles[b], and ECD b 29. 1.
is a right angle; therefore CEF is likewise a right angle; there-
fore CF is a rectangle[c]; therefore, the angles DFE, FEC are equal c Def. 1.
to two right angles[b]; therefore, DEF, FEB are less than two right
angles[d]; therefore, FD, EB will meet one another, which let be d. Cor. 17.
in G. But CEF is a right angle, and CEB half a right angle; there- 1.
fore, FEB is half a right angle, and EGF is likewise half a right an-
gle; therefore, EF is equal to FG[e], and the square of EG equal e 6. 1.
to the squares of EF, FG[a], or double the square of EF, or CD; therefore, the squares of AE, EG are double the squares of AC, CD; but the square of AG is equal to the squares of AE, EG, and

Book II. and likewise equal to the squares of AD, DG; for the angles AEG, and ADG are each right ones; therefore, the squares of AD, DG, or DB its equal, are double the squares of AD, CD. Wherefore, &c.

PROP. XI. PROB.

To cut a given right line so, that the rectangle contained under the whole line, and one of the parts, be equal to the square of the other part.

a 46. 1. Upon any given right line, as AB, describe the square ABDC [a]; bisect AC in E; join EB, and produce EA to F; make EF equal to EB; upon AF, describe the square FGHA, and produce GH to K; then AB is so cut in the point H, that the rectangle under AB, BH is equal to the square of AH.

b 6. c Const. d 47. 1. For the rectangle under CF, FA, together with the square of AE, is equal to the square of EF [b]; but EF is equal to EB [c], and the square of EB is equal to the squares of BA, AE [d]; therefore, the rectangle under CF, FA, together with the square of AE, are equal to the squares of BA, AE. Take the square of AE from both, there remains the rectangle under CF, FA, that is, the rectangle under CF, FG, that is, FK, equal to the square of AD. Take AK from both, there remains FH equal to HD; but FH is the square of AH, and HD the rectangle under AB, BH, for BD is equal to AB. Wherefore, &c.

PROP. XII. THEOR.

In every obtuse angled triangle, the square of the side subtending the obtuse angle, is greater than the squares of the sides containing the obtuse angle, by twice a rectangle under one of the sides containing the obtuse angle, and that part of the side produced, lying betwixt the obtuse angle, and perpendicular let fall from the opposite angle.

a 12. 1. Let BAC be the obtuse angle of the triangle ABC; produce the side CA till it meet the peopendicular BD, let fall from the point B [a]. The square of BC is greater than the squares of BA, AC, by twice the rectangle under CA, AD.

b 47. 1. For the square of BC is equal to the squares of BD, DC [b]; but the square of DC is equal to the squares of DA, AC, and twice

twice the rectangle under AD, AC [c]; but the square of AB is equal to the squares of BD, DA [b]; therefore the square of BC is equal to the squares of BA, AC, and twice the rectangle under BA, AC; therefore the square of BC is greater than the squares of BA, AC, by twice the rectangle under DA, AC. Wherefore, &c. Book II. c 4. b 47. 1.

PROP. XIII. THEOR.

In every acute angled triangle, the square of the side subtending the acute angle, is less than the squares of the side containing the acute angle, by twice a rectangle contained under one of the sides about the acute angle, and that part of the side lying between the acute angle and the perpendicular let fall from the opposite angle.

Let B be an acute angle in the triangle ABC; from the angle A let fall the perpendicular AD [a], cutting BC in D; the square of AC is less than the squares of AB, BC, by twice the rectangle under CB, BD. a 12. 1.

For the square of AC is equal to the squares of AD, DC [b]; and the square of AB is equal to the squares of AD, DB [b]; but the squares of BC, BD, are equal to twice the rectangle under BC, BD, together with the square of DC [c]; therefore the squares of AB, BC, are equal to the squares of AD, DC, and twice the rectangle under CB, BD; but the square of AC is equal to the squares of AD, DC; therefore the square of AC is less than the squares of AB, BC, by twice the rectangle under CB, BD. Therefore, &c. b 47. 1. c 7.

PROP. XIV. PROB.

To make a square equal to a given right lined figure.

Make the rectangle BCDE equal to a given right lined figure A [a]; If BE be equal to ED, then BCDE is a square; and what was required is done. If not, produce BE to F; make EF equal to ED, and bisect BF in G [b]; with the center G, and distance GB, describe a semicircle BHF; produce DE to H, and join GH. a 45. 1. b 10. 1.

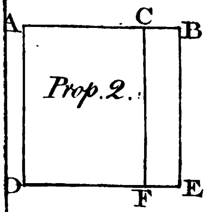
Then

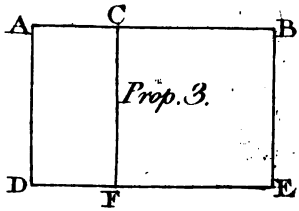
Book II. Then the rectangle under BE, EF, together with the square of GE, are equal to the square of GF[c], or GH[d]; but the square of GH is equal to the squares of GE, EH[e]; therefore the rectangle under BE, EF, together with the square of GE, are equal to the squares of HE, EG. Take the square of GE from both, and the rectangle under BE, EF, that is, BD, is equal to the square of EH. Wherefore, &c.

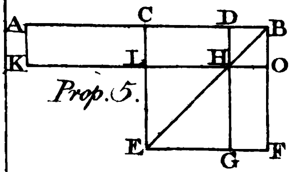
c 5.
d Def. 15. 1.
e 47. 1.

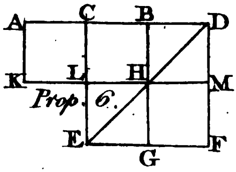
THE

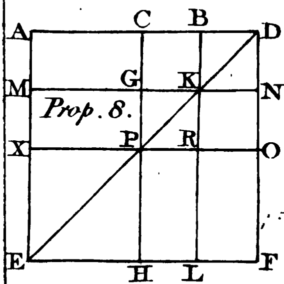
Book. II.

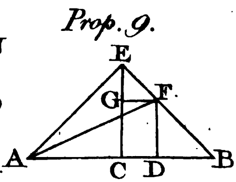

A C B
Prop. 2.
D F E

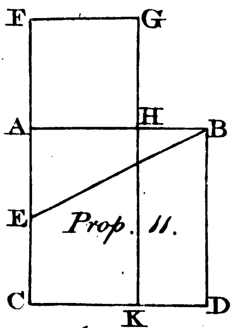

A C B
Prop. 3.
D F E

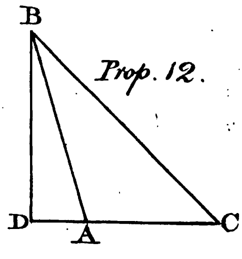

A C D B
K L H O
Prop. 5.
E G F

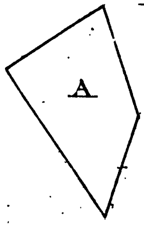

A C B D
K L H M
Prop. 6.
E G F

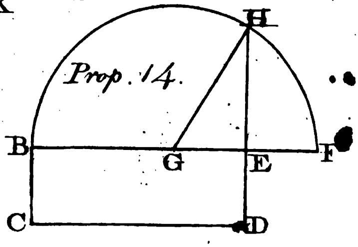

A C B D
M G K N
Prop. 8.
X P R O
E H L F


Prop. 9.
E
G F
A C D B


F G
A H B
E
Prop. 11.
C K D


B
Prop. 12.
D A C


A


H
Prop. 14.
B G E F
C D

THE

ELEMENTS

OF

EUCLID.

BOOK III.

DEFINITIONS.

I.

EQUAL circles are ſuch whoſe diameters are equal.

II.

A right line is ſaid to touch a circle, when drawn to the ſame, and being produced, does not cut the circle.

III.

Circles are ſaid to touch each other, which, meeting, do not cut one another.

IV.

Right lines in a circle are ſaid to be equally diſtant from the center, when perpendiculars drawn from the center to each of them are equal, and that line upon which the greateſt perpendicular falls is the leaſt line.

V.

Definition 19th, 1.

VI.

An angle of a ſegment is the angle contained by the right line and circumference of the circle.

VII.

An angle is ſaid to be in a ſegment, when right lines are drawn from ſome point in the circumference to the ends of that line which is the baſe of the ſegment, which lines contain the angle.

VIII. But,

Book III.

VIII.

But, when the right lines containing the angle do receive any part of the circumference, then the angle is said to stand upon that circumference.

IX.

A sector of a circle is that figure which is contained by two right lines, drawn from the center, and the circumference between them.

X.

Similar segments of circles are those which include equal angles or whereof the angles in them are equal.

PROP. I. PROB.

To find the centre of a circle.

Required to find the centre of the given circle ABC.

a 10. 1. b 11. 1. Draw in it the line AB, which bisect in D[a], by the right line CD at right angles to AB[b], and produce CD to E; bisect EC in F; which point is the centre of the circle ABC.

c def. 15. 1. d 8. 1. e Ax. 9. 1. If not, let G be the centre; join GB, GA, GD; then, because AD is equal to DB, DG is common, the base AG is equal to GB[c], and the angle ADG to GDB[d]; therefore, each of them is a right angle; but FDB is a right angle; therefore, GDB is equal to FDB, a part to the whole, which is impossible[e]; therefore, no point but F can be the centre. Wherefore, &c.

Cor. Hence, if, in a circle, any right line cut another right line into two equal parts, the centre of the circle will be in that line which cuts the other into two equal parts.

PROP. II. THEOR.

If any two points be assumed in the circumference of a circle, the right line joining these points will fall within the circle.

a 1. b def. 15. 1. c Ax. 9. 1. Let the circle be ABC, A and B the points in its circumference, the right line AB, joining these points, will fall within the circle. Find D the centre of the circle[a]; join DA, DB, and draw DF, cutting the right line AB in the point E; then the right lines DA, DB, DF, are equal[b]. But DF is greater than DE[c]; therefore, DA, DB, are likewise greater than DE; but DB, DA, reach the circumference; therefore DE does not reach

reach the circumference; therefore the right line AB is within the circle. Wherefore, &c.

Cor. Hence, if a right line touches a circle, it will touch it only in one point.

PROP. III. THEOR.

If, in a circle, a right line be drawn through the centre, cutting another line not drawn through the centre, into two equal parts, it shall cut it at right angles; and if it cut it at right angles, it shall cut it into two equal parts.

Let ABC be the given circle, CD the line passing through the centre, cutting the right line AB, not passing through the centre, into two equal parts, it will cut it at right angles; and if the angles AFE, EFB, be right angles, AF will be equal to FB.

For, find the centre E[a]; join EA, EB; for, because AF is equal to FB, and FE is common, the two sides AF, FE, are equal to BF, FE, and the base AE equal to EB[b]; therefore, the angle AFE is equal to EFB[c], and each is a right angle[d]; therefore, CD cuts the right line AB at right angles. If EFA, EFB, are right angles, AF is equal to FB; for, because AE is equal to EB, and EF common, the two sides AE, EF, are equal to BE, EF, and AFE, EFB, right angles[e], and the angle EAB equal to EBA[f]; therefore, the remaining angle AEF is equal to FEB[g]; therefore the base AF is equal to FB[h]. Wherefore, &c.

a 1. 1.
b def. 15. 1.
c 8. 1.
d Def. 10. 1.
e Hyp.
f 5. 1.
g Cor. 32. 1.
h 4. 1.

PROP. IV. THEOR.

If two right lines are drawn in a circle, neither of them passing through the centre, they will not mutually bisect each other.

Let two right lines AC, BD, not passing through the center, be drawn in the circle ABCD, cutting each other in the point E, they will not mutually bisect each other; for, if possible, let AE be equal to EC, and BE to ED; find the center F, and join FE; then, because FE, passing thro' the center, cuts the right line AC, not passing through the center, into equal parts, the angles FEA, FEC, are right angles[a]; and, because FE bisects BD, FEB is a right angle[a]; therefore, FEA is equal to FEB, a part to the whole, which is impossible; therefore, AC, BD, do not mutually bisect each other. Wherefore, &c.

a 3.

PROP.

Book III.

PROP. V. THEOR.

If two circles cut each other, they cannot have the same center.

Let the two circles ABC, CDG, cut each other, they cannot have the same center.

If possible, let E be the center of both, draw CE to the point of section C, and EFG through any other point; then, because E is the center of the circle ABC, EC is equal to EF[a]; and, because E is the center of the circle CDG, EC is equal to EG; therefore, EF is equal to EG[b], a part to the whole, which is impossible; wherefore E is not the center of both. Wherefore, &c.

a Def. 15. 1.
b Ax. 11.

PROP. VI. THEOR.

If two circles touch each other inwardly, they have not the same center.

Let the two circles ABC, CDE, touch each other inwardly in the point C, they have not the same center. If possible, let it be F; join FC, and draw FB through any other point.

Then, because F is the center of the circle ABC, CF is equal to FB[a]; for the same reason CF is equal to FE; therefore FE is equal to FB[b], that is, a part to the whole, which is impossible. Wherefore, &c.

a def. 15. 1.
b ax. 1.

PROP. VII. THEOR.

If some point is taken in the diameter of a circle, which is not the center, from that point if several right lines are drawn to the circumference, the greatest of these right lines is that part of the diameter in which the center is, and the remainder of the diameter is the least; of the other lines, the nearest to that passing through the center is greater than that more remote; and, on each side of the diameter, only two right lines can be drawn from that point to the circumference equal to one another.

Let F be a point in the diameter of the circle ABCD, which is not the center, and from it be drawn FA thro' the center, and FB, FC, FG, any how to the circumference, FA is the greatest line, FA is greater than FB, FB greater than FC, FC great-

er

er than FG, and FD the least. And from the point F only two right lines can be drawn equal to one another on each side of diameter; for, find the center E; join BE, CE, and GE. Then, because E is the center of the circle, EA is equal to EB; add EF to both; then AF is equal to BE, EF; but BE, EF, are greater than BF[a]; therefore AF is greater than BF; but BE is equal to CE, and EF common; therefore BE, EF, are equal to CE, EF; but the angle BEF is greater than CEF[b]; therefore, the base BF is greater than CF[c]. For the same reason CF is greater than GF; likewise the two sides GF, FE, of the triangle GEF, are greater than GE, that is, than ED; take EF from both, there remains GF greater than FD[d]; therefore, AF is the greatest right line, and FD the least.

a 20. 1.
b Ax. 9.
c 24. 1.
d Ax. 5. 1.

Lastly, on each side of the diameter, from the point F, only two right lines can be drawn equal to one another; for, at the point E, with the right line EF, make the angle FEH equal to FEG, then the base FH is equal to GF[e]. If any other right line can be equal to FG, let it be FK, that is, a line nearer to that passing through the center, equal to one more remote, which cannot be. Wherefore, &c.

e 4. 1.

PROP. VIII. THEOR.

If a point be taken without the circle, and from it right lines be drawn, one of which passing through the center, and the other falling upon the concave part of the circumference, the greatest of these lines is that passing through the center; and the line nearer to that, passing through the center, is greater than that more remote; of those falling upon the convex part of the circumference, that which lies betwixt the point and the diameter is the least line, and that line nearer to that passing through the center, is less than that more remote; and, on each side of the diameter, only two lines can be drawn from that point, falling either on the concave or convex part of the circumference equal to one another.

Let any point D be taken without the circle ABC; draw DA, DE, DF, DC, to the concave part of the circumference; of these lines DA, which passes through the center, is the greatest; DE is greater than DF, and DF than DC. Of these that fall upon the convex part of the circumference, DG is the least, DK is less than DL, and DL than DH; on each side of DG only two right lines can be drawn equal to each other, either on the convex or concave part of the circumference. For, find the center M; draw ME, MF, MC, MH, ML, MK. Now, because MA

is

Book III. is equal to ME, add MD, which is common to both, then AD is equal to DM, ME; but DM, ME, are greater than DE[a]; therefore DA is greater than DE; but DM, ME, are equal to DM; MF, and the angle DME greater than DMF; therefore DE is greater than DF[b]. For the same reason, DF is greater than DC; wherefore DA is the greatest of the right lines falling on the concave part of the circumference. Again, because DK KM are greater than DM[a], take the equal lines KM, GM[c], from both, there remains DG less than DK[d]; but K is a point taken within the triangle DLM; therefore DK, KM, are less than DL, LM[e]; take MK, ML[c] from both, there remains DK less than DL[d]. For the same reason, DL is less than DH; wherefore DG is the least line, and DK less than DL, &c.

a 20. 1.
b 24. 1.
c Def. 15. 1.
d Ax. 5. 1.
e 21. 1.

Likewise, from the point D on each side of the least line, only two right lines can be equal to each other, falling on the convex part of the circumference. For, make the angle DMB equal to the angle DMK, and join DB; then, because DM, MB, are equal to DM, MK, and the angle DMB to DMK, the base DB is equal to DK[f]. If any other right line can be equal to DB, let it be DN; that, is a line nearer to the least line equal to one more remote, which cannot be. Neither can more than two equal right lines fall upon the concave part of the circumference on each side of the diameter from the same point.

f 4. 1.

For, let the angle AMO be made equal to AME, join MO, DO; then the angles AMO, DMO, are equal to two right angles[g], and AME, DME, likewise equal to two right angles; but AMO is equal to AME; therefore, the angle DMO is equal to DME[h]; but DM, ME, are equal to DM, MO, and the angle DME to DMO; therefore, the base DO is equal to DE[i]. If any other right line can be equal to DO, let it be DP, that is, one nearer to that passing through the center, equal to one more remote. Wherefore, &c.

g 13. 1.
h Ax. 3. 1.
i 4. 1.

PROP. IX. THEOR.

If a point be assumed in a circle, and from it be drawn more than two right lines to the circumference equal to one another, that point is the center of the circle.

Let the point D be assumed in the circle ABC, and from it be drawn, to the circumference, the right lines DB, DC, DA, equal to one another, D is the center of the circle. If not, let E be the center, join D, E, which produce to F and G; then is FG the diameter, and D is some point in it, not the center; therefore

therefore DG is greater than DC, DC than DB, and DB than DA[a]; but DC, DB, DA, are equal[b]; and likewise not equal; which is impossible; therefore no point but D is the center of the circle. Wherefore, &c.

a 7.
b Hyp.

PROP. X. THEOR.

ONE circle cannot cut another in more than two points.

For, if possible, let the circle ABC cut the circle DEF in the points B, G, F; let K be the center of the circle ABC; join BK, KG, KF. Now, because K is a point within the circle DEF, from which there is drawn to the circumference the right lines BK, KG, KF, equal to one another; therefore K is the center of both circles[a]; which is impossible[b]. Wherefore, &c.

a 9.
b 5.

PROP. XI. THEOR.

IF two circles touch one another inwardly, a line joining their centers will fall on the point of contact.

Let the two circles ABC, ADE, touch each other inwardly in the point A; let F and G be the centers of the circles ABC, ADE; then the line joining the centers F, G, will pass through the point A. If not, let the right line joining the centers F, G, cut the circles in the points D, H; join GA; then because F is the center of the circle ABC, FA is equal to FH[a]. For the same reason, GD is equal to GA; but GA, GF, are greater than AF[b]; therefore DF is greater than AF; therefore greater than HF; and likewise less[c]; which is impossible; therefore the line joining the centers will not pass through any other point than A. Wherefore, &c.

a def. 15. 1.
b 20. 1.
c Ax. 9. 1.

PROP. XII. THEOR.

IF two circles touch one another outwardly, a line joining their centers will pass through the point of contact.

Let the two circles ABC, ADE, touch one another outwardly in the point A; a right line, joining their centers,

F will

Book III. will pass through the point A. If not, let F, G, be the centers of the two circles, and the right line FG joining them; cut the circles in C, D; join FA, AG. Because F is the center of the circle ABC, FA is equal to FC [a]; and, because G is the center of the circle ADE, GA is equal to GD [a]; add DC; then the whole FG is greater than FA, AG; and likewise less [b]; which is impossible. Wherefore, &c.

a def. 15. 1.
b 20. 1.

PROP. XIII. THEOR.

ONE circle cannot touch another, either outwardly or inwardly, in more than one point.

The circles ABC, BFD, cannot touch one another inwardly in more than one point; for, if possible, let them touch in B, D; let G be the center of the one circle, and H of the other; then BG is equal to GD, and greater than HD; therefore BH is much greater than HD; but H is the center of the circle BDF [a]; therefore BH is equal to HD; and likewise greater; which is impossible: Therefore the circles ABC, BFD, cannot touch one another inwardly in the points B, D. Let the circle AKC touch the circle ABC outwardly in the points A, C, if possible; join AC; then is AC within the circle ACK [b], and likewise without the same [c]; which is impossible. Wherefore two circles, &c.

a 11.
b 2.
c hyp.

PROP. XIV. THEOR.

ANY number of equal right lines, drawn in a circle, are equally distant from the center; and, if they are equally distant from the center, they are equal to one another.

Let AB, CD, be two equal right lines, drawn in the circle ABD; find E, the center of the circle, and from it let fall the perpendiculars EF, EG, they will be equal to one another; for, join EA, EC; then are the right lines AB, CD, bisected by the right lines EF, EG [a]; the square of AE is equal to the squares of AF, FE [b]; and the square of EC equal to the squares of CG, GE; therefore the squares of AF, FE, are equal to the squares of CG, GE [c]; but the square of AF is equal to the square of CG [d]; therefore the square of FE is equal to the square of EG [e]; therefore AB, CD, are equally distant from the center.

a 3.
b 47. 1.
c Ax. 1.
d Ax. 7.
e Ax. 5.

2dly,

2dly, if EF be equal to EG, then AB will be equal to CD; Book III. for the squares of AF, FE, are equal to the squares of CG, GE; but the square of EF is equal to the square of EG; therefore the square of AF is equal to the square of CG [e]; but c Ax. 5. AB is double AF [a], and CD double CG; therefore AB is equal a 3. to CD [f]. Wherefore, &c. f Ax. 6.

PROP. XV. THEOR.

THE diameter of a circle is the greatest right line in it, and the line nearest to the diameter is greater than that more remote; and on each side of the diameter only two right lines can be drawn equal to one another.

Let ABC be a circle, whose diameter is AD; let MN, FG, be drawn any how in the circle; then AD is the greatest line; AD greater than MN, and MN greater than FG. Find the center E; draw EM, EN, EF, EG; then AE, ED, are equal to ME, EN; but ME, EN, are greater than MN [a]; therefore a 20. 1. AD is greater than MN; likewise ME, EN, are equal to FE, EG, and the angle MEN greater than FEG; therefore MN is greater than FG [b]: So likewise on each side of AD only b 24. 1. two right lines can be drawn equal to one another, viz. upon which the equal perpendiculars fall. For, let fall a perpendicular EL upon MN, and draw EH equal to it, and BC at right angles to EH; then BC is equal to MN [c]. If any other c 14. right line can be equal to MN, or BC, let it be FG, a line nearer to the diameter equal to that more remote. Wherefore, &c.

PROP. XVI. THEOR.

A Line drawn from the extreme point of the diameter of a circle, at right angles to that diameter, shall fall without the same; and between that right line and the circumference no right line can be drawn.

Let ABC be a circle, whose diameter is AB; at the extremity of which, if a right line is drawn at right angles, it shall fall without the circle.

If not, let it fall within the circle, as AC; find the center, and join CD. Then, because DAC is a right angle, DCA will be

Book III. be likewise a right angle, for DA is equal to DC a, that is, two angles in a triangle equal to two right angles; which cannot
a 5. 1. be b; neither can it fall upon the circle c; therefore it must fall
b 17. 1. without the circle, which let be AE; and betwixt the right
c Def. 4. 1. line AE, and circumference CHA, no right line can be drawn. If possible, let FA be drawn; then DAF is less than a right angle. From the point D, to the right line FA, a line can be drawn at right angles to FA, falling without the circle; which let be DG; then, because DGA is a right angle, and DAG less than a right angle, DA is greater
d 19. 1. than DG d; but DA is equal to DH; therefore DH is greater than DG, and likewise less; which is impossible: Therefore, betwixt the circumference and right line AE no other right line can be drawn. Wherefore, &c.

Cor. I. Hence the angle between the right line and circumference is the least of all acute angles; and the angle betwixt the diameter and circumference is the greatest acute angle possible.

II. Hence, likewise, a right line, drawn at right angles, at the extreme point of the diameter of a circle, touches the circle only in one point; for, if it meet it in two points, it
c 2. would fall within the circle c.

PROP. XVII. PROB.

To draw a right line that will touch a given circle from a given point without the same.

Let BCD be the circle, and A the point without it; it is required to draw a right line from the point A, that will
a 1. touch the circle BCD. Find E the center of the circle a; join AE, cutting the circle BCD in D. About the center E, with the distance EA, describe the circle AFG; from the point
b. 11. 1. D draw DF at right angles to DE b, cutting the circle AFG in F; join EF; cutting DBC in B, and join AB; then is AB the tangent required.

For, because E is the center of both circles, the right lines AE, EB, are equal to FE, ED, and the angle E common;
c 4. 1. therefore the triangle ABE is equal to FDE c; and the angle EBA to EDF; but EDF is a right angle; therefore ABE is likewise a right angle: Therefore AB is a tangent to the
d 4. cor. 16. circle in the point B d, and drawn from the point A. Which was required.

PROP.

PROP. XVIII. and XIX. THEOR.

If any right line touches a circle, and from the center to the point of contact a right line be drawn, that line will be at right angles to the tangent; and if, from the point of contact, a right line be drawn, passing through the circle, at right angles to the tangent, the center of the circle will be in that line.

Let ABC be a circle, and DE a right line touching it in the point C; and if, from the center F, there be drawn a right line FC, that line will be perpendicular to the tangent. If not, let FG be drawn from the center F, at right angles to DE[a]. a 12. 1.

Now, because FGC is a right angle, FCG will be less than a right angle[b]; therefore FC is greater than FG[c]; that is, FB greater than FG, a part greater than the whole; which is impossible. For the same reason, no right line but FC can be perpendicular to DE. b 17. 1. c 19. 1.

2dly, If, from the point of contact C, of the tangent DE, AC be drawn through the circle ABC, at right angles to DE, the center of the circle will be in AC. If not, let it be in H; join HC; then HCE is a right angle[d]; but ACE is a right angle[e]; therefore HCE is equal to ACE, a part to the whole; which is absurd. Wherefore, &c. d 18. e Hyp.

PROP. XX. THEOR.

The angle at the center of the circle is double the angle at the circumference, when the same arc is the base of both.

Let ABC be a circle, and E its center, the angle BEC, at the center, is double the angle BAC, at the circumference; the arc BC being the base of both.

For, join AE, and produce it to F; then, because EA is equal to EB, the angle EAB is equal to EBA[a]; but EAB, EBA, are double EAB; and BEF is equal to EAB, EBA[b], or double EAB. For the same reason, FEC is double EAC; therefore the whole angle BEC is double BAC. Again, let there be another angle EDC; join EC, and produce DE to B; then the outward angle BEC is equal to EDC, ECD, or double EDC[a]. For the same reason, the angle BEF is double the angle BDF; but the whole angle BEC is double BDC, and a part BEF is double a part BDF; therefore, the remainder FEC is double the remainder FDC. Wherefore, &c. a 5. 1. b 32. 1.

PROP.

PROP. XXI. PROB.

A Segment of a circle being given, to deſcribe the circle whereof it is the ſegment.

See fig. 25. It is required to deſcribe the circle, whereof ABC is a ſegment, biſect AC in D, draw DB at right angles to AC, and join AB; then the angle BAD will be either equal, greater, or leſs than the angle ABD: *Firſt*, let them be equal; then a 6. 1. the ſide AD is equal to DB [a], and DC to AD [b]; therefore D [c] b Conſt. is the center of the circle. *2dly*, If the angle BAD is greater c 9. or leſs than ABD, make the angle BAE equal to ABD, and join AE, EC; then the ſide AE is equal to BE [a], and becauſe AD is equal to DC, and DE common, the angles ADE, c. 4. 1. CDE are right ones; therefore the ſide AE is equal to EC [c]; therefore the three lines EA, EB, EC are equal; therefore E is the center of the circle [c]. If the center E is within the ſegment, then the ſegment is greater than a ſemicircle, if without, leſs; if upon the baſe of the ſegment, then it is a ſemicircle. Wherefore, &c.

PROP. XXII. THEOR.

ANGLES that are in the ſame ſegment of a circle are equal to each other.

See fig. 21. Let BAED be the ſegment of a circle, either greater or leſs than a ſemicircle; and BAD, BED, angles in the ſame, theſe angles are equal to one another. Complete the circle a 21. ABCDE [a], and to the center F; draw BF, FD; then the angle BFD is double the angle BAD [b], and likewiſe double b 20. c Ax. 7. 1. BED; therefore BAD, BED, are equal to one another [c]. If the ſegment is leſs than a ſemicircle, join AE, complete the circle [a], and, to the center F, draw AF, FE; then the angle AFE is double the angle ABE or ADE [b]; but the d 15. 1. angle AGB is equal to EGD [d]; therefore the remaining e Cor. 32. 1. angle BAD is equal to BED [e]. Wherefore, &c.

PROP.

PROP. XXIII. THEOR.

THE opposite angles of every quadrilateral figure inscribed in a circle, are equal to two right angles.

Let ABDC be a quadrilateral figure inscribed in the circle ABDC, the opposite angles BAC, BDC are equal to two right angles; as also ABD, ACD; join DA, BC. See fig. 22.

Then in the segment DBAC, the angle DBC is equal to DAC[a]; for the same reason the angle BAD is equal to BCD[a]; therefore the whole angle BAC is equal to the two angles DBC, DCB; add BDC to both; then the two angles BAC, BDC are equal to the three angles in the triangle BDC, that is, equal to two right angles[b]: But all the inward angles of any quadrilateral figure are equal to four right angles[c]; therefore the angles ABD, ACD are equal to two right angles. Wherefore, &c.

a 22.
b 32. 1.
c Cor. 32. 1.

PROP. XXIV. THEOR.

TWO similar and unequal segments of circles cannot be placed upon the same right line, either on the same or opposite sides.

For, if possible, let the similar segments ADB, ACB, be placed upon the same right line AB; if not on the same side, there can be drawn, on the same side, a segment equal to one of them; let this be ACB; then, because they are similar, the angle ACB is equal to ADB[a]; which cannot be[b]. Wherefore, &c. See fig. 23.

a Def. 11.
b 16. 1.

PROP. XXV. THEOR.

SImilar segments of circles, being upon equal right lines, are equal to one another.

Let AEB, CFD be similar segments, constitute on the equal right lines AB, CD; they are equal to one another. See fig. 24.

For, let the segment AEB be applied to the segment CFD, so as the point A coincide with C, and B with D; then AB will coincide with CD, and the segment AEB with CFD: If not, they will cut one another, which let be in G; then

the

Book III. the segment CGD cuts the segment CFD in the points C, G, D; therefore a circle will cut another circle in more than two points, which cannot be[a]. Wherefore, &c.

a 10.

PROP. XXVI. THEOR.

In equal circles, the circumferences, upon which equal angles stand, are equal to one another, whether the angles are at the center or circumferences.

Let ABC, DEF, be equal circles, and BGC, EHF, equal angles at the centers, and BAC, EDF at the circumferences; then the circumference BKC is equal to ELF; join BC, EF; then, since BG, GC are equal to EH, HF[a], and the angle BGC to EHF, the base BC is equal to EF[b]; and because the angle BAC, is equal to EDF, and the right line BC to EF, the segment BAC is similar, and equal to EDF[c]; but the whole circle BAC is equal to the circle EDF, and the circumference BAC to the circumference EDF; therefore the remainder BKC is equal to ELF. Wherefore, &c.

a Def. 1.
b 4 1.
c Def. 10. and Prop. 25.

PROP. XXVII. THEOR.

Angles, that stand upon equal circumferences in equal circles, are equal to each other, whether they be at the centers or circumferences.

Let the angles at the centers of the circles ABC, DEF, be BGC, EHF, and the angles BAC, EDF, at their circumferences, standing on the equal circumferences BC, EF; then the angle BAC is equal to the angle EDF, and BGC to EHF.

For, if the angle BGC be not equal to the angle EHF, let one of them be greater, as BGC, and make BGK equal to EHF; then the circumference BK is equal to EF[a]; but EF is equal to BC[b]; therefore, BK is equal to BC, a part to the whole, which is impossible; therefore, the angle BGK is not equal to EHF; therefore, no angle but BGC can be equal to EHF at the center, and BAC to EDF at the circumference. Wherefore, &c.

a 26.
b Hypoth.

PROP.

PROP. XXVIII. and XXIX. THEOR.

IN equal circles, equal right lines cut off equal circumferences, the greater equal to the greater, and the lesser to the lesser, and the right lines, in equal circles, which cut off equal circumferences, are equal.

Let ABC, DEF be equal circles, in which are the equal right lines BC, EF, which will cut off the greater circumference, viz. BAC equal to EDF, and the lesser BGC to EHF.

For, find the centers K and L of the two circles, and join BK, KC, EL, LF; then, because the two circles are equal, the two sides BK, KC are equal to the two sides EL, LF [a], and the base BC equal to EF [b]; therefore the angle BKC is equal to ELF [c], and the circumference BGC equal to EHF; but the whole circumference BGCA is equal to the whole circumference EHFD; therefore the remaining circumference, BAC, is is equal to the remaining circumference EDF. Wherefore, &c.

a Def. 1.
b Hyp.
c 8. 1.

And, if the circumference BGC be equal to EHF, the right line BC will be equal to EF; for, the same construction remaining, because BK is equal to KC; and EL to LF [a], and the angle BKC to ELF [d], the base BC is equal to EF [e]. Wherefore, &c.

d 27.
e 14. 1.

PROP. XXX. PROB.

TO cut a given circumference into two equal parts.

It is required to cut the given circumference ADB into two equal parts. Join AB, which bisect in C [a]; from which draw the right line CD at right angles to AB [b], and join AD, DB.

a 10. 1.
b 11. 1.

Now, because AC is equal to CB, and CD common, the two sides AC, CD are equal to the two sides BC, CD, and the angle ACD to BCD [b]; therefore, the base AD is equal to DB [c], and the circumference AD to DB [d]. Wherefore, &c.

c 4. 1.
d 28.

Book III.

PROP. XXXI. THEOR.

THE angle in a semicircle is a right angle, and the angle in a segment greater than a semicircle is less than a right angle, and the angle in a segment less than a semicircle, is greater than a right angle.

Let the angle BAC be an angle in a semicircle, standing on the diameter BC; find the center E, and join AE; the angle BAC will be a right angle: Let ADC be a segment cut off by the right line AC, join AD, DC; then the angle ADC is greater than a right angle; if the circle be compleated, the segment ABC is greater than a semicircle, and the angle ABC in it less than a right angle.

For, because E is the center of the circle, the angle ABE is equal to BAE [a], and the angle EAC to ACE [a]; therefore, the whole angle BAC is equal to the two angles ACB, ABC; therefore, BAC is a right angle [b], and the angle BAC is greater than ABC; for BAE is equal to ABC. But the angles ABC, ADC are equal to two right angles [c], and ABC is less than a right angle; therefore ADC is greater than a right angle. Wherefore, &c.

a 5. 1.
b Cor. 32. 1.
c 22.

Cor. Hence the angle of a segment greater than a semicircle is greater than a right angle, and the angle of a segment less than a semicircle, is less than a right angle. For the angle that the circumference BA makes with the right line AC is greater than a right angle; for it contains the right angle BAC. And the angle that the circumference AC makes with the right line AC, is less than a right angle; for, if BA be produced to F, the right angle FAC contains it.

PROP. XXXII. THEOR.

IF a right line touch a circle, and from the point of contact a right line be drawn to the circle, the angles that right line makes with the tangent are equal to the angles in the alternate segments of the circle.

Let the right line EF touch the circle ABCD in the point B; from any point D, in the circle, draw the right line DB; then the angle DBF is equal to the angle in the alternate segment DAB; and the angle DBE equal to DCB; for, from the point of

of contact B, draw BA at right angles to EF [a]; take any point C in the circumference, and join AD, DC, CB.

Now, because BA is drawn from B, at right angles, to EF, the center of the circle is in AB [b]; and, because ADCB is a semicircle, the angle ADB is a right angle [c]; therefore ADB is equal to the two angles DBA, DAB [d]; but ABF is likewise a right angle; therefore the angle ABF is equal to the angles DBA, DAB; but the angle ABF is likewise equal to the angles DBF, DBA; therefore the angles DBA, DBF, are equal to the angles DAB, DBA [e]. Take the common angle DBA from both, there remains the angle DBF equal to DAB, the angle in the alternate segment.

a 11. 1.
b 19.
c 31.
d cor. 32. 1.
e ax. 1. 1.

Likewise the angle DCB is equal to the angle DBE; for, DCB, DAB, are equal to two right angles [f], and DBF, DBE [g], equal to two right angles; but DAB is proved equal to DBF; therefore the remainder, DCB, is equal to DBE. Wherefore, &c.

f 22.
g 13. 1.

PROP. XXXIII. PROB.

Upon a given right line to describe a segment of a circle, that will contain an angle equal to a given right lined angle.

It is required, upon AB, to describe a segment of a circle, that will contain an angle equal to a given angle, C.

At the point A, with the right line AB, make the angle BAD equal to C [a]; draw AE at right angles to AD [b]; bisect AB in F [c], and draw FG, at right angles, to AB, cutting AE in the point G; join GB; with the center G, and distance GA, describe the circle ABE, which will pass through the point B; for, because AB is bisected in F, and GF drawn, at right angles, to AB, the right lines AF, FG are equal to BF, FG; and the angle AFG equal to BFG; therefore AG is equal to GB [d]. Now, because AD is a tangent to the circle [e], the angle BAD is equal to the angle in the alternate segment BEA [f]; but the angle DAB is equal to the angle C [g]; therefore the angle AEB is equal to the angle C. Wherefore, &c.

a 23. 1.
b 11. 1.
c 10. 1.
d 4. 1.
e 16.
f 32.
g Const.

PROP. XXXIV. PROB.

To cut off a segment from a given circle that shall contain an angle equal to a given right lined angle.

Book III. It is required to cut off a segment from the given circle ABC, that shall contain an angle equal to the given angle D.

a 17. Draw the line EF, touching the circle in B[a]; from which
b 23. 1. draw BC, making an angle FBC equal to the angle D[b]; then the angle FBC will be equal to the angle in the alternate seg-
c 32. ment, viz. BAC[c]; but FBC is equal to the angle D; therefore BAC is equal to the angle D. Wherefore, &c.

PROP. XXXV. THEOR.

If two right lines in a circle mutually cut each other, the rectangle contained under the segment of the one, is equal to the rectangle contained under the segments of the other.

Let the two right lines AC, DB, in the circle ABCD, mutually cut each other in E; then the rectangle under AE, EC, is equal to the rectangle under DE, EB; if AC, DB, pass each through the center, then the rectangle under AE, EC, is equal to the rectangle under DE, EB, for the lines are e-
a def. 15. 1. qual[a].

2dly, If AC, passing through the center, cut BD, not passing through the center, at right angles, in the point E, find the
b 3. center F, and join FD; for, because BE is equal to ED[b], and the angle DEF is a right one, the squares of DE, EF, are
c 47. 1. equal to the square of FD[c]; but the rectangle under AE, EC,
d 5. 2. together with the square of EF, is equal to the square of FC[d], or FD. Take the square of FE, which is common, from both, there remains the rectangle under AE, EC, equal to the square of ED, that is, the rectangle under BE, ED. If the right line, AC, passing through the center, cut BD, not passing through the center, and not at right angles, draw FG at right angles to BD, and join FD; then BG is equal to GD[b]; the rectangle under BE, ED, together with the square of GE, is equal to the square of GD[d]. Add the square of GF to both, then the rectangle under BE, ED, with the squares of EG, GF, or the square of EF[c], are equal to the square of FD; but the rectangle under AE, EC, together with the square of EF, are likewise equal to the square of FD. Take the square of EF from both, then the rectangle under AE, EC, is equal to the rectangle under BE, ED.

3dly, If

3dly, If neither pass through the center, draw GH, passing through the center F, and cutting AC, BD, in E; then the rectangle under AE, EC, is equal to the rectangle under BE, ED; for each is equal to the rectangle under GE, EH. Wherefore, &c.

PROP. XXXVI. THEOR.

If some point be taken without a circle, and from that point two right lines be drawn, one of which touches the circle, and the other cuts it, the rectangle under the whole secant line, and the part between the point and convexity of the circle, is equal to the square of the tangent line.

Let ABC be the circle, D the given point, and DCA, DB, the two given right lines, of which DB touches the circle, and DCA cuts it; the rectangle under AD, DC, is equal to the square of DB.

Now, DCA either passes thro' the center, or not. First, let it pass thro' the center E, and join BE; then, because AC is bisected in E, and DC added, the rectangle under AD, DC, together with the square of CE, are equal to the square of DE[a]; [a 6. 2.] but the square of DE is equal to the squares of DB, BE[b]; for [b 47. 1.] the angle DBE is a right angle[c]; therefore the rectangle under [c 18.] AD, DC, together with the square of CE, are equal to the squares of DB, BE. Take the equal squares of BE, CE, from both, there remains the rectangle AD, DC, equal to the square of DB.

2dly, Let DA not pass through the center of the circle ABC; find the center E[d], and join ED, EC, EB; draw EF, at right [d 1.] angles, to AC, cutting it in F[e]; then AF is equal to FC[f]; [e 12. 1.] [f 3.] therefore the rectangle under AD, DC, together with the square of CF, are equal to the square of FD. Add the square of FE to both; then the rectangle under AD, DC, with the squares of CF, FE, are equal to the squares of DF, FE; but the square of CE is equal to the squares of CF, FE[b], and the square of DE equal to the squares of DF, FE[b]; therefore the rectangle under AD, DC, with the square of CE, are equal to the square of DE; but the square of DE is equal to the squares of DB, BE[b]; therefore the rectangle under AD, DC, with the square of CE, are equal to the squares of DB, BE. Take the equal squares of BE, CE, from both, and the rectangle under AD, DC, is equal to the square of DB. Wherefore, &c.

PROP.

Book III.

PROP. XXXVII. THEOR.

If, from a point without a circle two right lines be drawn, one of which cuts the circle, and the other falls upon it; and, if the rectangle under the whole secant line, and part betwixt the point and circle, be equal to the square of the other line, this last line shall be a tangent to the circle.

Let some point D, be assumed without the circle ABC, and from it draw the lines DCA, DB, so that DCA cut the circle, and DB fall upon it; and, if the rectangle under AD, DC, be equal to the square of DB, then DB will touch the circle in the point B.

a 17. For, let DE be drawn a tangent to the circle in the point E [a]; b 1. find the center F [b], and join BF, FE, and DF.

c 18. d 36. e Hyp. Then the angle DEF is a right angle [c]; therefore the rectangle under AD, DC, is equal to the square of DE [d]; but the rectangle under AD, DC, is equal to the square of DB [e]; therefore the square of DB is equal to the square of DE, and DB equal to DE; therefore the right lines DE, EF, are equal to DB, BF; and FD common; therefore the angle DBF is f 8. 1. equal to the angle DEF [f]; but DEF is a right angle; therefore DBF is likewise a right angle: Therefore DB is a tangent to g 16. the circle [g]. Wherefore, &c.

Cor. I. Hence, if any number of right lines, as DA, DG, be drawn from the point A, cutting the circle in C and H, the rectangles under AD, DC, and GD, DH, are equal to one another; for each of them is equal to the square of BD.

II. If, from any two points in the circumference of a circle, two tangents be drawn, so that, being produced, they will meet one another; then these tangents will be equal to one another; for each of their squares, viz. of BD, DE, is equal to the rectangle contained under AD, DC.

THE

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BOOK IV.

DEFINITIONS.

I.

A Right lined figure is ſaid to be inſcribed in a right lined figure, when every one of the angles of the inſcribed figure touches every one of the ſides of the figure wherein it is inſcribed. Book IV.

II.

A right lined figure is ſaid to be deſcribed about a right lined figure, when every one of the ſides of the circumſcribed figure touches each of the angles of the right lined figure.

III.

A right lined figure is inſcribed in a circle when each of the angles of the inſcribed figure touches the circumference of the circle.

IV.

A right lined figure is deſcribed about a circle when each of the ſides of the circumſcribed figure touches the circumference of the circle.

V.

A circle is inſcribed in a right lined figure, when the circumference of the circle touches all the ſides of the figure in which it is inſcribed.

VI.

A circle is deſcribed about a right lined figure when the circumference of the circle touches all the angles of the figure.

VII. A

Book IV.

VII.

A right line is applied in a circle when its extremes are in the circumference of the circle.

PROP. I. PROB.

To apply a right line in a circle, equal to a given right line, not greater than the diameter of the circle.

It is required to apply a right line in the circle ABC, equal to a given right line D, not greater than the diameter of the circle.

Draw the diameter BC; if equal to D, what was required is done; if not, the diameter BC is greater than D; put CE equal to D [a]; about the center C, with the distance CE, describe the circle AEF; then CA is equal to CE; but CE is equal to D; therefore CA is equal to D. Wherefore, there is drawn, &c.

a 3. 1.

PROP. II. PROB.

In a given circle, to inscribe a triangle equiangular to a given triangle.

It is required to inscribe a triangle, in a given circle ABC, equiangular to a given triangle DEF: Draw the right line GAH, touching the circle in the point A [a]; with the right line AH, at the point A, make the angle HAC equal to the angle DEF [b], and the angle GAB equal to DFE; join BC.

a 17. 3. b 23. 1.

Then the angle HAC is equal to the angle ABC [c]; but the angle HAC is equal to DEF; therefore, ABC is equal to DEF: And BAG is equal to ACB [c]; but BAG is equal to DFE; therefore ACB, is equal to DFE; therefore the remaining third angles BAC, EDF [d] are equal; therefore the triangle ABC, is inscribed in the circle ABC, and equiangular to the triangle DEF, which was required. Wherefore, &c.

c 32. 3. d Cor. 32. 1.

PROP.

PROP. III. PROB.

About a given circle, to describe a triangle equiangular to a triangle given.

It is required to describe a triangle about the given circle ABC equiangular to the given triangle DEF; produce the side EF both ways to G, H; find the center of the circle K, and draw KB any how; at the point K, with the right line KB, make the angles BKA, BKC, equal to the angles DEG, DFH[a], each to each; at the points A, B, C, draw the right lines LAM, MBN, LCN, tangents to the circle, in the points A, B, C[b]. a 23. 1. b 16. 3.

Then the angles that LM, MN, LN, make with the right lines KA, KB, KC, are right angles[c]; therefore the angles AKB, AMB, are equal to two right angles[d], and equal to DEF, DEG[e]; but BKA was made equal to DEG; therefore the remainder DEF is equal to AMB[f]. For the same reason DFE is equal to LNM, and the remaining angle MLN equal to EDF; wherefore the triangles LMN, DEF, are equiangular. Which was required. c 18. 3. d 32. 1. e 13. 1. f Ax. 8. 1.

PROP. IV. PROB.

To inscribe a circle in a given triangle.

It is required to inscribe a circle in the given triangle ABC. Bisect the angles ABC, ACB, by the right lines BD, DC[a], meeting each other in the point D; from which let fall DF, DE, DG, perpendiculars, upon the right lines AB, BC, AC[b]. Then, because the two angles DFB, DBF, in the triangle DBF, are equal to the two angles DEB, DBE, in the triangle DBE, and the side DB common to both; the remaining sides BE, ED, are equal to BF, FD, each to each[c]. For the same reason DF is equal to DG; therefore D is the center, and with any of the distances the circle EFG[d] may be inscribed in the given triangle ABC: Which was required. a 9. 1. b 12. 1. c 26. 1. d 9. 3.

PROP. V. PROB.

To describe a circle about a given triangle.

Book IV.

It is required to describe a circle about the triangle ABC.

a 10. 1.

Bisect the sides AB, AC, in the points D, E [a]; from which draw DF, FE, at right angles to AB, AC, which will meet one another, either within the triangle, upon one of the sides BC, or without the triangle, in the point F.

b 11. 1.
c 4. 1.
d Def. 6.

In either case, because AD is equal to DB, and DF common, the two sides AD, DF, are equal to BD, DF; and the angle BDF to ADF [b]; therefore the base BF is equal to the base AF [c]. For the same reason AF is equal to FC: Therefore, if, with the center F, and either of the distances AF, FB, or FC, a circle is described, it will touch all the angles of the triangle [d]: Which was required.

PROP. VI. PROB.

To inscribe a square in a given circle.

It is required to inscribe a square in the given circle ABC.

Draw the diameters BD and AC at right angles to, and bisecting each other in E; join BA, AD, DC, CB; then ADCB is a square.

a 11. 1.
b 4. 1.
c 31. 3.
d def. 30. 1.

For the two sides BE, EA, are equal to DE, EA [a], and the angle BEA to AED [a]; therefore the base BA is equal to AD [b]. For the same reason AD is equal to DC, and DC to CB; therefore the four sides are equal: But the angle BAD is a right angle [c]; therefore ADC, DCB, ABC, are each right angles [c]; therefore the figure is a square [d]: Which was required.

PROP. VII. PROB.

To describe a square about a given circle.

It is required to describe a square about the circle ABCD.

a 11. 1.

Draw the two diameters AC, BD, of the circle, cutting each other at right angles [a]; and through the points A, B, C, D, draw FG, GH, HK, KF, tangents to the circle ABCD.

b 16. 3.
c 29. 1.
d 30. 1.

For, because the angles GAE, AEB, are right angles [b], GF is parallel to BD [c]. For the same reason HK is parallel to BD; therefore GF is parallel to HK [d]. For the same reason, GH is paral-

parallel to FK; therefore GHKF is a parallelogram; but GF, BD, are equal[e], and likewise BD equal to HK; therefore GF is equal to HK[f]. For the same reason, GH is equal to FK; for each is equal to AC; but AC is equal to BD; therefore GH is equal to GF; therefore the four sides are equal; but the angles at G, F, are right ones; for each is equal to GAE, or FAE[b]; therefore all the angles are right ones; therefore GHKF is a square, described about the circle ABCD: Which was to be done.

e 34. 1.
f Ax. 1. 1.
b 16. 3.

PROP. VIII. PROB.

To inscribe a circle in a given square.

It is required to inscribe a circle in the square ABCD.

Bisect the sides AB, AD, in the points E, F[a]; through E draw EH parallel to AB[b], and through F draw FK parallel to AD or BC[b]; then AE, FG, are equal to one another[c], and likewise ED, GK; but AE is equal to ED[a]; therefore FG is equal to GK; but AF is equal to AE[d]; therefore EG is equal to FG, and FB to GH[c]; therefore GE, GF, GH, GK, are equal; therefore if, with the center G, and either of these distances, a circle is described, it will touch the square in the points E, F, H, K; for, if not, let it cut the square, then a right line drawn at right angles to the diameter of a circle will fall within the circle; which cannot be[e]; therefore the circle EFHK is inscribed in the square ABCD: Which was required.

a 10. 1.
b 31. 1.
c 34. 1.
d Ax. 7. 1.
e 16. 3.

PROP. IX. PROB.

To describe a circle about a given square.

It is required to describe a circle about the given square ABCD.

Join AC, BD, mutually cutting each other in the point E.

For, because AB is equal to AD, and AC common, the two sides BA, AC, are equal to the two sides DA, AC, and the base BC equal to DC; therefore the angle BAC is equal to DAC[a]; therefore the angle BAD is bisected by the right line AC. For the same reason the angle ADC is bisected by the right

a 8. 1.

Book IV. right line DB; but the angles BAD, ADC, DCB, ABC[b] are equal, therefore their halfs are equal. Again, because BA is equal to AD, and AE common, and the angle BAE to DAE, the base BE is equal to ED. For the same reason, AE is equal to EC: Therefore the right lines AC, BD, are bisected in E; but the two sides AD, DC, are equal to DC, CB[b]; and the angle ADC to DCB[b]; therefore the base AC is equal to BD[c]; therefore their halfs are equal: Therefore, if, with the center E, and distance AE, a circle is described, it will pass through the points A, B, C, D, of the square: Which was required. Wherefore, &c.

b 46. 1.

c 4. 1.

PROP. X. PROB.

To make an isosceles triangle, having each of the angles at the base double to the other angle.

Cut any given right line AB in the point C, so that the rectangle under AB, BC, be equal to the square of AC[a]; about the center A, and distance AB, describe the circle BDE; in which apply the right line BD equal to AC[b], and join DA, DC; then the triangle ADB is the isosceles triangle; having each of the angles at the base BD double the angle at A.

a 11. 2.

b 1.

For, describe a circle ACD about the triangle ADC[c].

c 5.

Then, because the rectangle under AB, BC, is equal to the square of AC, and BD is equal to AC, the rectangle under AB, BC, is equal to the square of BD; therefore BD is a tangent to the circle ACD[d]; therefore the angle CDB is equal to the angle in the alternate segment CAD[e]. To each add the angle CDA; then the whole angle ADB, or its equal ABD[f], is equal to the two angles CAD, CDA; but the angle BCD is likewise equal to the angles CDA, CAD[e]; therefore the angle BCD is equal to CBD[g]; therefore CD is equal to BD[h]; but BD is equal to CA; therefore CD, CA, are equal[g]; therefore the angle CAD is equal to CDA[f], but CAD, CDA, are double CAD; therefore ABD or ADB are each double BAD: Which was required.

d 37. 3.

e 32. 3.

f 5. 1.

g Ax. 1. 1.

h 6. 1.

PROP. XI. PROB.

To inscribe an equilateral and equiangular pentagon in a given circle.

It is required to inscribe an equilateral and equiangular pentagon in the given circle ABCDE. Book IV.

Make an isosceles triangle FGH, having each of the angles at the base GH double the other angle F[a]; inscribe the triangle ADC in the circle ABCDE, equiangular to the triangle FGH[b]; then each of the angles ACD, ADC, are double the angle at A; bisect the angles ACD, ADC[c], by the right lines CE, DB; join AB, BC, DE, EA; then ABCDE is the equilateral and equiangular pentagon required. [a] 10. [b] 2. [c] 9. 3.

For, because the angles ACD, ADC, are bisected by the right lines CE, DB, the five angles DAC, ADB, BDC, DCE, ECA, are equal; therefore the five circumferences AB, BC, CD, DE, AE, are equal[d]; and the right lines AB, BC, CD, DE, AE, equal to one another[e]; the figure is therefore equilateral; but it is also equiangular; for the circumference AB is equal to DE[e]. Add the circumference BCD to both, then the whole circumference ABCD is equal to the whole circumference EDCB; therefore the angle AED is equal to the angle BAE[f]. For the same reason, the other angles ABC, BCD, CDE, are equal to BAE, or AED; wherefore the pentagon ABCDE is equilateral, and likewise equiangular. Which was required. [d] 26. 3. [e] 29. 3. [f] 27. 3.

PROP. XII. PROB.

TO describe an equilateral and equiangular pentagon about a given circle.

It is required to describe an equilateral and equiangular pentagon about the given circle ABCDE.

Let the points A, B, C, D, E, be the angular points of an equilateral and equiangular pentagon inscribed in the circle; then the circumferences AB, BC, CD, DE, EA, are equal; draw the right lines GH, HK, KL, LM, MG, tangents to the circle in the above points[a]. From F, the center of the circle, draw to the same points the right lines FB, FC, FD, and join FK, FL; then, because FB, FC, FD, are at right angles to HK, KL, LM[b], the squares of FB, BK, are equal to the square of FK[c], and the squares of FC, CK, equal to the square of FK, and therefore equal to each other[d]; but the squares of BF, FC[e], are equal; which being taken from both, the square of BK is equal to the square of KC; that is, BK equal to KC. For the same reason, CL is equal to LD; and because BF, FK, are equal [a] 17. 3. [b] 18. 3. [c] 47. 1. [d] Ax. 1. 1. [e] def. 15. 1.

Book IV. equal to CF, FK, and the base BK equal to KC, the angle BFK is equal to CFK [f]. For the same reason, CFL is equal to LFD; but the whole angle BFC is equal to the whole angle CFD [g]; therefore the angle KFC is equal to the angle CFL [h]; and the base KC to CL [i]; but KC is equal to BK; therefore HK is equal to KL. For the same reason, KL is equal to LM; therefore the figure is equilateral.

f 8. 1.
g 27. 3.
h Ax. 7. 1.
i 4. 1.

It is likewise equiangular; for the angles B and C are each right ones; and the two angles BFC, BKC, equal to two right angles [k]. For the same reason, CFD, CLD, are equal to two right angles; but the angles CFD, CFB, are equal; therefore CLD, BKC, are likewise equal. For the same reason, the whole angle at L is equal to the angle at M; therefore the figure is equiangular; and likewise proved equilateral. Wherefore, &c.

k Cor. 7. 32. 1.

PROP. XIII. PROB.

To inscribe a circle in a given equilateral and equiangular pentagon.

It is required to inscribe a circle in the equilateral and equiangular pentagon ABCDE.

Bisect the sides BC, CD, DE, in the points H, K, L [a]; from which draw the right lines HF, KF, LF, at right angles to BC, CD, DE [b]; from the point F, where the right lines HF, KF, intersect each other, draw FC, FD, FE; then, because the angles FHC, FKC, are right angles, the square of FC is equal to the squares of FH, HC [c], and likewise to the squares of FK, KC; therefore the squares of FH, HC, are equal to the squares of FK, KC [d]. Take the equal square of HC, CK, from both, there remains the square of HF equal to the square of FK; that is, HF equal FK; therefore HF, FC, are equal to FC, FK, and the base HC to CK; therefore the angle HFC is equal to KFC [e]; but the angles FHC, FKC, are right ones; therefore the remaining angle HCF is equal to KCF; therefore the angle BCD is bisected by the right line FC. For the same reason, FK is equal to FL, and the angle CDE bisected by FD; therefore, because FH, FK, FL, are equal, if, with the center F, and either of these distances, a circle is described, it will touch the sides of the pentagon in the points G, H, K, L, M; wherefore the circle GHKLM is inscribed in the equilateral and equiangular pentagon ABCDE: Which was required.

a 10. 1.
b 11. 1.
c 47. 1.
d Ax. 1. 1.
e 8. 1.

Cor.

Cor. If two of the nearest sides of an equilateral and equiangular figure be bisected, and from the point where these lines cut each other, there be lines drawn to all the angles of the figure, these lines will bisect all the angles of the figure.

PROP. XIV. PROB.

To describe a circle about a given equilateral and equiangular pentagon.

It is required to describe a circle about the equilateral and equiangular pentagon ABCDE.

Bisect the right lines AB, BC [a], in the points H and G; from which draw the right lines HF, GF, intersecting each other in the point F, and at right angles, to AB, BC [b]; from the point F draw the right lines BF, FA, FE, FD, FC, they will bisect the angles at A, B, C, D, E [c]. Then, because AB is equal to AE, and AF common, and the angle BAF equal to EAF, the base BF will be equal to EF [d]. For the same reason, EF is equal to FC; therefore, if, with the center F, and distance B, E, or C, a circle is described, it will pass through the points A, B, C, D, E, of the equilateral and equiangular pentagon: Which was required.

a 10. 1.
b 11. 1.
c cor. 13.
d 4. 1.

PROP. XV. PROB.

To inscribe an equilateral and equiangular hexagon in a given circle.

It is required to inscribe the equilateral and equiangular hexagon in the given circle ABCDEF.

Draw AD a diameter to the circle ABCDEF, whose center is G; with the point D as a center, and distance DG, describe a circle EGCH; join EG, GC; which produce to the points B, F; join AB, BC, CD, DE, EF, FA; then ABCDEF is an equilateral and equiangular hexagon.

For, since G is the center of the circle ABCDEF, GC is equal [a] to GD; and since D is the center of the circle CGEH, GD is equal to DC; therefore CGD is an equilateral triangle [b]; but it is likewise equiangular [c]. For the same reason, GDE is an equilateral and equiangular triangle, and equal to the triangle

a def 15. 1.
b 1. 1.
c cor. 5. 1.

Book IV. triangle CGD; and, becauſe BG, GA, are equal to DG, GE, and the angle BGA to DGE [d], the baſe AB is equal to DE. For the ſame reaſon, AF is equal to CD; but CD is equal to DE; therefore AB is equal to AF. Again, becauſe CG falls upon BE, the angles CGB, CGE, are equal to two right angles [e]; but CGD, DGE, are each one third of two right angles [f]; therefore CGB is likewiſe one third of two right angles; therefore BG, GC, are equal to CG, GF, and the angle BGC equal to the angle CGD; therefore the baſe BC is equal to CD; but likewiſe BG, GC, are equal to FG, GE, and the angle BGC to FGE [d]; therefore the baſe BC is equal to FE; but BC is proved equal to CD, and CD to DE; therefore the ſix ſides BC, CD, DE, EF, FA, AB, are equal; therefore the figure is equilateral; it is likewiſe equiangular; for the two angles GDC, GDE, are equal to the two angles GCD, GCB; for each is one third of two right angles [f]; therefore the whole angle BCD, is equal to the whole CDE. For the ſame reaſon, all the other angles are equal to one another; therefore the figure is likewiſe equiangular. Wherefore, &c.

d 15. 1.
e 13. 1.
f cor. 32. 1.

Cor. Hence the ſide of a hexagon is equal to the ſemi-diameter of the circle. And, if through the points A, B, C, D, E, F, tangents to the circle, be drawn, an equilateral and equiangular hexagon will be deſcribed about the circle, as may be proved in the ſame manner as the pentagon: And ſo likewiſe a circle may be inſcribed and deſcribed about a given hexagon.

PROP. XVI. PROB.

To inſcribe an equilateral and equiangular quindecagon in a given circle.

It is required to inſcribe an equilateral and equiangular quindecagon in the given circle ABCD.

Let AC be the ſide of an equilateral and equiangular triangle inſcribed in the circle [a], and AB the ſide of an equilateral and equiangular pentagon [b], drawn from the point A; then, if the circle is divided into fifteen parts, the ſide of the triangle AC will ſubtend five of them, and the ſide of the pentagon AB will ſubtend three; therefore BC will be two of ſaid parts; therefore biſect BC in E; BE or CE will be one fifteenth part of the circum-

a 15.
b 11.

cumference; if BC, CE, &c. be joined, the equilateral and equiangular quindecagon will be inscribed: Which was required. Book IV.

Cor. If, from what has been said of the pentagon, right lines be drawn through the divisions of the circle, tangents to the same, an equilateral and equiangular quindecagon will be described about the circle; or a circle may be inscribed or described about the quindecagon.

THE

THE

ELEMENTS

OF

EUCLID.

BOOK V.

DEFINITIONS.

I.

A PART is a magnitude of a magnitude; the leſs of the greater, when the leſs meaſures the greater.

II.

A multiple is a magnitude of a magnitude; the greater of the leſs, when the leſs meaſures the greater.

III.

Ratio is a certain mutual habitude of magnitudes of the ſame kind, according to quantity.

IV.

Magnitudes have proportion to each other; which, being multiplied, can exceed one another.

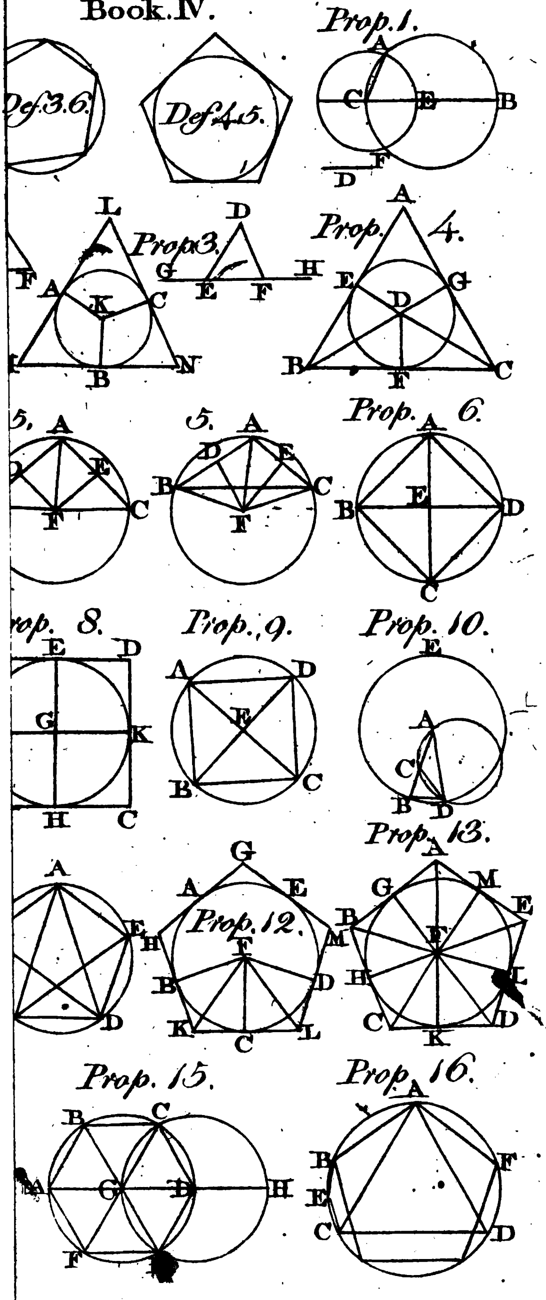
V.

Magnitudes have the ſame ratio to each other, viz. the firſt to the ſecond, and third to the fourth, when there are taken any equimultiples of the firſt and third, and likewiſe any equimultiples of the ſecond and fourth; if the multiple of the firſt be equal to the multiple of the ſecond, then the multiple of the third will be equal to the multiple of the fourth; if greater, greater; and, if leſs, leſs.

VI.

Magnitudes which have the ſame proportion are called Proportionals.

Book. IV.


Prop. 1.
Def. 3. 6.
Def. 4. 5.
Prop. 3.
Prop. 4.
5.
5.
Prop. 6.
rop. 8.
Prop. 9.
Prop. 10.
Prop. 12.
Prop. 13.
Prop. 15.
Prop. 16.

VII.

When, of equimultiples, the multiple of the firſt exceeds the multiple of the ſecond, but the multiple of the third does not exceed the multiple of the fourth; the firſt to the ſecond is ſaid to have a greater ratio than the third to the fourth.

VIII.

Analogy is a ſimilitude of proportions.

IX.

Analogy, at leaſt, conſiſts of three terms.

X.

When three magnitudes are proportionals, the firſt has to the third a duplicate ratio of what it has to the ſecond.

XI.

When four magnitudes are proportional, the firſt has to the fourth a triplicate ratio of what it has to the ſecond; and always one more in order as the proportionals ſhall be extended.

XII.

Homologous magnitudes, or magnitudes of a like ratio, are ſuch whoſe antecedents are to the antecedents and conſequents to the conſequents in the ſame ratio.

XIII.

Alternate ratio is the comparing the antecedent with the antecedent, and conſequent with the conſequent.

XIV.

Inverſe ratio is, when the conſequent is taken as the antecedent, and compared with the antecedent as a conſequent.

XV.

Compounded ratio is, when the antecedent and conſequent, taken as one, are compared with the conſequent itſelf.

XVI.

Divided ratio is, when the exceſs, by which the antecedent exceeds the conſequent, is compared with the conſequent.

XVII.

Converſe ratio is, when the antecedent is compared with the exceſs by which the antecedent exceeds the conſequent.

XVIII.

Ratio of equality is when there are taken more than two magnitudes in one order, and a like number of magnitudes in another order, comparing two to two, being in the ſame ratio; it ſhall be in the firſt order of magnitudes, as the firſt is to the laſt; ſo, in the ſecond order of magnitudes, is the firſt to the laſt.

XIX.

Ordinate proportion is, the ratio being, as in the laſt, as the antecedent is to the conſequent, in the firſt order of magnitudes;

ſo

so is the antecedent to the consequent in the second order of magnitudes; and as the consequent is to any other, so is the consequent to any other.

XX.

Perturbate proportion is, when there are three or more magnitudes, and others equal to them in number, taken two and two in the same ratio; in the first order of magnitudes, as the antecedent is to the consequent; so, in the second order of magnitudes, is the antecedent to the consequent; and, as in the first order, the consequent is to some other, so, in the second order, is some other to the antecedent.

AXIOMS.

I.

EQUIMULTIPLES of the same, or of equal magnitudes, are equal to each other.

II.

These magnitudes that have the same equimultiples, or whose equimultiples are equal, are equal to each other.

PROP. I. THEOR.

IF there be any number of magnitudes, equimultiples of a like number of magnitudes, each of each, whatever multiple any one of the former magnitudes is of its correspondent one, the same multiple are all the former magnitudes of all the latter.

Let AB, CD, be magnitudes, equimultiples of E, F, whatever multiple AB is of E, and CD of F, the same multiple AB, CD, together, is of E, F, together.

For, let the magnitudes in AB, equal to E, be AG, GB; and the magnitudes in CD, equal to F, be CH, HD; then AG, CH, are equal to E, F; and BG, HD, likewise equal to E, F; therefore, as often as AB contains E, and CD, F, so often AB, CD, contains E, F: Wherefore, if there are, &c.

PROP. II. THEOR.

IF the first be the same multiple of the second, as the third is of the fourth; and if the fifth be the same multiple of the second, that the sixth is of the fourth; then shall the first, added to the fifth, be the same multiple of the second, that the third, added to the sixth, is of the fourth.

Let

Let the first AB be the same multiple of the second C, that the third DE is of the fourth F; and let the fifth BG be the same multiple of the second C, that the sixth EH is of the fourth F; then AG will be the same multiple of C that DH is of F.

For, because AB is the same multiple of C that DE is of F, there are as many magnitudes in AB equal to C, as in DE, equal to F. For the same reason, there are as many magnitudes in BG equal to C, as there are in EH, equal to F; therefore there are as many magnitudes in AG equal to C, as there are in DH equal to F [a]. Wherefore, &c. a. 1.

PROP. III. THEOR.

If the first be the same multiple of the second, that the third is of the fourth, and there be taken equimultiples of the first and third, then will the magnitudes so taken be equimultiples of the second and fourth.

Let the first A be the same multiple of the second B that the third C is of the fourth D; and let EF, GH, be equimultiples of A, C; then EF is the same multiple of B, that GH is of D. For, let the magnitudes in EF, equal to A, be FK, KE; and the magnitudes in GH, equal to C, be HL, LG; then there are as many magnitudes equal to B in FE, as there are magnitudes equal to D in GH [a]; wherefore FE is the same multiple of B, a 2. that GH is of D. Wherefore, &c.

PROP. IV. THEOR.

If the first have the same ratio to the second that the third has to the fourth, then shall also the equimultiples of the first have the same ratio to the equimultiple of the second that the equimultiple of the third has to that of the fourth.

Let there be four magnitudes, A, B, C, D, such, that A is to B as C to D. Let E, F, be taken the same multiples of A, C; and G, H, the same multiples of B, D; then E is to G as F is to H. For, take K, L, any equimultiples of E, F, and M, N, any equimultiples of G, H; then K is the same multiple of A that L is of C [a]. For the same reason, M is the a 3. same multiple of B that N is of D [a]; but, because A is to B as C is to D, if K be equal to M, L will be equal to N; if great-

er,

Book V. er, greater, and, if less, less; but K, L, are equimultiples of E, F, and M, N, of G, H; wherefore E is to G as F is to H [b]; but it is proved, that, if K be equal to M, L is equal to N; but, if K is equal to M, M is equal to K, and N to L; if greater, greater, and, if less, less; wherefore G is to E as H is to F. Wherefore, if four magnitudes be proportional, they will also be inversely proportional. Wherefore, &c.

b def. 5.

PROP. V. THEOR.

If one magnitude be the same multiple of another magnitude, that a part taken from the one is of a part taken from the other; then the residue of the one shall be the same multiple of the residue of the other that the whole is of the whole.

Let AB be the same multiple of CD, that a part taken away AE is of a part taken away CF; then the residue EB shall be the same multiple of the residue FD, that the whole AB is of the whole CD.

For, let BE be the same multiple of CG that AE is of CF; then, because AE is the same multiple of CF that BE is of CG, AE will be the same multiple of CF that AB is of GF [a]; but AE is the same multiple of CF that AB is of CD, and AB is the same multiple of GF that it is of CD; therefore GF is equal to CD [b]; take CF, which is common, from both, there remains GC equal to FD; therefore AE is the same multiple of CF that EB is of FD. Wherefore, &c.

a 1.

b Ax. 3.

PROP. VI. THEOR.

If two magnitudes be equimultiples of two magnitudes, and some magnitudes, equimultiples of the same, be taken away, the residue shall either be equal to these magnitudes or equimultiples of them.

For, first, let GB be equal to E; if HD is not equal to F, let KC be equal to F; then AB is the same multiple of E that KH is of F [a]; but AB, CD, are put equimultiples of E, F; therefore HK is the same multiple of F that CD is of F; therefore KH is equal to CD [b]. Take CH, which is common, from both, there remains KC equal to HD; but KC was put equal to F; therefore HD is equal to F; after the same manner it may be demonstrated that, if GB is any equimultiple of E, HD will be the like equimultiple of F. Wherefore, &c.

a 2.

b Ax. 1.

PROP.

PROP. VII. THEOR.

EQUAL magnitudes have the same proportion to the same magnitude, and one and the same magnitude has the same proportion to equal magnitudes.

Let A, B, be two equal magnitudes, and C any third magnitude, then A, B, will have the same proportion to C. For, let D, E, be any equimultiples of A, B, and F any equimultiple of C; then, if D is equal to F, E is likewise equal to F; if greater, greater; and, if less, less; therefore A is to C as B is to C[a]. Again, C is to A as C is to B; for, the same construction remaining, if F is equal to D, it is likewise equal to E; wherefore C is to B as C is to A. Wherefore, &c.

a def. 5.

PROP. VIII. THEOR.

THE greater of two unequal magnitudes has a greater proportion to some third magnitude than the less has, and that third magnitude has a greater proportion to the lesser magnitude than it has to the greater.

Let AB, C, be two unequal magnitudes, of which AB is the greater; and let D be any third magnitude; then AB will have a greater proportion to D than C has to D; but D has a greater proportion to C than it has to AB.

For, because AB is greater than C, make BE equal to C, then AB will exceed C by AE; if AE is not greater than D, let it be multiplied till it exceed D; and let this multiple be FG; let GH be the same multiple of EB that FG is of AE; then FH will be the same multiple of AB that GH is of EB[a]; and make K the same multiple of C that GH is of EB; but EB is equal to C; therefore K is equal to GH[b]; wherefore FH is the same multiple of AB that K is of C. Now, of D let L be taken a double, M triple, and so on, till a multiple of D is found next greater than K, which let be N; and let M be the next multiple of D, less than N; then M will not be greater than K, that is, K will not be less than M; but M and D are equal to N; and because FG exceeds D, and GH is equal to K, FH will be greater than N; that is, FH, the multiple of AB, exceeds N, the multiple of D; but K, the multiple of C, does not exceed N, the multiple of D; therefore AB has to D a greater proportion than C has to D[c]; but likewise D has to C a greater proportion than it has to AB; for, the same construction remaining, N, the multiple of D, exceeds K, the multiple of C; but does not exceed FH, the multiple of AB. Wherefore, &c.

a 1.

b Ax. 1.

c Def. 7.

PROP.

Book V.

PROP. IX. THEOR.

MAGNITUDES which have the same proportion to one and the same magnitude are equal to one another; and if a magnitude has the same proportion to other magnitudes, these magnitudes are equal to one another.

Let the magnitudes A, B, have the same proportion to C, then A is equal to B. If not, let A be greater or less than B; if greater, then A has a greater proportion to C than B has to
a 8. C[a]; but it has not; therefore A is not greater than B; if less, then B has a greater proportion to C than it has to C; but it has not[a]; therefore A is not less than B; and, since neither greater nor less, it must be equal.

Again, if C have the same proportion to A that it has to B, A is equal to B; if not, C will have a greater proportion to the lesser magnitude, than it has to the greater[a]; but it has not; therefore A is not greater than B, or B is not greater than A; therefore A is equal to B. Wherefore, &c.

PROP. X. THEOR.

OF magnitudes having proportion to the same magnitude, that which has the greater proportion to some third is the greater magnitude; and that magnitude to which the same has a greater proportion is the lesser magnitude.

Of the magnitudes A, B, if A have a greater proportion to a third magnitude C, than B has to C, A is greater than B; for, if not, it will be either equal or less. If A is equal to
a 7. B, then they would have the same proportion to C[a]; but they have not; therefore A is not equal to B; neither is it less; for then B would have a greater proportion to C than it has to
b 8. C[b]; but it has not: Therefore, since A is neither equal nor less than B, it must be greater.

Again, if C have a greater proportion to B than it has to A, then B is less than A; if not, let it be equal or greater; if e-
c 9. qual, then C has the same proportion to B that it has to A[c]; but it has not; therefore B is not equal to A. If greater, then C will have a greater proportion to A than it has to B; but it has not; therefore, since B is not equal or greater than A, it must be less. Wherefore, &c.

PROP.

PROP. XI. THEOR.

PROPORTIONS that are the same to any third, are the same to one another.

Let A be to B, as C is to D, and C to D as E to F; then A will be to B as E to F.

For, let G, H, K, be any equimultiples of A, C, E; and L, M, N, any other equimultiples of B, D, F; now, because A is to B as C is to D; and G, H are equimultiples of A, C; and L, M any other equimultiples of B, D; if G is equal to L, H will be equal to M [a]; if greater, greater; and, if less, less: Likewise, because C is to D as E is to F; if H be equal to M, K will be equal to N [a]; if greater, greater; and, if less, less. Wherefore, if G be equal to L, K will be equal to N; for they are equal to H, M [b]; wherefore A is to B as E to F [a]. Wherefore, &c.

a Def. 5.

b Ax. 1. 1.

PROP. XII. THEOR.

IF any number of magnitudes be proportional, as one of the antecedents is to one of the consequents, so are all the antecedents to all the consequents.

Let the magnitudes be A, B, C, D, E, F; and, as A is to B, so is C to D, and E to F; then as A is to B, so are A, C, E, all the antecedents, to B, D, F, all the consequents. For, let let G, H, K be equimultiples of A, C, E; and L, M, N be any other equimultiples of B, D, F; then, because A is to B as C is to D, and C to D as E to F, and G, H, K equimultiples of A, C, E, and L, M, N any other equimultiples of B, D, F; if G be equal to L, H will be equal to M, and K to N [a]; if greater, greater; and, if less, less; wherefore, if G be equal to L; G, H, K, will be equal to L, M, N; if greater, greater; and, if less, less [b]; but G; G, H, K, are equimultiples of A; A, C, E, together; and L; L, M, N, equimultiples of B; B, D, F, together; such, that, if G be equal to L; G, H, K will be equal to L, M, N, together; wherefore A, C, E, together, are to B, D, F, together, as A is to B [a]. Wherefore, &c.

a Def. 5.

b 1.

Book V.

PROP. XIII. THEOR.

If the first is in the same proportion to the second, as the third to the fourth; and if the third has a greater proportion to the fourth, than the fifth to the sixth; then shall also the first have a greater proportion to the second, than the fifth to the sixth.

Let the first A have the same proportion to the second B, that the third C has to the fourth D; but let the third C have a greater proportion to the fourth D, than the fifth E to the sixth F; then the first A will have a greater proportion to the second B, than the fifth E has to the sixth F.

For, because C has a greater proportion to D, then E to F; let G, H be equimultiples of C, E; and K, L equimultiples
a Def. 7. of D, F; such, that, if G exceeds K, but H does not exceed L[a]; and let M be the same multiple of A that G is of C; and N the same multiple of B that K is of D; then, because G exceeds K,
b Def. 5. M will exceed N[b]; but G exceeds K, and H does not exceed L; and M exceeds N, and H does not exceed L; and M, H are equimultiples of A, E; and N, L, of B, F; wherefore A has a greater proportion to B, than E has to F[a]. Wherefore, &c.

PROP. XIV. THEOR.

If the first has the same proportion to the second, that the third has to the fourth; if the first be greater than the third, the second will be greater than the fourth; but, if the first be equal to the third, the second will be equal to the fourth; if the first is less than the third, the second will be less than the fourth.

Let the first A have the same proportion to the second B, that the third C has to the fourth D; if A is greater than C, then B is greater than D.

For, if A is greater than C, and B any third magnitude, A
a 8. has a greater proportion to B than C has to B[a]; but A is to B as C is to D; therefore C has to D a greater proportion than
b 13. C has to B[b]; therefore D is less than B[c]; that is, B is greater
c 10. than D. In the same manner it is proved, that, if A is equal to C, B is equal to D; and, if less, less. Wherefore, &c.

PROP. XV. THEOR.

Parts have the same proportion as their like multiples, if taken correspondently.

Let AB be the same multiple of C that DE is of F; then C will be to F as AB is to DE.

For, let AG, GH, HB be each equal to C; and DK, KL, LE, each equal to F; then AG, GH, HB are equal to one another[a]; and likewise DK, KL, LE equal to one another; therefore AG is to DK as GH is to KL, and as HB is to LE[b]; therefore AB is to DE as AG is to DK[c]; that is, as C to F. Wherefore, &c.

a Ax. 1. 1.
b 11.
c 12.

PROP. XVI. THEOR.

If four magnitudes of the same kind are proportional, they shall also be alternately proportional.

Let the four magnitudes A, B, C, D, be proportional, viz. as A is to B, so is C to D; they will likewise be proportional when taken alternately; that is, as A is to C, so is B to D; for, take E, F equimultiples of A, B; and G, H any equimultiples of C and D; then, because E is the same multiple of A that F is of B, and G the same multiple of C that H is of D, A is to B as E is to F[a]; but A is to B, as C is to D; therefore C is to D as E is to F[b]; and as C is to D, so is G to H[a]; therefore E is to F as G is to H[b]; therefore, since E, F, G, H are four magnitudes proportional, equimultiples of other four, A, B, C, D; therefore, if E is equal to G, F is equal to H; if greater, greater; and, if less, less[c]; wherefore A is to C as B is to D[d]; Wherefore, &c.

a 15.
b 11.
c 14.
d Def. 5.

PROP. XVII. THEOR.

If magnitudes compounded are proportional, they shall also be proportional when divided.

Let

Book. V.

Let the compounded magnitudes AB, BE, CD, DF, be proportional; that is, let AB be to BE as CD is to DF; these magnitudes shall be proportional when divided; that is, AE shall be to EB as CF is to FD.

For, take GH, HK, LM, MN, equimultiples of AE, EB, CF, FD, and KX NP, any equimultiples of EB, FD; now, because GH is the same multiple of AE that HK is of EB; and GH the same multiple of AE that LM is of CF; and LM the same multiple of CF that MN is of FD; therefore GK is the
a 1. same multiple of AB that GH is of AE [a]. But GH is the same multiple of AE that LM is of CF; therefore GK is the same
b 11. multiple of AB that LM is of CF. [b] But LM is the same multiple of CF, that MN is of FD; therefore LN is the same multiple of CD, that LM is of CF [a]; therefore GK is the same multiple of AB, that LN is of CD [b]. But HK, MN, are the same multiples of EB, FD; and KX, NP any other equimultiples of EB, FD; wherefore HX is the same multiple
d 2. of EB that MP is of FD [d]. But GK, LN are equimultiples of AB, CD; and XH, MP, any other equimultiples of EB, FD; if GK be equal to HX, LN will be equal to MP; take HK, MN, from both; then, if GH be equal to KX, LM will be equal to NP; if greater, greater; and, if less, less; wherefore AE is to
e Def. 5. EB as CF is to FD [e]. Wherefore, &c.

PROP. XVIII. THEOR.

If magnitudes divided be proportional, they shall also be proportional when compounded.

Let AE, EB, CF, FD, be the divided magnitudes, viz. as AE is to EB, so is CF to FD; they shall likewise be proportional when compounded, viz. as AB is to BE, so is CD to DF; if not, let AB be to BE as CD is to some magnitude, either greater or less than FD; first, to a less, as DG, viz. AB to BE,
a 17. as CD to DG; therefore AE is to EB as CG is to GD [a]; but
b Hyp. AE is to EB as CF is to FD [b]; therefore CG is to GD as CF
c 11. is to FD [c]; but CG is greater than CF; therefore DG is great-
d 14. er than FD [d]; but it is also less, which is impossible; therefore AB is not to BE as CD to DG. In the same manner it is proved, that AB is not to BE as CD to one greater than DF; therefore AB is to BE as CD is to DF. Wherefore, &c.

PROP. XIX. THEOR.

If the whole be to the whole, as a part taken from the one is to a part, taken from the other, then shall the residue of the one be to the residue of the other, as the whole is to the whole: and if four magnitudes be proportional, they shall be conversely proportional.

Let the whole AB be to the whole CD, as a part taken away AE, is to the part taken away CF; then the residue EB is to the residue FD as the whole AB is to the whole CD; for alternately as AB is to AE, so is CD to CF[a]; then BE is to AE as a 16.
DF is to FC[b]; and BE is to DF, as AE to CF[a]; but as AE is b 17.
to CF, so is AB to CD[c]; therefore EB is to the residue FD as c Hyp.
the whole AB is to the whole CD[d]: Again, if AB be to BE as d 11.
CD to DF, then they shall be conversely proportional; for AE e 17.
is to BE as CF is to FD[e]; and BE is to AE as DF is to CF[f]; f Cor. 4.
therefore as AB is to AE, so is CD to DF[g]; therefore the first g 18.
AB is to AE, its excess above the second, as CD, the third, is to DF, its excess above the fourth[h]. Wherefore, &c. h Def. 17.

PROP. XX. THEOR.

If there be three magnitudes, and others equal to them in number, which being taken two and two in each order, are in the same ratio; and if the first magnitude be equal to the third, then the fourth will be equal to the sixth; and, if the first be greater than the third, then the fourth will be greater than the sixth; and, if the first be less than the third, then the fourth will be less than the sixth.

Let A, B, C be three magnitudes, and D, E, F, others equal to them in number; which being two in two in each order, are in the same proportion, viz. A to B as D to E, and B to C as E to F; and if the first A be equal to the third C, then the fourth D shall be equal to the sixth F; if greater, greater; and, if less, less; for if A is equal to C, and B some other magnitude, A has the same proportion to B that C hath to B[a]; but A is to B a 7.
as D is to E[b]; therefore D hath the same proportion to E that b Hyp.
A has to B; but B is to C as E to F; and, inversely, C is to B as F is to E; therefore F has to E the same proportion that C

has

Book V. has to B; but A has the same proportion to B that C has to B; therefore D has the same proportion to E that F has to E; c 9. therefore D is equal to F[c]; if greater, greater; and, if less, less. Wherefore, &c.

PROP. XXI. THEOR.

If there be three magnitudes, and others equal to them in number, which, taken two and two in each order, are in the same ratio; and, if the proportion be perturbate; if the first magnitude be greater than the third, then the fourth will be greater than the sixth; and if the first be equal to the third, then the fourth will be equal to the sixth; if less, less.

Let the three magnitudes A, B, C, and others D, E, F, equal to them in number, be taken two and two in the same ratio, and if their analogy be perturbate, viz. as A is to B, so is E to F, and B to C as D to E; and if the first A be greater than the third C, then the fourth D will be greater than the sixth F; if equal, equal; and, if less, less.

For, if A is greater than C, A has a greater ratio to B than a 8. C has to B[a]; but A is to B as E is to F; therefore E has to F a greater ratio than C hath to B; and inversely as C is to B, so is E to D; therefore E has to F a greater ratio than E to D. But that magnitude to which the same has a greater ratio, is the b 10. lesser magnitude[b]; therefore F is less than D; that is, D is greater than F; if equal, equal; and, if less, less. Wherefore, &c.

PROP. XXII. THEOR.

If there be any number of magnitudes, and others equal to them in number, which, taken two and two, are in the same ratio; then they shall be in the same proportion by equality.

Let there be any number of magnitudes A, B, C, and others, D, E, F, equal to them in number, which, taken two and two in the same ratio, viz. A to B as D to E, and B to C as E to F; then they shall be in the same proportion by equality; that is, A to C, as D to F.

For, let G, H be equimultiples A, D, and K, L any equimultiples of B, E, and M, N any equimultiples of C, F; then, because

because A is to B as D to E, G is to K as H is to L [a]; but B is to C as E is to F; therefore K is to M as L to N [a]; wherefore, if G is equal to M [b], H will be equal to N; if greater, greater; and, if less, less; but G, M are equimultiples of A, C and H, N of D, F; wherefore A is to C as D to F [c]. Wherefore, &c.

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a 4.
b 20.
c Def. 5.

PROP. XXIII. THEOR.

If there be three magnitudes, and others equal to them in number, which, taken two and two, are in the same ratio; and if their analogy be perturbate, they shall be in the same proportion by equality.

Let there be three magnitudes A, B, C, and others D, E, F, equal to them in number, which, taken two and two, are in the same ratio; and if their analogy be perturbate, that is, as A is to B, so is E to F, and as B is to C, so is D to E; then they shall be in the same proportion by equality; that is, A is to C as D to F.

For, let G, H, L be equimultiples of A, B, D; and K, M, N, any equimultiples of C, E, F; then as A is to B, so is G to H [a]; and as E to F, so is M to N; but A is to B as E is to F; therefore G is to H as M to N [b]; and, because B is to C as D to E, H is to K as L to M; therefore, if G is equal to K, L is equal to N [c]; but G, K are equimultiples of A, C; and L, N of D, F; therefore A is to C [d] as D to F. Wherefore, &c.

a 15.
b 11.
c 21.
d Def. 5.

PROP. XXIV. THEOR.

If the first magnitude has the same proportion to the second that the third has to the fourth; and if the fifth has the same proportion to the second that the sixth has to the fourth; then the first, compounded with the fifth, shall have the same proportion to the second, that the third, compounded with the sixth, has to the fourth.

Let the first magnitude AB, have the same proportion to the second C, that the third DE has to the fourth F, and the fifth BG have the same proportion to the second C, that the sixth EH has to the fourth F.

For, because BG is to C as EH is to F; inversely, C is to BG, as F is to EH; but AB is to C as DE is to F; therefore AB is to BG [a], as DE is to EH; and AG is to GB as DH is to HE [b]; but as GB is to C, so is EH to F [c]; therefore AG is to C as DH is to F [a]. Wherefore, &c.

a 22.
b 18.
c Hyp.

PROP.

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PROP. XXV. THEOR.

If four magnitudes be proportional, the greatest and least will be greater than the other two.

Let four magnitudes AB, CD, E, F, be proportional, viz. AB to CD as E to F; of which let AB be the greatest, and F the least; then AB and F together, will be greater than CD and E; for, cut off AG equal to E, and CH to F; then AB is to CD as AG is to CH; therefore the remainder BG, will be to the remainder DH, as the whole AB is to the whole DC [a]; but AB is greater than CD; therefore GB is greater than HD; and, because AG is equal to E, and CH to F, then AG and F are equal to CH and E; but BG is greater than HD; therefore AB and F are greater than DC and E. Wherefore, &c.

a 19.

THE

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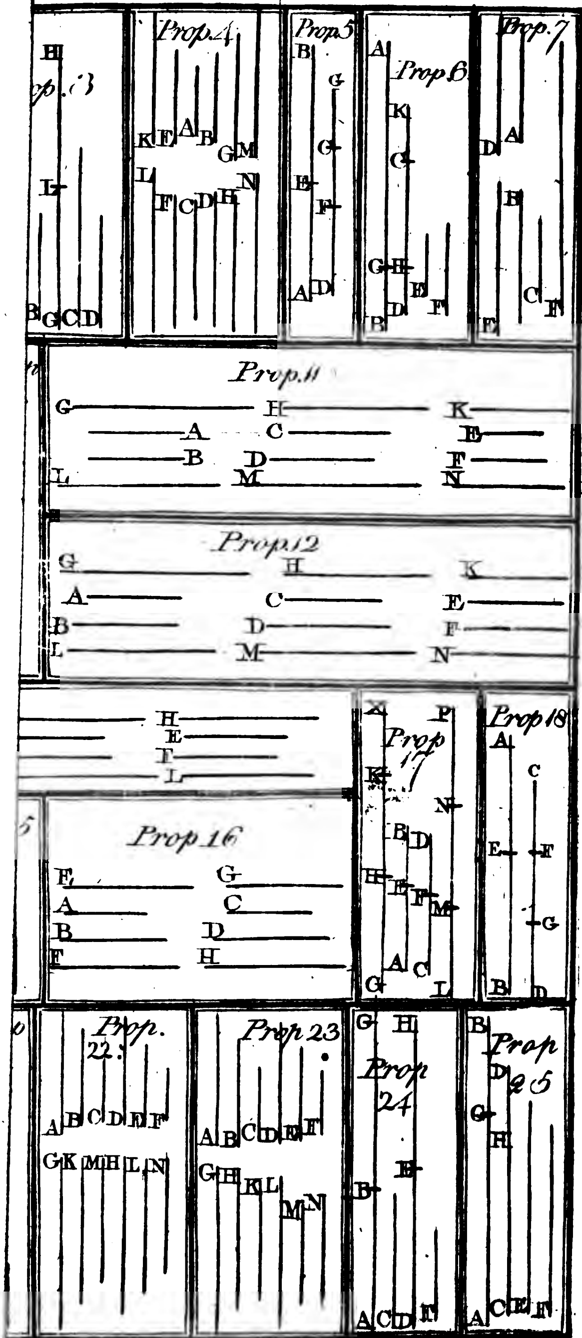
Prop. 3
Prop. 4
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Prop. 11
Prop. 12
Prop. 16
Prop 17
Prop 18
Prop. 22.
Prop 23
Prop 24
Prop 25

THE

ELEMENTS

OF

EUCLID.

BOOK VI.

DEFINITIONS.

I.

Similar right-lined figures are ſuch as have each of their ſeveral angles equal to one another, and the ſides about the equal angles proportional to each other.

II.

Figures are reciprocally proportional to each other, when the antecedent and conſequent terms of the ratio are in each figure.

III.

A right line is cut into extreme and mean ratio, when the whole is to the greater ſegment as the greater ſegment is to the leſſer.

IV.

The altitude of any figure, is a line drawn from the vertex perpendicular to the baſe.

V.

Ratio is ſaid to be compounded of ratios, when the ratio of the firſt term to the laſt is produced from the quantities of the ratios of the intermediate terms, either by multiplication, diviſion, or both.

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PROP. I. THEOR.

TRIANGLES and parallelograms that have the ſame altitude, are to each other as their baſes.

Let the triangles ABC, ACD, and the parallelograms EB, FD have the ſame altitude, they are to one another as their baſes; viz. as BC to CD; for, produce BC both ways to H, M; take BG, GH each equal to BC; and DK, KL, LM each equal to CD; and join AG, AH, AK, AL, AM; then the triangles ABC, ABG, AGH are equal to one
a 38. 1. another[a]; and ACD, ADK, AKL, ALM equal to one another[a]; then, becauſe HC is taken any multiple of BC, and CM any other multiple of CD, the triangle AHC is the ſame multiple of the triangle ABC that HC is of BC; and the triangle CAM the ſame multiple of CAD that CM is of CD; If HC be equal to CM, the triangle AHC will be equal to the triangle ACM; if greater, greater; and if leſs, leſs; therefore ABC
b Def. 5. 5. is to ACD as BC is to CD[b]; but the parallelogram EB is double
c 41. 1. the triangle ABC[c], and FD double ACD; therefore the parallelogram EB is to the parallelogram DF, as the triangle ABC is
d 15. 5. to the triangle ACD[d]; therefore the parallelograms EB, DF
e 11. 5. are to each other as their baſes BC, CD[e]. Wherefore, &c.

PROP. II. THEOR.

IF a right line be drawn parallel to one of the ſides of a triangle, it will cut the other ſides proportionally; and if a line cut the two ſides of a triangle proportionally, that right line ſhall be parallel to the other ſide of the triangle.

Let DE be drawn parallel to BC, one ſide of the triangle ABC; then AD will be to DB as AE is to EC; for join DC, BE, then
a 37. 1. the triangles BDE, DEC are equal[a]; and ADE is ſome other
b 7. 5. triangle; therefore BDE is to ADE as DEC is to ADE[b]: But
c 1. BDE is to ADE as BD is to AD[c]; and DEC is to ADE as
d 11. 5. EC is to AE[c]; therefore BD is to DA as CE is to EA[d]; and if BD is to DA as CE is to EA, then DE is parallel to BC.

For the ſame conſtruction remains: As BD is to DA, ſo is BDE to ADE[c]; and CE is to EA as DEC is to ADE; therefore the triangle BDE is to the triangle ADE as DEC is to
e 9. 5. ADE[d]; therefore the triangles BDE, CDE are equal[e]; there-
f 39. 1. fore DE is parallel to BC[f]. Wherefore, &c.

PROP.

PROP. III. THEOR.

If one angle of a triangle be bisected by a right line, which likewise cuts the opposite side, then the segments of that side will have the same proportion to one another that the other sides of the triangle have: And if the segments of that side have the same proportion to one another that the other sides of the triangle have, then a right line, drawn from the point of section to the vertex, will bisect the opposite angle.

Let there be a triangle ABC, and let one of its angles, as BAC, be bisected by the right line AD; then, as BD is to DC, so is BA to AC; for through C draw CE parallel to AD[a], and produce BA till it meet CE in the point E; then, because AC falls on the parallels DA, CE, the angle DAC is equal to the angle ACE[b]: But the angle BAD is equal to CAD[c]; therefore the angle BAD is equal to ACE[d]: But BAD is equal to AEC[b]; therefore AEC is equal to ACE[d]; therefore AE is equal to AC[e]: But BD is to DC as BA is to AE[f]; that is, to AC; and if BD is to DC as BA is to AC, then the right line DA bisects the angle BAC.

a 31. 1.
b 29. 1.
c Hyp.
d Ax. 1. 1.
e 6. 1.
f 2.

For the same construction remains; BD is to DC as BA is to AC[e]; and BD is to DC as BA is to AE[f]; therefore BA is to AC as BA is to AE[g]; therefore AE is equal to AC[h]; therefore the angles ACE, AEC are equal[i]: But ACE is equal to DAC[b]; therefore the angle AEC is equal to DAC[d]: But AEC is equal to BAD[b]; therefore BAD is equal to DAC[d]; therefore the angle BAC is bisected by the right line AD. Wherefore, &c.

g 11. 5.
h 9. 5.
i 5. 1.

PROP. IV. THEOR.

The sides about the equal angles of equiangular triangles are proportional; and the sides subtending the equal angles are homologous, or of like ratio.

Let the two equiangular triangles be ABC, DCE, viz. the angle ACB equal to the angle DEC; BAC to CDE; and ABC to DCE; then the sides that are about the equal angles are proportional, and the sides subtending the equal angles homologous, or of like ratio.

Let

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Let the sides BC, CE be placed in the same right line; then, because the two angles ABC, ACB are less than two right angles [a], and the angles ACB, DEC are equal, the angles ABC, DEC are less than two right ones: The right lines AB, DE being produced, will meet [b] in some point; which let be F; then, because the angle DCE is equal to ABC, DC is parallel to AB [c]: But the angle BAC is equal to CDE [d]; and BAC to ACD [e]; therefore the angles CDE, ACD are equal; therefore ED is parallel to AC [f]; therefore ACDF is a parallelogram, and FD is equal to AC, and AF to DC [g]; and, because AC is parallel to FE, BA is to AF as BC is to CE [h]: Alter. BA is to BC as AF or CD is to CE. Again, because [h] BC is to CE as FD or AC is to DE, alter. BC is to AC as CE to ED: But AB is to BC as CD to CE; and BC to AC as CE to ED; therefore AB is to AC as CD to DE [i]. Wherefore, &c.

a 17. 1.
b Cor. 17. 1.
c 28. 1.
d Hyp.
e 29. 1.
f 27. 1.
g 34. 1.
h 2.
i 22. 5.

PROP. V. and VI. THEOR.

If the sides of two triangles are proportional, or if one angle of the one be equal to one angle of the other, and the sides about the equal angles proportional, the triangles will be equiangular, and the angles which the homologous sides subtend will be equal.

Let there be two triangles ABC, DEF, having their sides proportional; viz. AB to BC as DE to EF; and as BC is to CA, so is EF to FD; and as BA to AC, so is ED to DF; or, if the angle BAC is equal to the angle EDF, and BA to AC as ED to DF; then the triangles ABC, EDF are equiangular, and the angle ABC equal to DEF; BCA to EFD, and BAC to EDF; that is, the angles which the homologous sides subtend are equal.

First, Let the triangles ABC, DEF have their sides proportional (fig. 1.); at the points E, F, with the right line EF, make the angle FEG equal to the angle ABC [a]; and EFG to ACB; then the remaining angles EGF, BAC will be equal [b]; and because the two triangles ABC, EFG are equiangular, AB is to BC as GE to EF [c]: But as AB is to BC so is DE to EF [d]; therefore DE is to EF as GE is to EF [e]; therefore DE is equal to GE [f]; for the same reason FG is equal to FD [f], and EF is common; therefore the triangles DEF, GEF are equal [g]; and the remaining angles of the one equal to the remaining angles of the other, each to each: But the triangle EGF is equiangular to the triangle ABC; therefore DEF is likewise equiangular to ABC.

a 23. 1.
b Cor. 32. 1.
c 4.
d Hyp.
e 11. 5.
f 9. 5.
g 8. 1.

Secondly,

Secondly, Let the angle BAC be equal to the angle EDF, (fig. 2.) and BA to AC as ED to DF; at the point D, with the right line DF, make the angle FDG equal to the angle EDF, or BAC; and the angle DFG equal to the angle ACB; then the remaining angles at G, B, are equal [h]; because BA is to AC as ED to DF; and likewise, as GD is to DF; therefore DG is equal to DE [f]. For the same reason EF is equal to FG; and the triangles DEF, DFG, equiangular; but DFG is equiangular to ABC; therefore DEF is likewise equiangular to ABC. Wherefore, &c,

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h cor. 32. 1.

f 9. 5.

PROP. VII. THEOR.

If there are two triangles, having one angle of the one equal to one angle of the other, and the sides about a second angle of the one proportional to the sides about the correspondent angle of the other, and the remainimg third angles, either both less, or both not less than right angles; then the triangles will be equiangular, and have these angles equal, about which the sides are proportional.

Let the two triangles ABC, DEF, have an angle BAC in in the one equal to the angle EDF in the other; and the sides about the angles ABC, DEF, proportional, viz. AB to BC, as DE to EF; and the other angles at C, F, either both less, or both not less than right angles; then the triangles ABC, DEF, are equiangular; the angle ABC equal to DEF, and ACB to DFE.

For, if the angle ABC be not equal to the angle DEF, let one of them, as ABC, be the greater, and make the angle ABG equal to the angle DEF [a]; then the remaining angles AGB, DFE [b], are equal, and the sides about the equal angles proportional [c], viz. AB to BG as DE to EF; but AB is to BC as DE to EF; therefore AB is to BG as AB is to BC [d]; therefore BG is equal to BC [e], and the angle BCG to BGC [f]; therefore BGC, BCG, are each less than a right angle; therefore DFE is less than a right angle [g]; but BGA, BGC, are equal to two right angles [h], and BGC is proved less than a right angle; therefore BGA is greater than a right angle; therefore DFE is likewise greater than a right angle [g], and less; which is impossible; therefore ABG is not equal to DEF; nor can any angle but ABC be equal to DEF. Wherefore, &c.

a 23. 1.
b cor. 32. 1.
c 4.
d 11. 5.
e 9. 5.
f 5. 1.
g Hyp.
h 13. 1.

PROP.

PROP. VIII. THEOR.

If a perpendicular be drawn in a right angled triangle, from the right angle to the base, then the triangles on each side of the perpendicular will be similar to the whole, and to one another.

Let ABC be a right angled triangle; and from the point A of the right angle BAC, let fall the right line AD perpendicular to the base BC; then the triangles ABD, ADC, are similar to ABC, and to one another.

For the right angles BAC, ADB, are equal; and the angle at B common to the two triangles ABC, ABD; therefore the
a cor. 32. 1. remaining third angles C and BAD are equal [a]; therefore BC is
b 4. to BA as BA is to BD [b]. Again, because the right angles BAC, ADC, are equal, and C common to both, the remaining third angles B, DAC, are equal [a]; therefore BC is to AC as AC is to DC [b]; they are likewise similar to one another; for the angles ADC, ADB, are each right angles, and the angle C equal to BAD, and B to DAC; therefore BD is to AD as AD is to DC; therefore the triangles ADB, ADC, are simi-
c Def. 1. lar to the whole [c], and to one another. Wherefore, &c.

Cor. Hence, in a right angled triangle, if a perpendicular is let fall from the right angle to the base, that perpendicular is a mean proportional to the segments of the base; and each of the sides containing the right angle is a mean proportional to the whole base, and that segment next to the side.

PROP. IX. PROB.

To cut off any part required from a given right line.

Let AB be a given right line, it is required to cut off any part of it, as one third.

From the point A draw any right line AC, making any angle with the line AB; assume any point D in the line AC;
a 3. 1. and make DE, EC, each equal AD [a]; join BC; and through
b 31. 1. D draw DF parallel to BC [b].

Then, because FD is parallel to BC, a side of the triangle
c 2. ABC, AF is to FB as AD is to DC [c]; but AD is one third part of AC; therefore AF is one third part of AB; which was required. Wherefore, &c.

PROP. X. PROB.

TO divide a given undivided right line, as another right line is divided.

Let the given undivided right line be AB, and the divided line AC, it is required to divide AB as AC is divided.

Let AC be any how divided in the points D, E; and making any angle with AB; join BC; and through the points D, E, draw DF, EG[a], parallel to BC; and through D draw DHK parallel to AB. [a] 31. 1.

Then, because FH, HB, are parallelograms, their opposite sides are equal[b]; and, because FD is parallel to GE, AF is to FG as AD is to DE[c]. Again, because HE is parallel to BC, DH is to HK as DE is to EC[c]; but DH is equal to FG; and HK to GB; therefore DE is to EC as FG is to GB; but AD is to DE as AF is to FG; wherefore the given undivided line AB is cut in the same proportion as AC: Which was required. [b] 34. 1. [c] 2.

PROP. XI. PROB.

TWO right lines being given, to find a third proportional.

Let AB, AC, be two given right lines, making any angle with each other, it is required to find a third proportional to them.

Produce AB, AC, to the points D, E; make BD equal to AC[a]; join the points B, C; through D draw DE parallel to BC[b]; then AB is to BD as AC is to CE[c]; but BD is equal to AC; therefore AB is to AC as AC is to CE. Wherefore, &c. [a] 3. 1. [b] 31. 1. [c] 2.

PROP. XII. PROB.

THREE right lines given, to find a fourth proportional.

Let A, B, C, be the three given right lines, it is required to find a fourth proportional to them.

Let

Book VI. Let DE, EF, be two right lines, making any angle EDF with each other; make DG equal to A [a]; GE equal to B; and DH equal to C. Join GH; and through E draw EF parallel to GH [b]; then DG is to GE as DH is to HF [c]; therefore HF is the fourth proportional required.

a 3. 1.
b 31. 1.
c 8.

PROP. XIII. PROB.

TO find a mean proportional to two given right lines.

Let the two given right lines AB, BC, be placed in one right line, as AC; upon which describe a semicircle ADC; at the point B, draw BD at right angles to AC [a]; join AD, DC; then BD is the mean proportional required.

a 11. 1.

For, because the angle ADC is a right angle [b], AB is to BD as BD is to BC [c]; therefore BD is a mean proportional to the given right lines AB, BC: Which was to be done.

b 31. 3.
c 8.

PROP. XIV. and XV. THEOR.

EQUAL parallelograms and triangles, having one angle of the one equal to one angle of the other, have the sides about the equal angles reciprocally proportional; and these parallelograms and triangles that have one angle of the one equal to one angle of the other, and the sides about the equal angles reciprocally proportional, are equal, viz. the parallelogram to the parallelogram, and triangle to the triangle.

Let AB, BC, be equal parallelograms, and FBD, EBG, equal triangles, having the angles at B equal; and let the sides DB, BE, be in one right line, and FB, BG, in another; then the sides DB, BE, and GB, BF, that are about the equal angles at B, are reciprocally proportional, that is, DB is to BE as GB to BF.

Let the equal parallelograms be AB, BC, and compleat the parallelogram EF; then, as the parallelogram AB is to EF, so is BC to FE [a]; but, as AB is to FE, so is the base DB to BE [b]; and, as BC is to FE, so is GB to BF [a]; therefore DB is to BE as BC is to FE [c].

a 7. 5.
b 1.
c 11. 5.

And, if DB is to BE as BG is to BF; then the parallelogram AB is equal to BC; for, as DB is to BE, so is AB to FE [b]; and, as GB is to BF, so is BC to FE; therefore AB is to FE as BC is to FE [c]; therefore AB is equal to BC [d].

d 9. 5.

Se-

Secondly, let FD, EG, be joined; then FDB, EBG, are the equal triangles, and FBE is any third magnitude; therefore FDB is to FBE as EBG is to FBE; but the triangle FDB is to FBE as DB is to BE [b]; and EBG is to FBE as GB is to BF [b]; therefore DB is to BE as GB is to BF [c].

b 1.
c 11. 5.

And, if DB is to BE as GB is to BF, the triangle FDB is equal to EBG: For, as DB is to BE, so is the triangle FDB to FBE; and, as GB is to BF, so is the triangle EBG to FBE; therefore FDB is to FBE as EBG is to FBE; therefore the triangle FDB is equal to EBG [d]: Wherefore equal parallelograms and triangles, &c.

d 9. 5.

PROP. XVI. THEOR.

If four right lines are proportional, the rectangle contained under the extremes is equal to the rectangle under the means; and, if the rectangle contained under the extremes be equal to the rectangle contained under the means, then the four right lines are proportional.

Let the four right lines AB, CD, E, F, be proportional, so that AB be to CD as E is to F; then the rectangle under AB, F, is equal to the rectangle under CD, E: For, draw AG equal to F, and at right angles to AB [a], and CH equal to E, and at right angles to CD; and compleat the rectangles GB, HD: Then, because AB is to CD as CH is to AG [b], the rectangle BG is equal to HD [c], and if GB is equal to HD, AB is to CD as CH is to AG, that is, as E to F; for the angles at C, A, are equal, being each right ones. Wherefore, &c.

a 11. 1.
b 7. 5.
c 14.

PROP. XVII. THEOR.

If three right lines are proportional, the rectangle contained under the extremes is equal to the square of the mean; and, if the rectangle under the extremes be equal to the square of the mean, then the three right lines are proportional.

Let the three right lines A, B, C, be proportional, viz. as A is to B so is B to C; then the rectangle under A, C, is equal to the square of B: For, make D equal to B, and compleat the rectangles under A, C, and B, D; then, because AC, BD, are two rectangles, and A is to B as D is to C [a], AC is equal to BD [b]; and, because AC is equal to BD, A is to B as

a 7. 5.
b 14.

Book VI. D is to C; but B is equal to D; therefore the rectangle under A, C, is equal to the square of B. Wherefore, &c.

PROP. XVIII. PROB.

Upon a given right line to describe a right lined figure similar and similarly situated to a right lined figure given.

Let AB be the given right line, and CDEFG the right lined figure given; it is required upon AB to describe a figure similar and similarly situated to CDEFG. Join DG, DF; and, at the points A, B, of the right line AB, make the angles BAH,
a 23. 1. ABH, equal to the angles C and CDG [a], each to each; then
b cor. 32. 1. the remaining angles AHB, CGD, will be equal [b], and the
c 4. sides about the equal angle proportional [c], that is, AB to BH as CD to DG; and AH to HB as CG to GD. Again, at the points H, B, with the right line BH, make the angles BHK, HBK, equal to DGF, GDF, each to each; then the remaining third angles HKB, GFD, are equal [b], and the triangles HKB, FDG, equiangular, and the sides about the equal angles proportional. Again, make the angles BKL, KBL, equal to the angles DFE, FDE, each to each; then the remaining third angles at L, E, will be equal [b], and the sides about the equal angles proportional [c]; but all the triangles in the figure ABLKH are proved similar to all the triangles in the figure CDEFG; and, because the angles AHB, BHK, are proved equal to the two angles CGD, DGF, each to each, the whole angle AHK is equal to the angle CGF, and the sides about the equal angles proportional; for AH is to HB as CG to GD; and KH to HB as FG to GD; therefore, by equality, AH is to HK as CG is to GF. For the same reason, HK is to KL as GF to FE; and KL to LB as FE is to ED; therefore the figure ABLKH is
d def. 1. similar to CDEFG [d]. Wherefore, &c.

PROP. XIX. THEOR.

Similar triangles are to one another in the duplicate ratio of their homologous sides.

Let ABC, DEF, be similar triangles having the angles at B and E equal; and AB, to BC, as DE to EF, and BC the side homologous to EF; then the triangle ABC to the triangle DEF has a duplicate ratio that BC has to EF.

For,

For, take BG a third proportional to BC, EF [a], that is, BC to EF as EF to BG. Join AG; then, because AB is to BC as DE to EF, alter. as AB is to DE so is BC to EF; but BC is to EF as EF is to BG; therefore AB is to DE as EF is to BG [b]; that is, the sides about the equal angles B, E, of the triangles DEF, ABG, are reciprocally proportional; therefore equal to one another [c]; and, because BC is to EF as EF is to BG, BC has to BG a duplicate ratio of what it has to EF [d]; and, as BC is to BG so is the triangle ABC to the triangle ABG [e]; therefore the triangle ABC has to the triangle ABG a duplicate ratio of what BC has to EF [b]; but the triangle ABG is equal to DEF; therefore ABC is to DEF in the duplicate ratio of BC to EF. Wherefore similar triangles, &c.

a 11.
b 11. 5.
c 14.
d def. 10. 5.
e 1.

Cor. Hence, if three right lines be proportional, as the first is to the third, so is a triangle described on the first, to a similar one described on the second.

PROP. XX. THEOR.

Similar polygons can be divided into an equal number of similar triangles, each homologous to the whole; and polygon is to polygon in the duplicate ratio of one homologous side to the other.

Let ABCDE, FGHKL, be similar polygons, and AB, FG, two homologous sides; join BE, EC, GL, LH; then the number of triangles in the polygon ABCDE, are equal to the number of triangles in the polygon FGHKL, similar to one another, and homologous to the whole; and the polygon ABCDE will be to the polygon FGHKL in the duplicate ratio of the side AB to FG.

For, because the polygon ABCDE is similar to FGHKL, the angle BAE is equal to GFL; and BA is to AE as GF to FL [a]; and the angle ABE equal to FGL [b]; and AB to BE as FG to GL; but the whole angle ABC is equal to FGH, and a part ABE equal to FGL; therefore the remainder EBC is equal to LGH, and EB to BC as LG is to GH; but the angle BCD is equal to GHK; and a part BCE to a part GHL [b]; therefore the remainder ECD is equal to LHK, and the sides about the equal angles proportional.

a 4.
b 6.

Now, because the triangle ABE is equiangular to the triangle FGL, and the sides about the equal angles proportional, the two triangles are similar, and are to one another in the duplicate

ratio

ratio of AB to FG [c], or of EB to LG; but EBC is likewise similar to LGH, and are to one another in the duplicate ratio of EB to LG, or of EC to LH; and, for the same reason, ECD is to LHK in the duplicate ratio of CE to LH; therefore the triangles in the polygon ABCDE are equal in number to the triangles in the polygon FGHKL, and similar to one another; therefore, because the triangle ABE is to the triangle FGL in the duplicate ratio of BE to GL; and the triangle EBC to the triangle LGH, in the duplicate ratio of BE to GL; therefore the triangles ABE, EBC, are to FGL, LGH, as EBC is to LGH [d]. For the same reason, EBC, ECD, are to LGH, LHK, as EBC is to LGH: Therefore all the antecedents ABE, EBC, ECD, are to all the consequents FGL, LHG, LHK, as ABE is to FGL [e]; that is, in the duplicate ratio of AB to FG [c]; and polygon to polygon in the duplicate ratio of one homologous side to another. Wherefore, &c.

c 19.
d 11. 5.
e 12. 5.

Cor. Hence, if three right lines are proportional, the polygon described on the first is to the similar polygon described on the second as the first is to the third; for, if X be taken a third proportional to any two right lines, AB, FG; then AB is to X in a duplicate ratio of AB to FG; that is, any similar figures described on AB, FG, are to one another in the duplicate ratio of AB to FG.

PROP. XXI. THEOR.

Figures that are similar to the same right lined figures are also similar to one another.

Let each of the right lined figures A, B, be similar to the right lined figure C; then the right lined figure A will be similar to the right lined figure B.

For, because the right lined figure A is similar to C, it is equiangular to it [a]; and the sides about the equal angles proportional. For the same reason, B is equiangular to C, and the sides about the equal angles proportional; therefore each of the figures A, B, are equiangular to C; and therefore equiangular to one another [b], and the sides about the equal angles proportional [c]; wherefore A is similar to B. Wherefore, &c.

a def. 1.
b ax. 1. 1.
c 11. 5.

PROP.

PROP. XXII. THEOR.

If four right lines are proportional, the right lined figures similar, and similarly described upon them, are proportional; and, if similar right lined figures similarly described upon right lines be proportional, the right lines shall also be proportional.

Let four right lines AB, CD, EF, GH, be proportional, viz. as AB is to CD so is EF to GH; on AB, CD, let the similar figures KAB, LCD, be similarly described[a]; and upon EF, GH, let MF, NH, be described similar to one another; then KAB will be to LCD as MF is to NH. a 18.

For, to AB, CD, take X a third proportional[b], and O, a third proportional to EF, GH. Now, because AB is to CD as EF is to GH, and CD is to X as GH is to O, then AB is to X as EF is to O[c]; but, as AB is to X, so is the right lined figure KAB to the similar figure LCD[d]; and, as EF is to O, so is MF to NH[d]; therefore, as the right lined figure KAB is to the similar figure LCD, so is the right lined figure MF to the similar figure NH[e]; and, if KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH. b 11. c 22. 5. d cor. 20. e 11. 5.

For, if not, let AB be to CD as EF is to PR[f]; upon PR describe a figure SR similar to MF or NH; then KAB is to LCD as MF is to SR, and as MF is to NH; therefore SR, NH, have the same proportion to MF; therefore SR is equal to NH[g], and also similar to it; therefore PR is equal to GH; therefore AB is to CD as EF is to GH. Wherefore, &c. f 12. g 9. 5.

PROP. XXIII. THEOR.

Equiangular parallelograms have the proportion to one another that is compounded of their sides.

Let AC, CF, be equiangular parallelograms, having the angle BCD equal to the angle ECG; then the parallelogram AC, to the parallelogram CF, is in the proportion compounded of their sides, viz. of BC to CG, and DC to CE; for, place BC in a right line with CG, and DC in a right line with CE[a], and compleat the parallelogram DG; then, as BC is to CG, so let K be to L; and, as DC is to CE, so let L be to M[b]; but the ratio of K to M is compounded of the ratios of K to L, and L to M[c]; therefore the ratio of K to M is that compounded of BC to CG, and DC to CE; but BC is to CG as AC is to DG[d]; and DC is to CE as DG is to CF[d]; but BC is to CG as K to L, and DC to CE as L to M; therefore a 14. 1. b 12. c def. 5. d 1.

AC

Book VI. AC is to CF as K to M [c]; that is, as BC to CG, and DC to CE. Wherefore, &c.

c 22. 5.

PROP. XXIV. THEOR.

In every parallelogram the parallelograms that are about the diameter are similar to the whole, and also to one another.

In the parallelogram ABCD the parallelograms GE, KH, are similar to the whole ABCD, and likewise to one another. For, in the triangles ADC, AGF, the angle AGF is equal to the angle ADC [a], and the angle AFG to ACD [a]; and the angle GAF common to both; therefore the triangles AGF, ADC, are equiangular. For the same reason, the triangles AEF, ACB, are equiangular, and the sides about the equiangles proportional [b]. Again, in the triangles AGF, FKC, the angle GAF is equal to the angle KFG [a], and AGF equal to FKC; for each are equal to the angle ADC [a], and AFG to FCK; therefore the triangles AGF, FKC, are equiangular, and the sides about the equal angles proportional [b]. For the same reason, AEF is equiangular to FCH; and the sides about the equal angles proportional. Then, because the two angles KFC, HFC, are equal to the two angles GAF, EAF; that is, the whole angle KFH equal to the whole angle GAE, and the angle C common to both; and the angles at K, H, equal to the angles at D, B, each to each; the parallelogram KH is equiangular to DB; for the same reason GE is equiangular to DB; therefore KH is equiangular to GE; and the sides about the equal angles proportional: For, because the angles GAE, KFH, are equal, GA is to AE as KF is to FH; but KF is to FH as DA is to AB, for each are proportional to AF, FC [c]. For the same reason, the sides about the other angles are likewise proportional; therefore the parallelogram DB is similar to KH; but GE is likewise similar to KH; therefore GE is similar to DB. Wherefore, &c.

a 29. 1.

b 4.

c 11. 5.

PROP. XXV. PROB.

To describe a figure similar to a given right lined figure, and equal to another given right lined figure.

Let ABC and D, be two given right lined figures; it is required to describe a right lined figure similar to ABC, and equal to D. On

On the side BC, of the given figure ABC, make a parallelogram, BE, equal to it [a]; and on the side CE make the parallelogram CM equal to the right lined figure D [b]; and the angle FCE equal to the angle CBL [b]; then BC, CF, as also LE, EM, will be right lines [c]. Find GH a mean proportional to BC, CF [d]; and on GH describe the right lined figure KGH similar and alike situate to ABC [e]; then, because BC is to GH as GH is to CF; and, as BC is to CF, so is the right lined figure ABC to the right lined figure KGH [f]; but, as BC is to CF, so is the parallelogram BE to EF [g]; therefore, as the right lined figure ABC is to the right lined figure KGH, so is the parallelogram BE to the parallelogram EF [h]; altern. as ABC is to BE, so is KGH to EF: But the right lined figure ABC is equal to the parallelogram BE; therefore the right lined figure KGH is equal to the parallelogram FE [i]; but FE is equal to the right lined figure D; therefore the right lined figure KGH is equal to D: Which was required.

a 42. 1.
b 44. 1. & 29. 1.
c 14. 1.
d 13.
e 18.
f Cor. 20.
g 1.
h 11. 5.
i 14. 5.

PROP. XXVI. THEOR.

If, in a parallelogram, be constitute another parallelogram similar to the whole, and alike situate, and having an angle common with it, they shall be about the same diameter.

Let the parallelogram AF be constitute in the parallelogram ABCD, similar to it, and alike situate, having the angle DAB common to both; then the parallelograms ABCD and AF are about the same diameter AC.

For, if not, let AHC be the diameter of the parallelogram BD; and produce GF to H; draw HK parallel to AD or BC; then, because the parallelograms ABCD, KG, are about the same diameter, they will be similar to one another [a]; and DA to AB as GA to KA [b]; but, because the parallelograms ABCD, GE, are likewise similar [c], DA is to AB as GA is to AE; therefore, as GA is to AE so is GA to AK [d]; therefore AE is equal to AK [e], the greater to the less, which is impossible; therefore the parallelograms AH, ABCD, are not about the same diameter AHC; therefore no other but AF can be about the same diameter with ABCD. Wherefore, &c.

a 24.
b Def. 1.
c Hyp.
d 11. 5.
e 9. 5.

PROP.

PROP. XXVII. THEOR.

Of all parallelograms applied to the same right line, and wanting in figure by parallelograms similar and alike situate to that described on half the line, the greatest is that which is applied to the half line, and similar to the defect.

Let AB be a right line bisected in the point C; and let the parallelogram AD be applied to the right line AB, wanting in figure by the parallelogram CE, similar and alike situate to that described on half the line AB; then AD is greater than a parallelogram applied to any other part of the right line AB, wanting in figure by a parallelogram similar and alike situate to CE. For, let the parallelogram AF be applied to the right line AB, wanting in figure by the parallelogram HK, similar and alike situate to CE; then the parallelogram AD is greater than AF. For, because the parallelogram CE is similar to HK, they will stand about the same
a 26. diameter [a]. Let DB, that diameter, be drawn, and the figure
b 43. 1. described; then the parallelograms CF, FE, are equal [b]; add HK, which is common to both; then the whole CH is equal to
c 36. 1. the whole KE; but CH is equal to GC [c]; add CF, which is common; then the whole AF is equal to the gnomon EKN; but the parallelogram CE is greater than the gnomon EKN; therefore CE, that is, AD, is greater than AF. Wherefore, &c.

PROP. XXVIII. PROB.

Upon a given right line to apply a parallelogram equal to a given right lined figure, and deficient by a parallelogram similar to a given parallelogram; but the right lined figure to which the parallelogram is to be made equal, must not be greater than that described on half the line, as the defect must be similar.

It is required, upon the given right line AB, to apply a parallelogram equal to the right lined figure C, and deficient by a parallelogram similar to D; and the right lined figure C not greater than the parallelogram described on half the line AB, which is similar to D. For, bisect AB in E, and on EB de-
a 28. scribe a parallelogram EF similar and alike situate to D [a], and compleat the parallelogram AG.

Now, AG is either equal or greater than C; if equal, what was required is done. If not, make the parallelogram KLMN

similar

similar and alike situate to D [a], and equal to the excess by which EF exceeds C [b]; then EF is equal to C, and KLMN together; therefore KLMN is less than EF; and, because they are similar, the side GF is greater than LM, and GE than LK; make GO equal to LM, and GX to LK; and compleat the parallelogram GP, which will be similar to, and about the same diameter with EF [c]; let this diameter GB be drawn, and produce XP to R, and OP to S; then TS will be equal to C, and wanting in figure by SR, which is similar to D [d]: Which was to be done.

Book VI.

a 18. b 25. c 26. d 27.

PROP. XXIX. PROB.

TO apply a parallelogram upon a given right line, equal to a given right lined figure, exceeding by a parallelogram similar to another given parallelogram.

Upon the given right line AB, it is required to apply a parallelogram equal to the given right lined figure C, exceeding by a parallelogram similar to the given parallelogram D.

Bisect AB in E; upon EB describe the parallelogram EL, similar and alike situate to D [a]; and the parallelogram GH equal to C and EL together [b], and similar and alike situate to D [a]; let KH be a side homologous to FL, and KG to FE; then, because the parallelogram GH is greater than the parallelogram EL, the right line KH is greater than FL, and KG than FE: Produce FL and FE to M and N, so that FM be equal to KH, and FN to KG; compleat the parallelogram NM; then MN is equal and similar to GH; but GH is similar to EL; therefore MN is similar to EL [c]; therefore EL is about the same diameter with MN [d]; let FX, their diameter, be drawn, and describe the figure: Then, since GH is equal to EL and C together, as also to MN; therefore MN is equal to EL and C together. Take KL, which is common, from both; then the gnomon MPE is equal to C; and, because the parallelograms AN, NB, are equal [e], AN is equal to LO [f]; and, if BX be added, AX is equal to the gnomon MPE; therefore AX is equal to C. Wherefore, &c.

a 18. b 25. c 21. d 26. e 36. 1. f 43. and Ax. 1. 1.

PROP. XXX. PROB.

TO cut a given right line into extreme and mean ratio.

Book VI. It is required to cut the given right line AB into extreme and mean ratio.

a 46. 1. Upon AB describe the square BC[a], and to AC apply the parallelogram CD, equal to the square BC, exceeding by the figure AD[b], similar to BC; but BC is a square; therefore AD is also a square. From the equal parallelograms BC, CD, take away the common parallelogram CE; then the remainder BF will be equal to AD; but BF is equal to AD; therefore FE is to ED as AE is to EB[c]; that is, AB is to AE as AE is to EB: Or, let AB be cut in E, so that the rectangle under AB, BE, be equal to the square of AE[d]. Wherefore, &c.

b 29.

c 14.

d 11. 2.

PROP. XXXI. THEOR.

IN every right angled triangle, any figure described upon the side subtending the right angle, is equal to the two similar figures described upon the sides containing the right angle.

Let ABC be the right angled triangle, the figure described on BC, subtending the right angle, is equal to the two similar figures described on BA, AC; for, from the point A, let fall the perpendicular AD; then the triangle ABC is divided into the two similar triangles ADB, ADC; then, because the triangle ABC is similar to the triangle ABD, CB is to BA as BA is to BD[a], and CB is to BD as the figure described on CB is to the similar figure described on BA[b]. For the same reason, as BC is to CD, so is the figure described on BC to the similar one described on AC: Wherefore, as BC is to BD, and DC together, so is the figure described on BC to the two similar figures described on BA, AC[c], together; but BC is equal to BD, and DC together; therefore the figure described on BC is equal to the two similar figures described on BA, AC. Wherefore, &c.

a 8.

b Cor. 20.

c 24. 5.

PROP. XXXII. THEOR.

IF two triangles having two sides proportional to two sides, be so compounded or set together at one angle, that their homologous sides be parallel; then the other sides of these triangles will be in one right line.

If the triangles ABC, DCE be so placed at the point C, that the side DE be parallel to AC, and DC to AB; then BCE will be a right line.

For,

For, because the homologous sides AB, DC, are parallel, and AC falls upon them, the alternate angles BAC, ACD, are equal [a]; for the same reason, CDE is equal to ACD; then, since the two triangles BAC, CDE, have the angles at A and D equal, and the sides about them proportional, viz. BA to AC as CD to DE, the triangles are equiangular [b], viz. the angle ABC equal to DCE, and ACB to DEC; but the angle ACD is proved equal to BAC; therefore, the whole angle ACE is equal to the two angles ABC, BAC. Add the common angle ACB to both, then the two angles ACE, ACB, are equal to the three angles ABC, ACB, BAC; that is, equal to two right angles [c]; therefore BCE is one right line [d]. Wherefore, &c.

a 29. 1.
b 6.
c 32. 1.
d 14. 1.

PROP. XXXIII. THEOR.

In equal circles, the angles are in the same proportion to one another as the circumferences on which they stand, whether the angles be at the centres or the circumference; so likewise are sectors, as being at the centres.

Let ABC, DEF, be equal circles, and the angles BGC, EHF, at the centres G, H; and BAC, EDF, angles at their circumferences; then the angle BGC will be to the angle EHF as the circumference BC is to the circumference EF; and likewise the angle BAC to the angle EDF, and the sector BGC to the sector EHF, as the circumference BC to EF. For, take any number of circumferences, as CK, KL, each equal to BC; and any number of circumferences, as FM, MN, each equal to EF; join GK, GL, HM, HN; then, because the circumferences BC, CK, KL, are equal, the angles BGC, CGK, KGL, are likewise equal [a]; therefore, BL is the same multiple of BC, that the angle BGL is of the angle BGC; for the same reason, EN is the same multiple of EF, that EHN is of EHF; therefore, if the circumference BL be equal to the circumference EN, the angle BGL is equal to the angle EHN, if greater greater, and if less less; therefore, as BC is to EF, so is BGC to EHF [b]; and so is BAC to EDF [c]. Again, as the circumference BC is to EF, so is the sector BGC to the sector EHF; for, join BC, CK, EF, FM, and assume the points X, O, in the circumference BC, CK, and join BX, XC, CO, OK; then, because BG, GC, are equal to CG, GK, and contain equal angles, the base BC is equal to the base CK [d], and the triangles equal; and, because the right line BC is equal

a 27. 3.
b def. 5. 5.
c 15. 5. and 20. 3.
d 4. 1.

Book VI. qual to the right line CK, the circumference BXC is equal to the circumference COK [e]: therefore the angle BXC is equal to the angle COK: Therefore, the segments BXC, COK, are equal and similar [f]; but the triangles BGC, CGK, are equal; therefore the whole sector BGCX is equal to the whole sector CGKO; in the same manner the sectors EHF, FHM, are proved equal; therefore BK is the same multiple of BC, that BGK is of BGCX; and EM the same multiple of EF that the sector EHMF is of the sector EHF; therefore, if the circumference BK is equal to the circumference EM, the sector BGK is equal to the sector EHM, if greater, greater, and if less, less; therefore, as BC is to EF, so is the sector BGCX to the sector EHF [b]. Wherefore, &c.

e 28. 3

f 24. 3. and def. 11. 3.

b def. 5. 5.

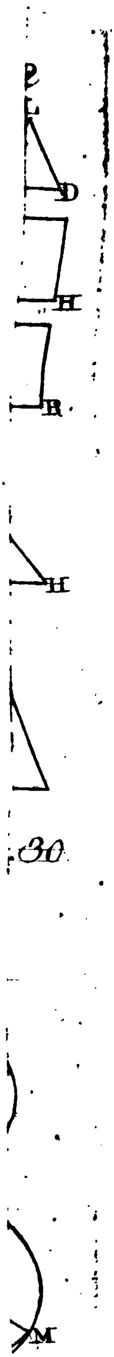
Cor. I. An angle at the centre of a circle is to four right angles, as the arch on which it stands is to the whole circumference; for, as the angle BAC is to a right angle, so is the arch BC to a quadrant, the consequents quadrupled; then BAC is to four right angles as BC is to the whole circumference.

Cor. II. The arches IL, BC, of unequal circles, which subtend equal angles, whether at the centres or circumferences, are similar: For IL is to the whole circumference ILE as the angle IAL, or BAC, is to four right angles; and so is the arch BC to the whole circumference BCF; therefore the arches IL, BC, are similar.

Cor. III. Two semidiameters AB, AC, cut off similar arches IL, BC, from concentric circumferences.

ERRATA for the first six Books.

B. II. pr. 3. l. 5. for AC, CD, r. AF, CE; pr. 10. l. 15. for DEF r. DFE.
B. III. pr. 23. note, for def. 11. r. def. 10; pr. 28. for def. 14. r. def. 4.
pr. 37. cor. 1. l 2. for point A. r. point D.
B IV. pr. 15 p. 64. l. 8. for CG, GF r. CG, GD.
B. V. pr. 9. l. 5. for it has to C, r. A has to C; the same pr. 10. l. 6.
B. VI. pr. 12. p. 88. l. 1. for EF r. DF. pr. 29. l. 17. for KL r. EL.



THE

ELEMENTS

OF

EUCLID.

BOOK XI.

DEFINITIONS.

I.

A Solid, is that which hath length, breadth, and thickneſs. Book XI.

II.

The term of a ſolid, is a ſuperficies.

III.

A right line is perpendicular to a plain, when it makes right angles with all the lines that touch it, and are drawn in the ſame plain.

IV.

A plain is perpendicular to a plain, when all the right lines in one plain, drawn at right angles to the common ſection of the two plains, are at right angles to the other plain.

V.

The inclination of a right line to a plain, is the acute angle contained under that line, and another right one drawn in the plain, from that end of the inclining line, which is in the plain, to the point where a right line falls from the other end of the inclining line, perpendicular to the plain.

VI.

VI.

The inclination of a plain to a plain, is the acute angle con-tained by the right lines drawn in both plains, to the sam point of their common section, and making right angles wi it.

VII.

Plains are inclined similarly, when their angles of inclinatio are equal.

VIII.

Parallel plains are such, which being produced, never meet.

IX.

Similar solid figures are such as are contained under an qual number of similar plains.

X.

Equal and similar solid figures are such as are contained by a equal number of similar and equal plains.

XI.

A solid angle is the inclination of more than two right lin that meet in one point, but are not in the same superficies.

XII.

A pyramid is a solid figure, contained by more than tw plains set upon one plain, and meeting at one point in t vertex.

XIII.

A prism is a solid figure contained by plains, whereof the tw opposite are equal, similar, and parallel; and the other pa rallelograms.

XIV.

A sphere is a solid figure, described by a semicircle revolvin about its diameter, which remains fixed in the same pos tion.

XV.

The axis of a sphere is that fixed right line about which th semicircle revolves.

XVI.

The centre of a sphere is the same with that of the semi circle.

XVII.

The diameter of a sphere is a right line drawn through th centre, and terminated on either side by the superficies of th sphere.

XVIII.

A cone is a solid figure described by a right angled triangle revolving about one of the sides, containing the right angle, remaining fixed. If the fixed right line be equal to the o-ther side containing the right angle, then it is a rectangular

cone;

cone; if less, an obtuse angled cone; and if greater, an acute angled cone. Book XI.

XIX.

The axis of a cone is that fixed right line about which the triangle is moved.

XX.

The base of a cone is the circle described by the revolving line.

XXI.

A cylinder is, a figure described by a right angled parallelogram, revolving about one of the sides, containing the right angle, remaining fixed.

XXII.

The axis of a cylinder is that fixed right line about which the parallelogram is moved.

XXIII.

The bases of a cylinder are the circles described by the motion of the two opposite sides of the parallelogram.

XXIV.

Similar cones and cylinders are such, whose axes and diameters of their bases are proportional.

XXV.

A cube is a solid figure contained by six equal squares.

XXVI.

A tetrahedron is a solid figure contained by four equal equilateral triangles.

XXVII.

An octahedron is a solid figure contained by eight equilateral triangles.

XXVIII.

A dodecahedron is a solid figure contained by twelve equal equilateral and equiangular pentagons.

XXIX.

An icosahedron is a solid figure contained by twenty equal equilateral triangles.

XXX.

A parallelopipedon is a solid figure contained by six quadrilateral figures, whereof those that are opposite are parallel.

PROP. I. THEOR.

ONE part of a right line cannot be in a plain superficies, and another part above it.

For,

Book XI.

For, if possible, let the part AB of the right line ABC be in a plain superficies, and the part BC above the same; there will be some right line in that plain which will make one right line with AB, which let be DB; then the two right lines ABC, ABD, will have one common segment AB; which is impossible[a]. Wherefore, &c.

a Cor. 14. 1.

PROP. II. THEOR.

If two right lines cut each other, they are both in one plain, and every triangle is in one plain.

Let the two right lines AB, CD, cut each other in the point E, they are both in one plain.

For, take any points F, G, in the right lines AB, CD, and join CB; then the right lines AB, CD, are in one plain, and the triangle ECB is in one plain. For the parts DF, AG, cannot be in one plain, and FC, GB, above it[a]; therefore DC, AB, are in one plain; and, because the points B, C, are in one plain[b], therefore the triangle ECB is in one plain. Wherefore, &c.

a 1.
b def, 4. 11.

PROP. III. THEOR.

If two plains cut each other, their common section will be a right line.

Let the two plains be AB, BC, cutting each other; and let BD be their common section; then BD is a right line. For, if not, let the right line BED be drawn in the plain CB, and BFD in the plain BA; then two right lines bound a figure; which cannot be[a]. Wherefore, &c.

a Ax. 10. 1.

PROP. IV. THEOR.

If a right line stand in the common section of two right lines, cutting one another, and at right angles to the same; then it shall be at right angles to the plain passing through these lines.

Let

Let the right line EF ſtand in the common ſection at right angles to the two right lines AB, CD; then EF is likewiſe at right angles to the plain paſſing through AB, CD. For, take the right lines AE, ED, EB, EC, equal to one another, and join AD, BC; through E draw GEH to the points G, H, in the right lines AD, BC; join FD, FC, FA, FB, FG, FH; then, becauſe the two ſides AE, ED, are equal to the two ſides BE, EC; and the angles AED, BEC[a], equal; the baſe AD is equal to the baſe BC; and the remaining angles EAD, EDA, equal to the angles EBC, ECB, each to each[b]; but the two angles AEG, EAG, in the triangle AGE, are equal to the two angles BEH, EBH, in the triangle HBE; and a ſide AE in the one equal to a ſide EB in the other; the remaining ſides AG, GE, in the one are equal to BH, HE, in the other, each to each[c]; but, becauſe AE is equal to EB, and FE common, and at right angles to AB, the baſe AF is equal to the baſe FB[b]. For the ſame reaſon, FD is equal to FC; but FA, AD, are proved equal to FB, BC, each to each; and the baſe FD equal to FC; therefore the angle FAD is equal to the angle FBC[d]. Again, becauſe FA, AG, are proved equal to FB, BH, each to each, and the angle FAG equal to FBH; the baſe FG is equal to FH[b]. Now, ſince FE, EH, are equal to FE, EG, and the baſe FH equal to FG; the angle FEH is equal to FEG; therefore each is a right angle[e]; therefore FE is at right angles to all the lines paſſing through AB, BC; and therefore at right angles to the plain paſſing through AB, DC[f]. Wherefore, &c.

a 15. 1.
b 4. 1.
c 26. 1.
d 8. 1.
e def. 10. 1.
f def. 3.

PROP. V. THEOR.

If a right line ſtand in the common ſection of three right lines, and at right angles to them, theſe three right lines ſhall be in the ſame plain.

Let the right line AB ſtand at right angles to the three right lines BC, BD, BE, in the point of contact B; theſe three lines ſhall be in the ſame plain.

For, if not, let BD, BE, be in the ſame plain, and BC above it; and let the plain paſſing through AB, BC, be produced, till it meet the plain paſſing through BD, BE; and let BF be their common ſection, then BF is a right line[a]; then the three right lines BE, BD, BF, are in one plain; but AB is at right angles to BD, BE; therefore at right angles to BF, meeting BD, BE, in B[b]; but the angle ABC is a right angle[c]; and the angles

a 3.
b 4.
c hyp.

Book XI. ABC, ABF, are in the same plain [d]; therefore the angle ABC is equal to ABF, a part to the whole; which is impossible; therefore BC is in the same plain with BD, BE. Wherefore, &c.

d def. 3.

PROP. VI. THEOR.

If there be two parallel lines, and a point taken in each of them, the right line joining these points shall be in the same plain with the parallel lines.

Let AB, CD, be two parallel lines, and E, F, points taken in them; then the right line EF joining these points is in the same plain with the parallels; if not, let it be elevated above the plain, as EGF; through which let some plain be drawn, whose common section with the plain in which the parallels are, let be EF [a]; then the two right lines EGF and EF bound a figure; which is impossible [b]; therefore the right line EF is not above, nor can it be below the plain, for the same reason; therefore it is in the same plain. Wherefore, &c.

a 3.
b ax. 10. 1.

PROP. VII. and VIII. THEOR.

If two right lines be perpendicular to the same plain, these right lines are parallel; and, if two right lines are parallel, and one of them is perpendicular to some plain, then the other is perpendicular to the same plain.

Let two right lines AB, CD, be perpendicular to the same plain, then AB is parallel to CD; and, if AB be parallel to CD, and AB be perpendicular to some plain, then CD is perpendicular to the same plain.

First, let AB, CD, be perpendicular to some plain, and let them meet it in the points B, D; join BD; and, in the point D, draw ED at right angles to BD, and equal to AB; join BE, AE, AD; then, because AB is perpendicular to the plain in which BDE is, it will be at right angles to all the lines drawn in it, and touching AB [a]; but AB touches BD, BE, in the same plain; therefore each of the angles ABD, ABE, is a right angle. For the same reason, each of the angles CDB, CDE, is a right angle; then, because AB is equal to DE, and BD common, the two lines ED, DB, are equal to AB, BD; and the angle

a def. 3.

angle ABD equal to the angle EDB; for each is a right one; therefore the base EB is equal to the base AD [b]; therefore EB, BA, are equal to ED, DA; and the base AE common; therefore the angles ABE, EDA, are equal [c]; but EBA is a right angle; therefore EDA is likewise a right angle; therefore ED is perpendicular to AD, and likewise perpendicular to BD, DC; therefore BD, DA, DC, are in one plain [d]; but BD, DA, are in the same plain with AB [e]; therefore AB, DC, are in one plain; and the angles ABD, CDB, right ones; therefore AB, DC, are parallel [f].

b 4. 1.
c 8. 1.
d 5.
e 2.
f 28. 1.

Second, If AB and CD are parallel, and AB perpendicular to some plain, CD is perpendicular to the same plain. For, the same construction remaining, AB is proved at right angles to BD, BE; and ED at right angles to DB, DA; and, because AB is parallel to CD, and DB joins them, CD, AB, are in the same plain with DB [g]; but ED is proved at right angles to DB, DA; therefore at right angles to DC [h]; for DC is in the same plain with DB, DA; therefore CD is at right angles to DE, DB [i]. Wherefore, &c.

g 6.
h 4.
i def. 3.

PROP. IX. THEOR.

RIGHT lines that are parallel to the same right line, altho' not in the same plain with it, are parallel to one onother.

Let the right lines AB, CD, be each parallel to the right line EF, but not in the same plain with it; then AB will be parallel to CD. For, in EF, assume any point G, and draw GH at right angles to EF, in the same plain passing through EF, AB; and likewise GK at right angles to EF, in the plain passing through EF, CD; then, because EF is at right angles to GH, GK, it is also at right angles to the plain passing through GH, GK [a]; but EF is parallel to AB; therefore AB is also at right angles to the plain passing through GH, GK [b]. For the same reason, CD is perpendicular to the same plain; therefore AB is parallel to CD [b]; for each is at right angles to the same plain. Wherefore, &c.

a 4.
b 7.

PROP. X. THEOR.

IF two right lines touching one another, be parallel to two other right lines touching one another, but not in the same plain; these right lines contain equal angles.

Let

Book XI. Let two right lines AB, BC, touching one another, be parallel to two right lines DE, EF, touching one another, but not in the same plain, the angle ABC is equal to the angle DEF. For, take BA, BC, ED, EF, equal to one another, and join AD, CF, EB, AC, DF; then, because AB, DE, are equal and parallel,
a 33. 1. AD, BE, that join them, are likewise equal and parallel [a]. For the same reason, CF, BE, are equal and parallel; then AD, CF,
b 9. are equal and parallel [b]; therefore AC, DF, that join them, are equal and parallel [a]; then, since AB, BC, are equal to DE, EF, and the bases AC, DF, are equal, the angle ABC is equal
c 8. 1. to DEF [c]. Wherefore, &c.

PROP. XI. PROB.

TO let fall a perpendicular on a given plain from a given point above it.

Let BH be the given plain, and A the point above it; it is required from the point A to let fall a perpendicular upon the given plain BH. In the plain BH take any right line BC; and
a 12. 1. from the point A draw AD perpendicular to BC [a]. If AD is perpendicular to BH, what was required is done; if not, draw
b 11. 1. DE in the plain at right angles to BC [b]; and from A draw AF
c 31. 1. perpendicular to DE [a]; and through F draw GH parallel to BC [c]. Then, because BC is perpendicular both to DA and DE, it is
d def. 3. perpendicular to the plain passing through DA, DE [d]; but GH is parallel to BC; therefore GH is perpendicular to the plain
e 7. passing through DA, DE [e]; therefore AF is perpendicular to
f def. 3. GH [f]; but AF is perpendicular to DE; therefore at right angles
g 4. to the plain passing through GH, ED [g]; that is, to BH. Wherefore, &c.

PROP. XII. PROB.

TO erect a right line perpendicular to a given plain from a given point in it.

It is required to draw a perpendicular to the plain MN, from a given point A in it. From some point B, above the plain, let
a 11. fall a perpendicular BC upon it [a]; and from the point A draw
b 31. 1. AD parallel to BC [b]. Then, because AD, BC, are two parallel right lines, BC, one of them, is perpendicular to the plain
c 7. MN; the other, AD, is perpendicular to the same plain [c]. Wherefore, &c.

PROP.

PROP. XIII. THEOR.

TWO *right lines cannot be drawn at right angles to a given plain, from a point given therein.*

For, if possible, let the right lines AB, AC, be drawn perpendicular to a given plain, from the given point A; let a plain passing through AB, AC, cutting the given plain through A, in the right line DAE[a]; but the right line DAE being in the given plain touches it; therefore AB, AC, DAE, are in one plain; then, because AC is perpendicular to the given plain, the angle CAE is a right angle[b]; for the same reason BAE is a right angle; therefore the angle BAE is equal to the angle CAE, a part to the whole, which is absurd. Wherefore, &c.

a 3.
b def. 3.

PROP. XIV. THEOR.

THOSE *plains to which the same right line is perpendicular, are parallel to each other.*

If the right line AB be perpendicular to each of the plains DC, EF; then these plains are parallel: For, if not, let them be produced till they meet each other; and let the right line GH be their common section; in which, take any point K, and join AK, BK; then, because AB is perpendicular to the plain EF, it is perpendicular to the right line[a] BK, being in the same plain produced; therefore ABK is a right angle; for the same reason BAK is a right angle; that is, two angles in a triangle equal to two right angles, which cannot be[b]; therefore the plains CD, EF, being produced, will not meet each other; therefore parallel. Wherefore, &c.

a def. 3.
b 17, 1.

PROP. XV. THEOR.

IF *two right lines, touching one another, be parallel to two other right lines, touching one another, and not in the same plain with them, the plains drawn through these right lines are parallel to each other.*

Let AB, BC, two right lines touching one another, be parallel to two right lines DE, EF, touching one another, but not in the same plain with them; then the plains passing through AB, BC, DE, EF, being produced, will not meet each other: For, from the point B, draw the right line BG[a], to the point

a 11.

G;

Book XI. G, in the plain passing through DE, EF, and perpendicular to it. From the point G, draw GH parallel to DE, and GK parallel to EF [b]; then, because BG is perpendicular to the plain passing through DE, EF, it is perpendicular to all the right lines touching it in that plain [c]; therefore BG is perpendicular to GH, GK; and, since BA is parallel to GH, the angles GBA, BGH, are equal to two right angles [d]; but BGH is a right angle; therefore ABG is likewise a right angle: For the same reason, GBC is a right angle; therefore BG is at right angles to the plain passing through BA, BC [e]; but it is likewise perpendicular to the plain passing through DE, EF; therefore BG is perpendicular to the plains passing through BA, BC, and ED, EF [f]: Therefore these plains are parallel. Wherefore, &c.

b 31. 1.
c def. 3.
d 29. 1.
e 4.
f 14.

PROP. XVI. THEOR.

If two parallel plains are cut by another plain, their common sections will be parallel.

Let the two parallel plains AB, CD, be cut by any plain EFGH, whose common sections are EF, GH; then EF is parallel to GH: For, if EF, GH, are not parallel, if produced, they will meet, either toward F, H, or E, G. Let them meet in K: Then, because EFK is in the plain AB, all points taken in it are in the same plain [a]; therefore K is in the plain AB: For the same reason, K is in the plain CD; therefore the plains AB, CD, meet each other; but they are parallel; therefore they cannot meet; for the same reason they cannot meet, if produced toward E, G; therefore the common sections EF, GH, are parallel. Wherefore, &c.

a 1.

PROP. XVII. THEOR.

If two right lines are cut by parallel plains, they will be cut in the same proportion.

Let the two right lines AB, CD, be cut by parallel plains GH, KL, MN, in the points A, E, B, C, F, D; then AE will be to EB as CF is to FD: For, let AC, BD, AD, be joined; and let AD meet the plain KL, in the point X, join EX, XF; then, because the plains KL, MN, are cut by the plain

plain EBDX, their common sections EX, BD, are parallel[a]; Book VI.
for the same reason, the common sections XF, AC, are parallel: then, since EX is parallel to BD, AE is to EB as AX is a 16.
to XD[b]; for the same reason AX is to XD as CF is to FD; b 2. 6.
therefore AE is to EB as CF is to FD[c]. Wherefore, &c. c 11. 5.

PROP. XVIII. THEOR.

If a right line is perpendicular to some plain, then all plains passing through that line will be perpendicular to the same plain.

Let the right line AB, be perpendicular to the plain CL; then all plains passing through AB are perpendicular to the same plain. For, let a plain DE pass through the right line AB, whose common section with the plain CL is the right line CE; from some point F, in CE, draw FG in the plain DE, perpendicular to the right line CE; then, because AB is perpendicular to the plain CL, it is perpendicular to the right line CE in it[a]; therefore the angle ABF is a right angle; but a def. 3.
GFB is likewise a right angle; therefore AB is parallel to FG[b]; but AB is at right angles to the plain CL; therefore b 28. 1.
FG is at right angles to the same plain[c]: For the same reason, c 7.
all other lines drawn perpendicular to the common section of any plain passing through AB, is perpendicular to CL: Then, because AB, FG, are drawn in one plain, perpendicular to CE, the common section of the plains CH, CL, the plain CH is perpendicular to the plain CL[d]. Wherefore, &c. d def. 4.

PROP. XIX. THEOR.

If two plains cutting each other, be perpendicular to some plain, their common section will be perpendicular to that same plain.

Let two plains AB, BC, cutting each other, be perpendicular to some third plain, ADC; their common section BD is perpendicular to the plain ADC: For, if BD is not perpendicular to ADC, from the point D, draw DE in the plain AB, a def. 4.
at right angles to AD[a]; and DF in the plain CB, at right angles to DC; then the two right lines DE, DF, are drawn from the same point, each at right angles to the same plain ADC[a]; which is impossible[b]; therefore BD is perpendicular b 13.
to ADC. Wherefore, &c.

PROP.

Book XI.

PROP. XX. THEOR.

If a solid angle be contained under three plain angles, any two of them, however taken, are greater than the third.

Let the solid angle A, be contained under three plain angles BAC, CAD, BAD, any two of them, however taken, are greater than the third.

If the three angles, or any two of them, are equal, then any two of them must be greater than the third; but, if not equal, let one of them, as BAC, be the greater: At the point A, make the angle BAE, with the right line AB, in the plain passing through BA, AC, equal to the angle DAB [a]; make AE equal to AD; through E, draw BEC, cutting the right lines AB, AC, in the points B, C; and join DB, DC; then, because DA is equal to AE, the two sides BA, AD, are equal to the two sides BA, AE; and the angle BAD equal to the angle BAE [b]; then the bases BD, BE, are equal [c]; but the two sides BD, DC, are greater than the third BC [d]; and BD is equal to BE; therefore the remainder DC, is greater than EC; and the sides CA, AD, are equal to the two sides CA, AE, and the base DC greater than CE; therefore the angle DAC is greater than EAC [e]; but the angles BAE, EAC, are equal to BAC; therefore the angles BAD, DAC, are greater than BAC. After the same manner, any other two angles may be proved greater than the third. Wherefore, &c.

a 23. 1.
b constr.
c 4. 1.
d 20. 1.
e 25. 1.

PROP. XXI.

Every solid angle is contained under plain angles, together less than four right angles.

Let the angle A be a solid angle, contained under the plain angles BAC, BAD, DAC; these angles are less than four right angles. For, in the lines, AB, AD, AC, take any points B, D, C, and join BD, DC, BC; then, because the solid angle at B is contained under three plain angles, CBA, ABD, DBC, any two of which are greater than the third [a]; the two angles CBA, ABD, are greater than DBC; for the same reason, the angles BCA, ACD, are greater than BCD; and CDA, ADB, are greater than BDC: Therefore the six angles ABD, ABC, ACB, ACD, ADC, ADB, are greater than the three angles

a 20.

DBC,

DBC, BDC, BCD; but these three angles are equal to two right angles [c]; therefore the six angles ABD, ABC, ACB, ACD, ADC, ADB, are greater than two right angles: But the three angles of every triangle are equal to two right angles; therefore the nine angles CBA, BCA, BAC, ACD, CAD, ADC, ADB, ABD, DAB, are equal to six right angles: But six of which are proved greater than two right angles; therefore the remaining three angles BAC, DAC, BAD, which contain the solid angle A, are less than four right angles: In the same manner, it may be proved, if the angle is contained by more than three plain angles, that these are together less than four right angles. Wherefore, &c.

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c 32. 1.

PROP. XXII. THEOR.

If there be three plain angles, whereof any two, however taken, are greater than the third, and the right lines that contain them be equal, then it is possible to make a triangle of the right lines, joining the equal right lines, which form the angle.

Let ABC, DEF, GHK, be three given plain angles, any two of which are greater than the third; and let AB, BC, ED, EF, GH, HK, be the equal right lines that contain them; and join AC, DF, GK; then, of these three right lines, a triangle may be made. For, if the angles B, E, H, or any two of them, are equal, then any two of them must be greater than the third [a], and likewise their bases [a]; of which, let AC be greater than DF, or GK; then DF and GK are greater than AC. For, make the angle ABL equal to the angle GHK [b], and make BL equal to either AB, BC, DE, EF, GH, HK; and join AL, CL; then the two sides AB, BL, are equal to the two sides GH, HK, each to each; and they contain equal angles; therefore the base AL will be equal to the base GK [a]; and, since the angles at E and H are greater than the angle ABC, the angle GHK is equal to the angle ABL; therefore the angle at E is greater than LBC [c]; but the two sides LB, BC, are equal to DE, EF, each to each; and the angle DEF greater than LBC; then the base DF is greater than LC [d]; but GK is proved equal to AL; therefore DF, GK, are greater than AL, LC; but AL, LC, are greater than AC [e]; therefore DF, GK, are much greater than AC; therefore any two of the right lines AC, DF, GK, are greater than the third: Therefore a triangle may be made, whose sides are equal to the three given right lines. Wherefore, &c.

a 4. 1.
b 23. 1.
c hyp.
d 24. 1.
e 20. 1.

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PROP. XXIII. PROB.

To make a solid angle of three plain angles, any two of which are greater than the third; but these three angles must, together, be less than four right angles.

It is required to make a solid angle of three plain angles ABC, DEF, GHK, any two of which are greater than the third; and all the angles together less than four right angles. Let the right lines AB, BC, DE, EF, GH, HK, be made equal to one another; and join AC, DF, GK. Then a triangle may be made [a] of three right lines equal to AC, DF, GK; which let be LMN; make the side LM equal to AC, MN to DF, and LN to GK [b]; describe the circle LMN about the triangle [c]; its center, X, is either within the triangle, upon one of the sides, or without the triangle. First, let it be within the triangle, and join LX, MX, NX; then, if AB be not greater than LX, it will be either equal or less. First, let it be equal; then AB, BC, are equal to LX, XM, and the base LM equal to AC; then the angle LXM is equal to the angle ABC [d]. For the same reason, the angle MXN is equal to DEF, and NXL to GHK; but the three angles LXM, MXN, NXL, are equal to four right angles [e]; therefore ABC, DEF, GHK, are equal to four right angles; but they are less [f]; which is absurd; therefore LX, XM, are not equal to AB, BC; and they are not greater. For, if possible, let LX, XM, be greater than AB, BC, and cut off XO, XP, equal to AB, BC; join OP. Then, because AB is equal to BC, and XO to XP, the remainders LO, MP, will be equal; therefore OP is parallel to LM [g]; and the triangles LXM, OXP, are equiangular [h]; therefore XO is to OP as XL is to LM [i]: and, by altern. XO is to XL as OP is to LM. But LX is greater than XO; therefore LM is greater than OP. But LM is put equal to AC; therefore AC is greater than OP; therefore the angle ABC will be greater than the angle OXP [k]. For the same reason, DEF is greater than MXN, and GHK than NXL; but OXP, MXN, NXL, are equal to four right angles [e]; therefore the angles ABC, DEF, GHK, are greater than four right angles, and likewise less [f]; which is impossible: Therefore LX, XM, are not greater than AB, BC; but they are proved not equal; therefore they are less; therefore, on the point X, raise XR perpendicular to the plain of the circle LMN [l], and equal to the excess by which the square of AB exceeds the square of LX; and join RL, RM, RN. Then, because RX is perpendicular to the plain LMN, it is at right angles to LX, MX, NX [m]; therefore the squares of LX, XR, are equal to the square of

a 22.
d 22. 1.
c 5. 4.
d 8. 1.
e 15. 1.
f hyp.
g 2. 6.
h 29. 1.
i 4. 6.
k 25. 1.
l 12.
m def. 3.

of

of LR [n]. For the same reason, the squares of RX, XM, are equal to the square of RM; and the squares of RX, XN, equal to the square of RN; but LX, MX, NX, are equal, and RX common; therefore LR, RM, RN, are equal; but the square of AB is equal to the squares of LX, XR [o]; therefore LR is equal to AB. But BC, ED, EF, GH, HK, are each equal to AB; therefore RL, RM, RN, are each equal to AB or BC; and the base ML equal to AC; therefore the angle MRL is equal to the angle ABC; but MR, RN, are equal to DE, EF, and the base MN to DF, the angle MRN to DEF, and the angle LRN to GHK; therefore the solid angle at R is contained by the three plain angles LRM, MRN, LRN, equal to the three plain angles ABC, DEF, GHK.

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n 47. 1.

o const.

Now, let the center of the circle X be on one side of the triangle, viz. MN; join XL; then AB is greater than LX. For, if not, it will be either equal or less. First, let AB be equal to LX; then MX, XL, are equal to AB, BC; that is, MX, XL, are equal to MN; but MN is equal to DF; therefore DE, EF, are equal to DF; which is absurd [p]; much less can MX, XL, that is, MN, that is, DF, be greater than DE, EF; therefore AB is greater than LX; and, if XR is drawn perpendicular to the plain LMN, and equal to the excess by which the square of AB exceeds the square of LX, the figure can be constructed as before.

p 20. 1.

Lastly, let the center X of the circle be without the triangle LMN; join LX, MX, NX; then AB is greater than LX. If not, it is either equal or less. First, let it be equal; then the two sides AB, BC, are equal to the two sides MX, XL; and the base AC equal to ML; therefore the angle ABC is equal to the angle MXL [d]. For the same reason, GHK is equal to LXN; but the whole angle MXN is equal to the angles MXL, NXL; therefore MXN is equal to ABC, GHK; that is, DEF is equal to ABC, GHK; but ABC, GHK, is greater than DEF [q], and likewise equal; which is absurd; therefore AB is not equal to LX. Let AB be less than LX, and make OX, XP, equal to AB, BC; then the remainders OL, MP, will be equal; therefore OP is parallel to ML [g], and the triangles equiangular; therefore XO is to OP as XL is to LM; by altern. as XO is to XL, so is OP to LM; but XL is greater than XO; therefore LM is greater than OP; but LM is equal to AC; therefore AC is greater than OP; and the angle ABC greater than OXP [k]. Draw XV equal to XO or XP; and join OV; then the angle GHK is greater than OXV. At the point X, with the right line LX, make the angle LXS equal to ABC; and the angle LXT to GHK [r]; and XS, XT, each equal to XO; and join OS, OT, ST; then, because the two sides AB, BC, are equal

d 8. 1.

q 20.

g 2. 6.

k 25. 1.

r 23. 1.

to

Book XI. to the two ſides OX, XS, and the angle ABC to OXS; the baſe AC, that is, LM, will be equal to OS [s]. For the ſame
s 4. 1. reaſon, LN is equal to OT; and, ſince the two ſides ML, LN, are equal to the two ſides OS, OT, and the angle MLN, or POV, greater than SOT, for it contains it, the baſe MN is
t 24. 1. greater than ST [t]; but MN is equal to DF; therefore DF is greater than ST; therefore the angle DEF is greater than
k 25. 1. SXT [k]; but the angle SXT is equal to the angles ABC, GHK; therefore the angle DEF is greater than ABC, GHK. and likewiſe leſs; which cannot be; therefore AB is not leſs than LX; but it has been proved not equal to it; therefore muſt be greater. Then, make XR equal to the exceſs by which the ſquare of AB exceeds the ſquare of LX; and join RM, RN, RL; then, in the ſame manner, it may be proved, that the ſolid angle R is the angle required. Wherefore, &c.

PROP. XXIV. THEOR.

IF a ſolid be contained by ſix parallel plains, the oppoſite plains thereof are equal parallelograms.

Let the ſolid CDGH be contained by the parallel plains AC, GF, BG, CE, FB, AE, the oppoſite plains thereof are equal parallelograms. For, becauſe the parallel plains BG, CE, are cut by the plain AC, their common ſections AB, CD, are pa-
a 16. rallel [a]; and, becauſe the parallel plains BF, AE, are cut by the plain AC, their common ſections AD, BC, are parallel; therefore AC is a parallelogram. In the ſame manner, it is proved that GF is a parallelogram. Then, becauſe BH, AG, CF, DE, join the parallel lines AD, GE, BC, HF, they are equal to
b 23. 1. one another [b]. For the ſame reaſon, AB, HG, CD, EF, are equal to one another; therefore BG, CE, AC, GF, AE, BF, are parallelograms. Join AH, DF; then, becauſe AB, BH,
c 10. are parallel to DC, CF, the angle ABH is equal to DCF [c]; then, becauſe AB, BH, are equal to DC, CF, and the angles
d 4. 1. ABH, DCF, equal, the baſes AH, DF, are equal [d]; but the parallelogram BG is double the triangle ABH, and CE double
e 34. 1. CDF [e]; therefore the parallelograms BG, CE, are equal; in the ſame manner, the parallelograms AC, GF, are proved equal; and AE equal to BF. Wherefore, &c.

PROP.

PROP. XXV. THEOR.

If a solid parallelopipedon be cut by a plain parallel to opposite plains; then, as base is to base, so is solid to solid.

Let the solid ABCD be cut by a plain YE, parallel to the opposite plains RA, DH; then, as the base EFUA is to the base EHCF, so is the solid ABFY to the solid EGCD. For, produce AH both ways, and make AK, KL, each equal to AE; and HM, MN, each equal to EH; and compleat the parallelograms LO, KU, HX, MS, and the solids LP, KR, HQ, MT; then, because the right lines LK, KA, AE, are equal, the parallelograms LO, KU, AF, are equal [a]; as also the parallelograms KV, KB, AG, and the parallelograms LW, KP, AR. For the same reason, the parallelograms EC, HX, MS, are equal; as also, HG, HI, IN, and the parallelograms DH, MQ, NT. Then, because LK, KA, AE, are equal; and likewise HM, MN, each equal to HE; LE is the same multiple of AE that LF is of AF; and EN the same multiple of EH that ES is of EC; and LG of AG; LR of AR; ET of HG; and EQ, of EY. Wherefore the three plains in the solid LP, and the three opposite ones, which are equal to them, are equal to the three plains in the solid KR, or AY, and the three opposite plains which are equal to them [b]; therefore the three solids LP, KR, AY, are equal [c], and the same multiple of AY that LF is of AF. For the same reason, the solids ED, HQ, MT, are equal; therefore ET is the same multiple of ED that ES is of EC: Wherefore, if LF be equal to ES, the solid LY will be equal to the solid NY, if greater, greater, and, if less, less: Wherefore, as AF is to FH, so is the solid AY to ED [d]. Wherefore, &c.

a 36. 1.
b 24.
c def. 10.
d def. 5. 5.

PROP. XXVI. PROB.

At a given point, in a given right line, to make a solid angle equal to a solid angle given.

It is required, at a given point A, in a given right line AB, to make a solid angle equal to the solid angle contained by the plain angles EDC, EDF, FDC. In the right line DF assume any point F; from which draw FG perpendicular to the plain passing through ED, DC, meeting the plain in the point G;

Book XI. G [a]; join DG; at the point A, with the right line AB, make the angles BAL, BAK, equal to the angles EDC, EDG [b]; and make AK equal to DG; at the point K, in the plain BAL, raise a perpendicular HK [c]: which make equal to GF; and join HA; then the solid angle at A, which is contained by the plain angles BAL, BAH, HAL, is equal to the solid angle at D, contained by the plain angles EDC, EDF, FDC. For, take the right line AB, equal to DE; AL to DC; and join HB, KB, FE, GC, FC; then, because GF is perpendicular to the plain EDC [d], the angles FGD, FGE, FGC, are right angles; for the same reason, HKA, HKB, HKL, are right angles; and, because the two sides KA, AB, are equal to the two sides GD, DE, and contain equal angles, the bases BK, EG, are equal; and, because BK, KH, are equal to EG, GF, each to each, and contain equal angles, the bases HB, FE, are equal [e]. Again, because AK, KH, are equal to DG, GF, each to each, and contain equal angles, the base AH is equal to DF; but AB, AH, are equal to DE, DF, and the base BH equal to EF; therefore the angle BAH is equal to EDF [f]; but the angle BAL is equal to EDC, and a part BAK equal to EDG; therefore the remainders KAL, GDC, are equal, and the base KL to GC [e]; and, because HK, KL, are equal to FG, GC, each to each, and the angle HKL equal to FGC, the base HL is equal to FC; but HA, AL, are equal to FD, DC, and the base HL equal to FC; the angle HAL is equal to FDC [e]; therefore the plain angles BAL, BAH, HAL, containing the solid angle A, are equal to the plain angles EDC, EDF, FDC, containing the solid angle at D, each to each; therefore the solid angle at A is made equal to the solid angle at D [g]; which was to be done.

a 11. b 23. 1. c 12. d def. 3. e 4. 1. f 8. 1. g def. 10.

PROP. XXVII. PROB.

To describe a parallelopipedon from a given right line, similar and alike situate to a solid parallelopipedon given.

It is required to describe, from the right line AB, a solid parallelopipedon, similar and alike situate to the given solid parallelopipedon CD.

At the point A, in the given right line AB, make a solid angle A, contained by the plain angles BAH, HAK, KAB, equal to the solid angle at C [a], so that the angle BAH be equal to ECF; BAK to ECG; and HAK to FCG; and make BA to

a 26.

AK

AK as EC is to CG [b]; and KA to AH as GC is to CF; then [c], Book XI.
by equality, as BA is to AH, so is CE to CF. Compleat the parallelogram BH, and solid AL: Then, because the three plain angles, containing the solid angle at A, are equal to the three plain angles containing the solid angle at C, and the sides about the equal angles proportional, the parallelogram KB is similar to the parallelogram GE. For the same reason, KH is similar to GF, and HB to FE; therefore the three parallelograms of the solid AL are similar to the three parallelograms of the solid CD; but these three parallelograms are equal and similar to the three opposite ones [d]; therefore the solid AL is similar to the solid CD [e]: Which was to be done.

b 12. 6.
c 22. 5.
d 24.
e def. 9.

PROP. XXVIII. THEOR.

If a solid parallelopipedon be cut by a plain passing through the diagonals of two opposite plains, that solid will be bisected by the plain.

If the solid parallelopipedon AB be cut by the plain GAEF, passing through the diagonals GF, AE, of two opposite plains, then the solid AB is bisected by the plain GAEF. For, because the triangles CGF, GBF, are equal, and likewise the triangles ADE, AEH [a], and the parallelograms AC, BE [b], for they are opposite, and likewise GH equal to CE; the prism contained by the two triangles CGF, ADE, and the three parallelograms GE, AC, CE, is equal to the prism contained by the triangles GFB, AEH, and the three parallelograms GE, BE, AB [c]. Wherefore, &c.

a 34. 1.
b 24.
c def. 10.

PROP. XXIX. THEOR.

Solid parallelopipedons, constitute upon the same base, having the same altitude, and whose insistent right lines are in the same right line, are equal to one another.

Let the solid parallelopipedons CM, BF, be constitute upon the same base AB, having the same altitude, and whose insistent right lines AF, AG, LM, LN, CD, CE, BH, BK, are in the same right lines FN, DK; then the solid CM is equal to the solid CN. For, because CH, CK, are parallelograms, DH, EK, are each equal to CB [a]; therefore DH is equal to EK. Take EH from, or add to both, then there will remain

a 34. 1.

HK

Book XI. HK equal to DE, and the triangle DEC equal to HKB[b], and the parallelogram DG to HN; but the parallelogram CF is equal to BM[c], and CG to BN, for they are opposite; therefore the prism contained by the two triangles AFG, DEC, and the three parallelograms CF, DG, CG, is equal to the prism contained by the two triangles LMN, HKB, and the three parallelograms BM, HN, BN[d]; add, or take away the solid whose base is the parallelogram AB, opposite to the parallelogram GEHM; then the solid CM is equal to the solid CN. Wherefore, &c.

b 8. 1.
c 24.
d def. 10.

PROP. XXX. THEOR.

SOLID parallelopipedons, constitute upon the same base, having the same altitude, and whose insistent right lines are not in the same right line, are equal to one another.

Let there be solid parallelopipedons CM, CN, having equal altitudes, standing on the same base AB, and whose insistent right lines AF, AG, LM, LN, CD, CE, BH, BK, are not in the same right lines; then the solid CM will be equal to the solid CN. For, produce NK, DH, till they meet in R; and draw GE, FM, meeting in X; likewise produce GE, FM, to the points O, P; join AX, LO, CP, BR; then the solid CM, whose base is the parallelogram ACBL, opposite to the equal parallelogram FDHM[a], is equal to the solid CO[b], whose base is the same parallelogram AB, opposite to the equal parallelogram XR; for they stand upon the same base AB, and the insistent lines AF, AX, LM, LO, CD, CP, BH, BR, are in the same right lines FO, DR; but the solid CO is equal to the solid CN, for they have the same base AB, opposite to the parallelograms XR, GK, each equal to AB, and their insistent right lines AG, AX, CE, CP, LN, LO, BK, BR, are in the same right lines GP, NR; therefore the solid CM is equal to the solid CN. Wherefore, &c.

a 24.
b 29.

PROP. XXXI. THEOR.

SOLID parallelopipedons, constitute upon equal bases, and having the same altitudes, are equal.

Let AE, CF, be solid parallelopipedons, constitute upon the equal bases AB, CD; and having the same altitude, the solid AE is equal

equal to the solid CF. First, let the solids AE, CF, have the insistent lines AG, HK, LM, BE, OP, DF, CG, RS, at right angles to the bases AB, CD; and let the angle ALB be equal to the angle CRD. Produce CR to T; and make RT equal to LB; compleat the parallelogram DT, equiangular to AB or CD; and the solid RI, having its insistent right lines at right angles to DT, and of the same altitude with AE or CF. Then, because the right lines DF, RS, are at right angles to the plain OT, they are parallel [a] and equal [b]; therefore the parallelogram DS is equal and parallel to CP, TI: Therefore the solid CF is to the solid RI as the base OR is to the base DT [c]; but OR is equal to DT; therefore the solid CF is equal to the solid RI. But the solid RI is equal to the solid AE; therefore the solid AE is equal to the solid CF: But, if the angle ALB is not equal to CRD, at the point R, with the right line RF, make the angle TRY equal to the angle ALB; and make RY equal to AL; and compleat the parallelogram RX, and solid YW. Produce DR, VT, XY, to the points Q and *a*; and compleat the solid *ae*; then the parallelograms RX, RQ, are equal [d]; and, because RX is equiangular to AB, and the insistent lines at right angles to the base RX, and of the same altitude with the solid AE, the plains in the solid AE are equal and similar to these in the solid YW; therefore the solid YW is equal to the solid AE [d]. For the same reason, the solid *a*W, whose base is the parallelogram RW, and *ae*, that opposite to it, is equal to the solid YW, whose base is the parallelogram RW, and Y*f*, that opposite to it; for they stand upon the same base RW, have the same altitude, and their insistent lines R*a*, RY, TX, TQ, SZ, SN, W*e*, W*f*, are in the same right lines *a*X, Z*f*; but the solid YW is equal to the solid CF; therefore the solid *a*W is equal to the solid CF.

a 6.
b constr.
c 25.
d 35. 1.
d def. 10.

Now, let the insistent lines ML, EB, GA, KH, NO, SD, PC, FR, not be at right angles to the bases AB, CD, the solid AE will be equal to the solid CF. For, from the points G, K, E, M, P, F, S, N, let fall the right lines M*f*, ET, GY, K*g*, PX, FW, N*a*, SI, perpendicular to the plain of the bases AB, CD [e], meeting them in the points *f*, Y, *g*, T, X, W, I, *a*; and join *f*Y, Y*g*, *g*T, T*f*, X*a*, XW, WI, I*a*; then, because GY, K*g*, are at right angles to the same plain, they are parallel [f]. For the same reason, M*f* is parallel to ET. But MG is parallel to EK; therefore the plains MY, KT, of which the one passes through GY, Y*f*, and the other through K*g*, *g*T, which are parallel to GY, YF, and not in the same plain with them, are parallel to one another [g], and equal and parallel to their opposite plains; therefore *f*E is a parallelopipedon. It may be proved in the same manner, that *a*F is a parallelopipedon; but the

e 11.
f 6. 6.
g 15.

Book XI. the solid GT is equal to the solid PI; for they are upon equal bases, and of the same altitude, from what has been demonstra-
b 29. or 30. ted; and the solid GT is equal to the solid AE [b]; and the solid XF to the solid aS; therefore the solid AE is equal to the solid CF. Wherefore, &c.

PROP. XXXII. THEOR.

SOLID parallelopipedons that have the same altitude are to one another as their bases.

Let AB, CD, be solid parallelopipedons, having the same altitude; as the base AE is to the base CF, so is the solid AB to the solid CD.

For, to the right line FG, apply the parallelogram FH, equal to the parallelogram AE; upon the base FH, compleat the solid GK, of the same altitude as CD; then the solid AB is
a 31. equal to the solid GK [a]; but the solid CK is cut by the plain DG, parallel to the opposite plain; therefore the solid CD is to
b 25. the solid GK, as CF is to FH[b]; that is, AB is to CD as AE is to CF. Wherefore, &c.

PROP. XXXIII. THEOR.

SIMILAR solid parallelopipedons are to one another in the triplicate ratio of their homologous sides.

Let AB, CD be similar solid parallelopipedons, and let the side AE be homologous to the side CF; then the solid AB has to the solid CD a triplicate ratio of that which the side AE has to CF.

For, produce AE, GE, HE, to K, L, M; make EK equal to CF, EL to FN, and EM to FR; and the angle KEL is equal to CFN; for AEG is equal to CFN; and compleat the parallelogram KL, and the solid KO; then the parallelogram KL is similar and equal to the parallelogram CN. For the same reason, the parallelogram KM is equal and similar to the parallelogram CR, and OE to FD; therefore the whole solid
a 24. KO is equal and similar to the solid CD [a]. Likewise, compleat the parallelogram HL, and solids EX, LP, upon the bases GK, KL, having the same altitude as AB, for EH is an insistent line to both; but the solid OK is proved similar to CD; and AB is given

given similar to CD; therefore AB is similar to OK [b]. Then, because AB is similar to CD, and the base AG to CN, as AE is to CF, so is EG to FN; and so is EH to FR [c]; and, because FC is equal to EK, and FN to EL, and FR to EM, as AE is to EK, so is EG to EL; and so is the parallelogram AG to GK; but, as GE is to EL, so is the parallelogram GK to KL [c]; and, as HE is to EM, so is the parallelogram PE to KM [c]; therefore, as AG is to GK, so is GK to KL; and so is PE to KM; but, as AG is to GK, so is the solid AB to the solid EX [d]; for the plain GH is parallel to the opposite plains; and, as GK is to KL, so is the solid EX to PL [d]; and, as PE is to KM, so is the solid PL to the solid KO. Then, because the four solids AB, EX, P , KO, are proportionals, AB has to KO a triplicate proportion of what AB has to EX [e]; but, as AB is to EX, so is the parallelogram AG to GK [d]; and so is the right line AE to the right line EK [c]; therefore the solid AB has to the solid KO a triplicate ratio of what AE has to EK; but the solid KO is equal and similar to the solid CD; and the right line EK equal to CF: Therefore, the solid AB has to the solid CD a triplicate ratio of what the homologous side AE has to the homologous side CF. Wherefore, &c.

b 21. 6. and 12. 5.
c 1. 6.
d 25.
e def. 11.

Cor. Hence, if four right lines be proportional, as the first is to the fourth, so is a solid parallelopipedon described on the first, to a similar one described on the second.

PROP. XXXIV. THEOR.

The bases and altitudes of equal solid parallelopipedons are reciprocally proportional; and parallelopipedons, whose bases and altitudes are reciprocally proportional, are equal.

Let AB, CD, be equal solid parallelopipedons; then the base EH is to the base NP, as the altitude of the solid CD is to the altitude of the solid AB.

First, let the insistent right lines AG, EF, LB. HK, CM, NX, OD, PR, be at right angles to their bases; then, as the base EH is to the base NP, so is CM to AG. For, if the base EH is equal to the base NP; then the altitudes CM, AG, are equal. For, if the bases EH, NP, are equal, but the altitudes not equal, then the solids are not equal [a]; but the solid AB is put equal to the solid CD; therefore the altitudes CM, AG, are equal: Therefore, as the base EH is to the base NP, so is CM to AG [b]. Now, let the bases be unequal, and let EH be the greater, then the altitude CM will be greater than the altitude

a 31.
b 32.

AG;

Book XI. G: for the solids are equa Make CT equal to AG; and upon PN compleat the solid CV, whose altitude is CT. Then, because the solids AB, CD, are equal, and CV is some other solid, AB is to CV as CD is to CV [c]; but AB is to CV as the base EH is to the base NP [d]; and, as the solid CD is to CV, so is the base MP to TP, and so is MC to TC [e]; therefore the base EH is to NP as MC is to CT; but CT is equal to AG; therefore EH is to NP as MC is to AG. And, if the bases and altitudes are reciprocally proportional, then AB is equal to CD. For, again, let the insistent right lines be at right angles to the bases; then, if the bases are equal, because the altitudes are as their bases, the altitudes are equal; therefore the solids AB, CD [a], are equal. But, if the bases are not equal, let EH be greater than NP, then the altitude of the solid CD is greater than the altitude of the solid AB; that is, CM is greater than AG. Put CT equal to AG, and compleat the solid CV; then, because the base EH is to the base NP, as MC is to AG; and CT is equal to AG; and, as EH is to NP, so is MC to CT [f]; but, as MC is to CT, so is MP to PT [e]; and, as EH is to NP, so is AB to CV [d]; but, as AB is to CV, so is CD to CV; therefore AB is equal to CD [f].

c 7. 5.
d 32.
e 1. 6. and 32.
f 9. 5.

Secondly, If the insistent right lines FE, BL, GA, KH, XN, DO, MC, RP, are not at right angles to the bases; from the points F, G, B, K, X, M, D, R, let be drawn perpendiculars meeting the plain of the bases EH, NP, in the points S, T, V, Y, Q, Z, *a*, *f*, and compleat the solids FV, X*a*. Then, if the solids be equal, their bases and altitudes are reciprocally proportional, viz. as EH is to NP, so is the altitude of CD to the altitude of AB [f]. For, because the solids AB, CD, are equal, and the solid AB is equal to the solid BT, and the solid CD to the solid DZ [g]; therefore the solid BT is equal to the solid DZ; but the bases and altitudes of equal solids, whose insistent right lines are at right angles to their bases, are proved to be reciprocally proportional. Therefore, as the base KF is to the base RX, so is the altitude of the solid DZ to the altitude of the solid BT; but the solids DZ, DC, have the same altitude; and the solids BT, BA, have the same altitude; therefore, the base EH is to the base NP, as the altitude of the solid DC is to the altitude of the solid AB.

g 29. or 30.

Again, let the bases and altitudes of the solid parallelopipedons be reciprocally proportional, viz. as EH is to NP, so let the altitude of CD be to the altitude of AB; then the solids AB, CD, are equal. For, the same construction remaining, as the base EH is to the base NP, so is the altitude of CD to the altitude of AB; but the altitudes of the solids AB, BT, are the same, and likewise of the solids CD, DZ; therefore, as the

base

base FK is to the base XR, so is the altitude of the solid DZ to the altitude of the solid BT; therefore the base and altitudes of the solid parallelopipedons BT, DZ, are reciprocally proportional: But these solid parallelopipedons whose insistent right lines are at right angles to their bases, and their bases and altitudes are reciprocally proportional, are equal to each other; but the solid BT is equal to BA, for they stand upon the same base FK, and have the same altitude. For the same reason, the solid DZ is equal to the solid DC; therefore the solid AB is equal to the solid CD: Wherefore, solid parallelopipedons, whose bases and altitudes are reciprocally proportional, are equal. If the bases are not equal, and the insistent right lines not at right angles to the bases, the bases and altitudes may be proved reciprocally proportional, in the same manner as when the insistent right lines are at right angles to their bases; and in the same manner, if the bases and altitudes are reciprocally proportional, the solids are equal. Wherefore, &c.

PROP. XXXV. THEOR.

If there be two plain angles equal, and from their vertices two right lines be elevated above the plains in which the angles are, making equal angles with the lines containing the given angles; and if, in the elevated lines, any points be taken, from which perpendiculars are let fall to the plains passing through the given right lines; then these elevated lines shall be equally inclined to the given plain.

Let BAC, EDF, be two right lined plain angles, from whose vertices A, D, let two right lines AG, DM, be elevated above the plains of the said angles, making the angles MDE, MDF, equal to the angles GAB, GAC, each to each; then the angle MDN will be equal to GAL.

For, if AG is not equal to DM, take AH equal to DM; and from H draw HK parallel to GL; but GL is perpendicular to the plain passing thro' BAC; therefore HK is likewise perpendicular to the same plain [a]. From the points K, N, draw the right lines KB, KC, NE, NF, perpendicular to the right lines AB, AC, DE, DF. Join HC, CB, MF, EF, HB, EM; then the square of HA is equal to the squares of HK, KA [b], and the square of KA equal to the squares of KC, CA [b]; therefore the square of AH is equal to the squares of HK, KC, CA; but the squares of HK, KC, are equal to the square of HC; for the angle HKC is a right one: Therefore the square of HA is equal to the squares of HC, CA; therefore the angles HCA, HBA, are right ones.

a 7.

b 47. 1.

Book XI. ones [c]. For the ſame reaſon, the angles DFM, DEM, are right angles: Therefore the angle ACH is equal to DFM; but the angle HAC [d] is equal to MDF; therefore the two triangles HAC, MDF, have two angles, HAC, HCA, in the one, equal to MFD, MDF, of the other, each to each, and HA, a ſide of the one, equal to MD, a ſide of the other; therefore, the remaining ſides MF, FD, are equal to HC, CA, each to each [e]. In the ſame manner, AB is proved equal to DE, and BH to EM: Therefore, the two ſides CA, AB, are equal to the two ſides FD, DE, and the angle BAC equal to FDE; therefore the baſe BC is equal to EF, the angle ACB to DFE, and ABC to DEF. But the angles ACK, DFN, are equal; for each is a right one; therefore the remaining angle BCK is equal to EFN. For the ſame reaſon, the angle CBK is equal to FEN. And, becauſe the two triangles BCK, EFN, have the two angles CBK, BCK, in the one, equal to the two angles FEN, EFN, each to each, and a ſide BC equal to EF; therefore the ſide FN is equal to CK [e]. But AC is equal to DF; therefore the two ſides AC, CK, are equal to the two ſides DF, FN, and they contain right angles; therefore the baſe AK is equal to DN. And, ſince AH is equal to DM, their ſquares are equal; but the ſquares of AK, KH, are equal to the ſquare of AH, and the ſquares of DN, NM, equal to the ſquare of DM; for the angles AKH, DNM, are right angles; therefore the ſquares of AK, KH, are equal to the ſquares of DN, NM, and the ſquares of AK, DN, equal; therefore the ſquares of KH, MN, are equal; that is, the right line HK equal to MN; therefore the angle HAK is equal to MDN [f], that is, GAL equal to MDN: Which was to be demonſtrated.

c 48. 1.
d hyp.
e 26. 1.
f 8. 1.

Cor. Hence, if two right lined plain angles be equal, from whoſe point equal right lines are elevated on the plain of the angles containing equal angles with the given lines, each to each; perpendiculars drawn from the extreme points of theſe elevated lines to the plain of the given angles, are equal to one another.

PROP. XXXVI. THEOR.

If three right lines A, B, C, are proportional, the ſolid parallelopipedon made of them is equal to the ſolid parallelopipedon made of the middle line, it being equiangular to the former parallelopipedon.

Let

Let the three proportional right lines be A, B, C, viz. A to B as B is to C; then the solid made with A, B, C, is equal to the equiangular solid made from B. Book XI.

Let E be a solid angle contained under the plain angles DEF, GEF, GED; make GE, EF, DE, each equal to B; and compleat the solid parallelopipedon EK. Again, put LM equal to A; and at the point L, with the right line LM, make a solid angle contained under the three plain angles NLX, XLM, MLN, equal to the solid angle E [a]. a 26.

Then, because LM, LN, EF, EG, or ED, and LX, are equal to the three proportional lines A, B, C; therefore, as LM is to EF, so is GE to LX; therefore the sides about the equal angles GEF, MLX are reciprocally proportional; and the parallelograms MX, GF, equal [b]; and since the two plain angles GEF, XLM, are equal, and from the points N, D, are drawn the equal perpendiculars NL, DE, at right angles to the plain passing through XLM, GEF [c]; therefore the solids LH, EK, have the same altitude; but their bases MX, GF, are equal; therefore the solid HL is equal to the solid EK [d]; but HL is made of the three right lines A, B, C, and KE of the right line B. Wherefore, &c. b 14. 6. c 35. d 31.

PROP. XXXVII. THEOR.

If four right lines are proportional, the similar solid parallelopipedons described from them shall be proportional; and if the similar solid parallelopipedons be proportional, then the right lines they are described from shall be proportional.

Let the four right lines AB, CD, EF, GH, be proportional, viz. AB to CD, as EF is to GH; and let KA, LC, ME, NG, be the similar parallelopipedons described from them; then KA is to LC as ME is to NG.

For, because the solid parallelopipedon KA is similar to LC, KA is to LC in the triplicate proportion of AB to CD [a]. For the same reason, the solid ME is to NG in the triplicate proportion of EF to GH; but AB is to CD as EF to GH; therefore AK is to LC as ME to NG [b]. And, if AK be to LC as ME to NG, then AB is to CD as EF is to GH. For, because AK has to LC a triplicate proportion of AB to CD [a], and ME to NG, a triplicate proportion of EF to GH; therefore AB is to CD as EF to GH [b]. Wherefore, &c. a 33. b 11. 5.

PROP.

Book XI.

PROP. XXXVIII. THEOR.

If a plain be perpendicular to a plain, and a line be drawn from a point in one of the plains perpendicular to the other plain, that perpendicular shall fall in the common section of the plain.

Let the plain CD be perpendicular to AB; and, in the plain CD, any point E be taken, from which let fall the perpendicular EG; then EG shall be perpendicular to the common section AD.

For, if not, let it fall without the common section, as EF, and draw FG in the plain EGF, perpendicular to AD [a]; then, because FG is perpendicular to the plain CD, and the right line EG in the plain CD touches it, the angle FGE is a right angle[b]; but EF is also at right angles to the plain AB; therefore the angle EFG is a right angle: Therefore two angles in the triangle FGE are equal to the two right angles; which cannot be [c]; therefore the right line drawn from the point E perpendicular to AB, does not fall without the line AD; therefore must fall on it. Wherefore, &c.

a def. 4.
b def. 3.
c 17. 1.

PROP. XXXIX. THEOR.

If the sides of the opposite plains of a solid parallelopipedon be divided into two equal parts, and plains be drawn thro' their common sections, the common section of these plains, and the diameter of the solid parallelopipedon shall bisect each other.

Let the sides of CF, AH, the opposite plains of the solid parallelopipedon AF, be each bisected in the points K, L, M, N, X, O, P, R; let the plains KN, XR, be drawn through the sections; and let YS be the common section of the plains, and DG the diameter of the solid; then YS, DG, will bisect each other.

For, join DY, YE, BS, SG; then, because DX is parallel to OE, the alternate angles DXY, YOE, are equal [a]; and, because DX is equal to OE, and YX to YO, and they contain equal angles, the base DY is equal to YE, and the triangle DXY to YOE, the angles in the one equal to the angles of the other, each to each [b], viz. the angle XYD equal to the angle OYE, and OEY to XDY; therefore DYE is a right line [c]. For the same reason, BSG is a right line, and BS equal to SG. Then,

a 29. 1.
b 4. 1.
c 14. 1.

Then, because CA is equal and parallel to DB, and to EG [d], DB is parallel to EG [e], and DE to GB [d]; but D, E, G, B, Y, S, are points taken in each of them. Join DG, YS; then DG, YS, are in one plain [f]; and, since DE is parallel to GB, the angle EDT is equal to BGT [a]; but the angle DTY is equal to the angle GTS [g]; therefore DTY, GTS, are two triangles, having the angles YDT, DTY, equal to the two angles SGT, GTS, each to each, and the side YD equal to GS; therefore DT is equal to TG, and YT to TS [h]. Wherefore, &c.

d 34. 1.
e 9.
f 2.
a 29. 1.
g 15. 1.
h 26. 1.

PROP. XL. THEOR.

If two triangular prisms of equal altitudes, the base of one of which is a parallalelogram, and the other a triangle, and, if the parallelogram be double the triangle, the prisms are equal to each other.

Let ABCDEF, GHKLMN, be two prisms of equal altitude; let the parallelogram AF be the base of the one, and the triangle GHK the base of the other; and, if AF be double GHK, the prisms are equal.

For, compleat the solids AX, GO; then, because AF is double GHK, and the parallelogram HK double the triangle GHK [a]; the parallelograms AF, HK, are equal; therefore the solids AX, GO, are equal [b]; but the half of equal things are equal; therefore the prism GHKLMN is equal to the prism ABCDEF; for each is half the solids GO, AX [c]. Wherefore, &c.

a 41. 1.
b 31.
c 28.

THE

ELEMENTS

OF

EUCLID.

BOOK XII.

PROP. I. THEOR.

Book XII

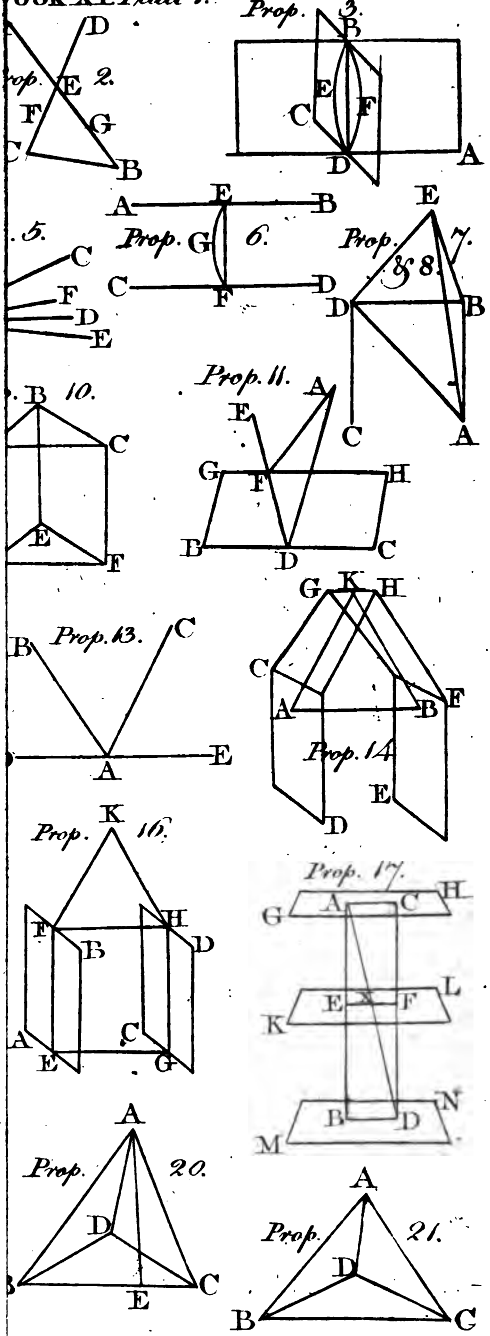
SIMILAR polygons inscribed in circles, are to one another as the squares of the diameters of the circles.

Let ABCDE, FGHKL, be circles, in which are inscribed the similar polygons ABCDE, FGHKL; let BM, GN, be the diameters of the circles; then the polygon ABCDE is to FGHKL as BM square is to GN square.

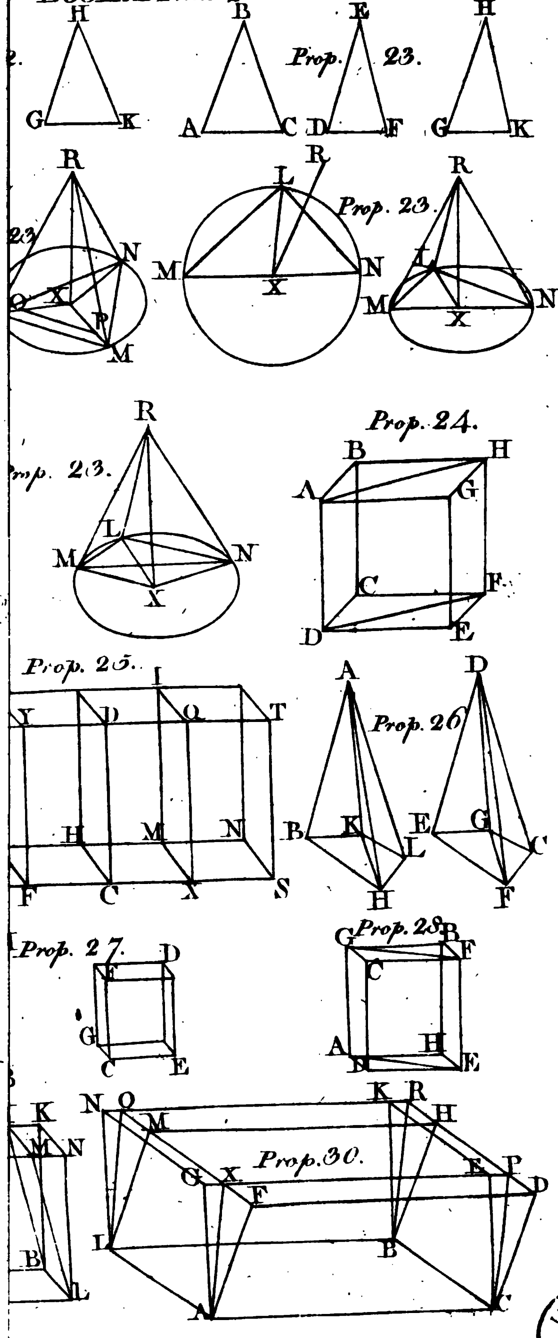
For, Join BE, AM, GL, FN; then, because the polygons are similar, the angles BAE, GFL, are equal; and BA is to AE as GF to FL; wherefore the triangles ABE, FGL, are e-
a 6. 6. quiangular[a]; that is, the angle AEB equal to FLG, and the sides about them proportional; but the angle AMB is equal to
b 21. 3. AEB[b], and FLG to FNG[b]: therefore the angle AMB is equal
c 31. 3. to FNG, the angle BAM to GFN[c], and the remaining angle ABM to FGN: Therefore the triangles AMB, GFN, are equi-
d 4. 6. angular, and BM is to GN as BA is to GF[d]; but the triangle ABM is to the triangle FGN in the duplicate ratio of AB to
e 19. 6. GF[e], and the polygon ABCDE is to the polygon FGHKL in
f 20. 6. the duplicate ratio of AB to GF[f]; but the triangle ABM is to the triangle FGN in the duplicate ratio of BM to GN; therefore the polygon ABCDE is to the polygon FGHKL in the du-
g 22. 5. plicate ratio of BM to GN[g]. Wherefore, &c.

PROP.

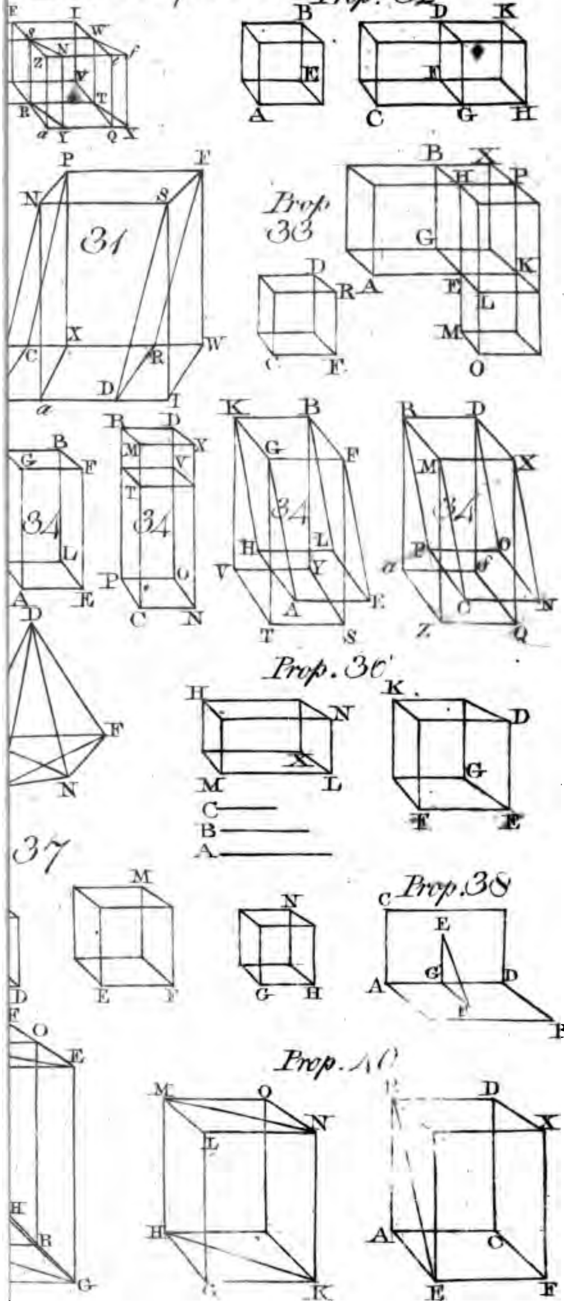
ook XI. Plate 1.



Book XI Plate 2



Book XI plate 3



PROP. I. BOOK X. LEMMA.

If there be two unequal magnitudes, from the greater be taken a part greater than its half, and from the remainder a part taken greater than its half; this may be done till the magnitude remaining be less than any proposed magnitude.

Let AB and C be two unequal magnitudes, of which AB is the greater; from AB let a part BH be taken greater than the half, and from the remainder AH a part KH greater than its half; and so on, till the remaining magnitude, which let be AK, be less than the assigned magnitude C. Let C be multiplied till it become greater than AB, which let be DE, and divide it into the parts DF, FG, GE, each equal to C. Then, because DE is greater than AB, and the part EG taken from it less than the half thereof, and the part BH greater than the half of AB, there remains DG greater than AH. Again, because GD is greater than HA, and GF, half of GD, is taken from it; and if from HA be taken HK greater than the half of HA, there will remain FD greater than KA; but FD is equal to C; therefore KA is less than C. Which was required.

PROP. II. THEOR.

Circles are to each other as the squares of their diameters.

Let ABCD, EFGH, be circles, whose diameters are BD, FH; then, as the square of BD is to the square of FH, so is the circle ABCD to the circle EFGH. If not, the circle ABCD will be to some figure either less or greater than the circle EFGH.

First, let it be to a figure S, less than the circle EFGH, in which inscribe the square EFGH, which will be greater than half the circle. For, if tangents are drawn to the circle, thro' the points E, F, G, H, the square EFGH will be half the square described about the circle; but the circle is less than the square described about it; therefore the square EFGH is greater than half the circle. Let the circumferences EF, FG, GH, HE, be bisected in the points K, L, M, N, and join EK, KF, FL, LG, GM,

Book XII GM, MH, HN, NE, and if tangents are drawn from the points K, L, M, N, and parallelograms compleated upon EF, FG, GH, and HE; then each of the triangles EKF, FLG, GMH, HNE, will be equal to half the parallelogram [a], and therefore greater than half the segment of the circle it stands in; if the remaining segments are bisected, and triangles drawn, as before; and this be continued till the segments are less than the excess by which the circle EFGH exceeds the figure S [b]. Let these be the segments cut off by the right lines EK, KF, FL, LG, GM, MH, HN, NE; then the remaining polygon EKFLGMHN will be greater than the figure S.

a 41. 1.

b Lem.

Describe the polygon AXBOCPDR, in the circle ABCD, similar to the polygon EKFLGMHN [c]; then, as the square of BD is to the square of FH, so is the circle ABCD to the figure S [d]; and, as the polygon AXBOCPDR to the polygon EKFLGMHN, so is the circle ABCD to the figure S [e]; but the circle ABCD is greater than the polygon in it; therefore the figure S is greater than the polygon EKFLGMHN [f]; but it is less; which is absurd; therefore the square of BD to the square of FH is not as the circle ABCD to some figure less than the circle EFGH. In like manner, it is proved that the square of FH to the square of BD is not to the circle EFGH as some figure less than the circle ABCD.

c 1.

d hyp.

e 11. 5.

f 14. 5.

Lastly, the square of BD to the square of FH, is not as the circle ABCD to some figure greater than the circle EFGH.

For, if possible, let it be to the figure T, greater than the circle EFGH; then, inversely, the square of FH is to the square of BD as the figure T is to the circle ABCD; but, because T is greater than the circle EFGH, T will be to the circle ABCD as the circle EFGH is to some figure less than the circle ABCD, which is proved impossible; therefore the square of BD to the square of FH is not as the circle ABCD to some figure less than the circle EFGH, nor to one greater; therefore, as the square of BD is to the square of FH, so is the circle ABCD to the circle EFGH. Wherefore, &c.

PROP. III. THEOR.

EVERY pyramid having a triangular base, may be divided into two pyramids, equal and similar to one another, having triangular bases, and similar to the whole pyramid; and into two equal prisms; which two prisms are greater than the half of the whole pyramid.

Let

Let there be a pyramid, whose base is the triangle ABC, and vertex the point M, then the pyramid ABCM may be divided into two pyramids, equal and similar to one another, having triangular bases, and similar to the whole; and into two equal prisms; which two prisms are greater than the half of the whole pyramid.

For, bisect AB, BC, AC, MA, MB, MC, in the points E, N, G, H, K, L; join EH, EG, EK, EN, HG, HK, HL, KL, KN, NG; then, because AE is equal to EB, and AH to HM, EH is parallel to MB [a], and KH to AB [b]; but AE is equal to KH; for each is equal to EB; therefore the two sides AE, AH, are equal to the two sides KH, HM, the angle MHK equal to HAE [c]; therefore the bases EH, MK, are equal [d], and the triangle AEH equal and similar to KHM. For the same reason, AHG is equal and similar to MHL. And, because AE, AG, are equal and parallel to HK, HL, each to each, the angle KHL is equal to EAG, and the base KL to EG [e]; therefore the triangles KML, EHG, are equal and similar; therefore the pyramid, whose base is the triangle AEG, and vertex the point H, is equal and similar to the pyramid whose base is the triangle HKL, and vertex the point M [f]; and, because HK is parallel to AB, the side of the triangle AMB, the triangles AMB, MHK, are similar [g]. For the same reason, the triangle MBC is similar to MKL, and the triangle AMC to MHL; but the angles KHL, BAC, are equal, and the triangles similar [e]; therefore the pyramid ABCM is similar to HKLM; but the pyramid AEHG is proved similar to HKLM, therefore similar to one another [h], and similar to ABCM.

a 2. 6.
b 34. 1.
c 29. 1.
d 4. 1.
e 10. 11.
f def. 10. 11.
g 2. 6.
h 21. 6.

Again, because BN is equal to NC, the parallelogram BG is double the triangle GNC [i]; therefore the prism contained by the two triangles BKN, EHG, and the three parallelograms BG, BH, KG, is equal to the prism contained by the two triangles NGC, KHL, and the three parallelograms KG, GL, NL, the one of which is constitute upon the parallelogram BG, and opposite to it the right line KH; the other upon the triangle GNC, and opposite to it the triangle KHL; and the parallelogram BG is double the triangle GNC, and have the same altitude; therefore they are equal [k]; but either of these prisms is greater than the pyramid AEGH, or HKLM; for the prism EBNGHK is greater than the pyramid EBNK, which is equal to the pyramid AEGH [f], or HKLM; wherefore the prism EBNGHK is greater than the pyramid AEGH, or HKLM; therefore the prism GNCHKL, is likewise greater than the pyramid HKLM; but the prisms are equal; therefore, together, are greater than the two pyramids; therefore the whole pyramid is divided into two equal pyramids similar to the whole, and to one another, and into

i 41. 1.
k 40. 11.

Book XII into two equal prisms, which two prisms together are greater than half the pyramid.

PROP. IV. THEOR.

If there are two pyramids of the same altitude, having triangular bases, and each of them divided into two pyramids equal to one another, and similar to the whole, and into two equal prisms; and, if each pyramid be divided in the same manner, and this be done continually; then, as the base of the one pyramid is to the base of the other, so are all the prisms in the one pyramid to all the prisms in the other, being equal in number.

Let there be two pyramids of the same altitude, having the triangular bases ABC, DEF, and vertices the points M, H, and each of them divided into two pyramids, equal to one another, and similar to the whole, and into two equal prisms; and if, in like manner, each of the pyramids made by the former division be supposed divided, and this be done continually; then, as the base ABC is to the base DEF, so are all the prisms in the pyramid ABCM to all the prisms in the pyramid DEFH, being equal in number.

For, let the pyramid DEFH be constructed similar to the pyramid ABCM; then, all the triangles described in the base ABC being similar to the whole, and to one another; and also those in DEF, being equal in number to the triangles in ABC; then ABC will be similar to NGC, and DEF to RQF; and, as BC is to NC, so is EF to FQ [a]; therefore ABC is to NGC as DEF is to RQF [b]. And, altern. as ABC is to DEF, so is NGC to RQF; but, as NGC is to RQF, so is the prism GNCLHK to the prism RQFYST [c]: But the two prisms in the pyramid ABCM are equal to one another [d], as also the two prisms in the pyramid DEFH; wherefore the prism, whose base is the parallelogram EGNB, and opposite base the right line KH, is to the prism, whose base is the triangle NGC, and opposite base the triangle HKL, so is the prism whose base is the parallelogram EPRQ, and opposite base the right line ST to the prism whose base is the triangle RQF, and opposite to it the triangle STY [c], compound. as the prisms EBNGKH, GNCLHK, together, are to the prism GNCLHK, so the prisms PEQRST, RQFSTY, together, are to RQFSTY; altern. as the prisms EBNGKH, GNCLHK, together, are to the prisms PEQRST, RQFSTY, together, so the prism GNCLHK to RQFSTY; but the prism GNCLHK is to the prism RQFSTY as

a 4. 6.
b 22. 6.
c 32. and 28. 11.
d 3.

as the base GNC to the base RQF and so is the base ABC to the base DEF; therefore, as the base ABC is to the base DEF, so are the two prisms in the pyramid ABCM to the two prisms in the pyramid DEFH. For the same reason, the prisms in the pyramids HKLM, STYH, or any other pyramids made by any of the former divisions, are to each other as their bases; wherefore, all the prisms in the pyramid ABCM are to all the prisms in the pyramid DEFH as the base ABC to the DEF. Wherefore, &c. Book XII

PROP. V. and VI. THEOR.

PYRAMIDS of the same altitude, having triangular or polygonous bases, are to one another as their bases.

First, let ABCM, DEFH, be pyramids of the same altitude, having the triangular bases ABC, DEF; then the pyramid ABCM is to the pyramid DEFH as the base ABC is to DEF; and so are any number of pyramids to their triangular bases.

If not, let the base ABC be to the base DEF as the pyramid ABCM is to some solid Z, less than the pyramid DEFH, which divide into two pyramids equal to each other, and into two prisms which are greater than half of the whole pyramid; and, if the pyramids made by the former division be divided in the same manner, till some pyramids in the pyramid DEFH is found less than the excess by which the pyramid DEFH exceeds Z. Let these pyramids be DPRS, STYH. Let the pyramid ABCM be divided into the same number of similar parts, as the pyramid DEFH; then are the prisms in the pyramid ABCM to the prisms in the pyramid DEFH [a], as the base ABC is to DEF; but the base ABC is to the base DEF as the pyramid ABCM to the solid Z [b]; therefore the pyramid ABCM is to the solid Z as the prisms in ABCM to the prisms in DEFH; but ABCM is greater than the prisms in it; therefore the solid Z is greater than the prisms in DEFH [c]; and likewise less [b]; which is absurd; therefore the base ABC is not to the base DEF as the pyramid ABCM to some solid less than DEFH. For the same reason, the base DEF is not to the base ABC as the pyramid DEFH to some solid less than ABCM; but ABC is not to DEF as ABCM is to some solid I, greater than DEFH. For, if possible, DEF is to ABC as I to ABCM [d]; but the solid I is greater than DEFH; then, as I is to ABCM, so is DEFH to some solid less than ABCM; which is proved absurd; therefore ABC to DEF is not as ABCM to some solid greater than DEFH; but it was also proved not to be to some solid less than DEFH; therefore ABC is to DEF as ABCM is to DEFH.

a 4.
b hyp.
c 14. 5.
d invers.

For

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For the same reason, in the pyramids ABCDEM, FGHKLN, fig. 6. the pyramid ABCM is to the pyramid ACDM as the base ABC is to the base ACD; and ACDM is to ADEM as ACD is to AED; therefore the whole ABCDEM is to ABCM as ABCDE to ABC [d]. For the same reason, as FGHKLN is to FGHKL as FGHN is to FGH; if ABCDE is equal to FGHKL, the pyramid ABCDEM is equal to FGHKLN; if greater, greater, and, if less, less; therefore, as the base ABCDE is to the base FGHKL, so is the pyramid ABCDEM to the pyramid FGHKLN [e]; Wherefore, &c.

d 12. 5.
e def. 5.

PROP. VII. THEOR.

EVERY prism, having a triangular base, may be divided into three pyramids, equal to one another, and having triangular bases.

Let there be a prism, whose base is the triangle ABC, and the opposite base to that the triangle DEF; then the prism ABCDEF may be divided into three equal pyramids, having triangular bases.

For, join BD, EC, CD; then, because ABCD is a parallelogram, whose diameter is BD [a], the pyramid whose base is the triangle ABD, and vertex the point C, is equal to the pyramid whose base is the triangle EBD, and vertex the point C [b]; but the pyramid whose base is the triangle EBD, and vertex the point C, is equal to the pyramid whose base is the triangle EBC, and vertex the point D; for they are contained by the same plains; therefore the pyramid whose base is the triangle ABD, and vertex the point C, is equal to the pyramid whose base is the triangle EBC, and vertex the point D.

a 34. 1.
b 5. and 6.

Again, because FCBE is a parallelogram whose diameter is CE, the triangle ECF is equal to the triangle CBE [a]; therefore the pyramid whose base is the triangle BEC, and vertex the point D, is equal to the pyramid whose base is the triangle CEF, and vertex the point D [b]: But the pyramid whose base is the triangle BEC, and vertex the point D, has been proved equal to the pyramid whose base is the triangle ABD, and vertex the point C; therefore, also the pyramid whose base is the triangle CEF, and vertex the point D, is equal to the pyramid whose base is the triangle ABD, and vertex the point C; therefore the prism ABCDEF is divided into three pyramids, equal to one another, and having triangular bases. And, because the pyramid whose base is the triangle ABD, and vertex the point C, is the same with the pyramid whose base is the triangle ABC,

and

and vertex the point D, for they are contained by the same plains; and the pyramid whose base is the triangle ABD, and vertex the point C, has been proved to be a third part of the prism, having the same base, viz. the triangle ABC, and the opposite base the triangle DEF: Which was required.

Cor. I. Hence every pyramid is a third part of a prism, having the same base, and an equal altitude; for, if the base of a prism be of any other figure, it can be divided into prisms, having triangular bases.

II. Prisms of the same altitude are to one another as their bases.

PROP. VIII. THEOR.

SIMILAR pyramids, having triangular bases, are in the triplicate ratio of their homologous sides.

Let the two pyramids whose bases are ABC, DEF, and vertices the points G, H, be similar and alike situate, then the pyramids ABCG, DEFH, are to one another in the triplicate ratio of BC to EF.

For, compleat the parallelopipedons BGML, EHPO, then each contain two equal prisms, having triangular bases [a]; and a pyramid is one third of a prism, having the same base and altitude [b]; but similar solid parallalelopipedons are to one another in the triplicate ratio of their homologous sides [c], and parts have the same proportion as their like multiples [d]; therefore the pyramids ABCG, DEFH, are to one another in the triplicate ratio of their homologous sides. Wherefore, &c.

a 28. 11.
b 7.
c 33. 11.
d 15. 5.

Cor. Hence similar pyramids, having polygonous bases, are to one another in the triplicate ratio of their homologous sides.

PROP. IX. THEOR.

THE bases and altitudes of equal pyramids, having triangular bases, are reciprocally proportional; and those pyramids, having triangular bases, whose bases and altitudes are reciprocally proportional, are equal.

Book XII Let the pyramids whose triangular bases are ABC, DEF, and vertices the points G, H, be equal; then the base ABG is to the base DEF, as the altitude of the pyramid DEFH is to the altitude of the pyramid ABCG.

For, compleat the solids BGML, EHPO; then, because the pyramids ABCG, DEFH, are equal, the solids ABGML, EHPO, are equal [a]; but equal solid parallelopipedons have their bases and altitudes reciprocally proportional [b]; therefore the pyramids ABCG, DEFH, have their bases and altitudes reciprocally proportional [c]; and, if their bases and altitudes are reciprocally proportional, they are equal. For, the same construction remaining, the solid parallelopipedons, whose bases and altitudes are reciprocally proportional, are equal; therefore pyramids of the same altitudes with the solids, having their bases and altitudes reciprocally proportional, are likewise equal [c]. Wherefore, &c.

a 28. 11. and 7.
b 34. 11.
c 15. 5.

PROP. X. THEOR.

EVERY cone is the third part of a cylinder, having the same base, and an equal altitude.

Let there be a cone and cylinder, having the same base, viz. the circle ABCD, and their altitudes equal, then the cone is one third of the cylinder; that is, the cylinder is triple the cone. If not, it will be either greater or less than triple the cone. First, let it be greater, and let a polygon AEBFCGDH be inscribed in the circle ABCD, and let the small segments AE, EB, BF, FC, CG, GD, DH, HA, the excess by which the circle exceeds the polygon, be less than any assigned magnitude [a]; and, upon the circle and polygon let a cylinder and pyramid be described, of the same altitude with the cone; and, upon the remaining segments, the remaining parts of the cylinder, which let be less than the excess by which the cylinder exceeds triple the cone; therefore the prism whose base is the polygon AEBFCGDH, and altitude the same of the cone, is greater than triple the cone; but the prism is triple the pyramid of the same base and altitude of the cone [b]; therefore the pyramid is greater than the cone, and likewise less, as included in it; which is absurd; therefore the cylinder is not greater than triple the cone, neither is it less; for then, inversely, the cone would be greater than one third of the cylinder; for, the same construction remaining, the pyramid, whose base is the polygon AEBFCGDH, and vertex the same of the cone, is greater than one third of the cylinder; but the pyramid

a 2.
b 7.

pyramid is one third of the prism constitute on the same base, and having the same altitude; therefore the pyramid whose base is the polygon AEBFCGDH, and altitude the same of the cone, is greater than the cone whose base is the circle ABCD; and likewise less, as contained in it; which cannot be; therefore the cylinder is not less than triple the cone. Therefore, since neither greater nor less, it must be triple the cone. Wherefore, &c. Book XII

PROP. XI. THEOR.

CONES and cylinders, of the same altitude, are to one another, as their bases.

Let there be cones and cylinders of the same altitude, whose bases are the circles ABCD, EFGH, and axes KL, MN, and diameters of their bases AC, EG; then, as the circle ABCD is to the circle EFGH, so is the cone AL to the cone EN. If not, the circle ABCD is to the circle EFGH, as the cone AL is to some solid greater or less than the cone EN. First, let it be to a solid X less than the cone; and let the solid I be equal to the excess of the cone EN above the solid X; then the cone EN is equal to the solid I and X together. Let a polygon HOEPFRGS be inscribed in the circle EFGH, of which the remaining circumferences HO, OE, EP, PF, FR, RG, GS, SH, are less than any assigned magnitudes. Upon the polygon HQEPFRGS let a pyramid be described, of the same altitude with the cone, and let the remaining segments of the cone described upon the circumferences HO, OE, EP, PF, FR, RG, GS, SH, and vertex the same as the pyramid be less than the solid I; therefore the pyramid HOEPFRGS, and altitude the same of the cone, will be greater than the solid X.

Upon the circle ABCD let the polygon DTAYBQCV be described similar and alike situate to HOEPFRGS, and let a pyramid EN be erected, of the same altitude as the cone AL; but the polygons DTAYBQCV, HOEPFRGS, are to one another as the squares of their diameters AC, EG [a], and the circles ABCD, EFGH, are to one another as the squares of their diameters AC, EG [b]; therefore, as the circle ABCD is to the circle EFHG, so is the polygon DTAYBQCV to the polygon HOEPFRGS; but, as the circle ABCD is to the circle EFGH, so is the cone AL to the solid X; therefore the polygon DTAYBQCV is to the polygon HOEPFRGS [c] as the cone AL is to the solid X [d]; but the pyramid DTAYBQCVL is to the

a 1.
b 2.
c 13. 5.
d hyp.

Book XII the pyramid HOEPFRGSN as their bases [e]; therefore the pyramid DTAYBQCVL is to the pyramid HOEPFRGSN as the cone AL is to the solid X; but the pyramid is greater than the solid X, and the cone AL graeater than the pyramid in it; therefore, likewise the cone EN is greater than the pyramid in it; but the pyramid in the cone EN is greater than X; therefore the cone EN is much greater than X; but it was put less; which is absurd; therefore the circle ABCD, to the circle EFGH, is not as the cone AL to a solid less than the cone EN; and it is proved, in the same manner, that the circle EFGH is not to the circle ABCD, as the cone EN is to a solid less than the cone AL.

e 5. and 6.

Again, the circle ABCD to the circle EFGH, is not as the cone AL to a solid Z greater than the cone EN; then, inversely, as the circle EFGH is to the circle ABCD, so is the solid Z to the cone AL; but the solid Z is greater than the cone EN. Then, as the solid Z is to the cone AL, so is the cone EN to some solid less than the cone AL; therefore, as the circle EFGH is to the circle ABCD, so is the cone EN to some solid less than the cone AL; which is impossible; therefore the circle ABCD to the circle EFGH is not as the cone AL to some solid greater or less than EN, therefore, to the cone EN; but, as cone is to cone, so is cylinder to cylinder [f]. Wherefore, &c.

f 15. 5.

PROP. XII. THEOR.

SIMILAR cones and cylinders are to one another, in the triplicate ratio of the diameters of their bases.

Let there be similar cones and cylinders, whose bases are the circles ABCD, EFGH, their diameters BD, FH, and axes of the cones and cylinders KL, MN; then the cone whose base is the circle ABCD, and vertex the point L, to the cone whose base is the circle EFGH, and vertex the point N, hath a triplicate ratio of BD to FH.

For, if the cone ABCDL be not to the cone EFGHN, in the triplicate ratio of BD to FH, let it be in the triplicate ratio to some solid greater or less than the cone EFGHN. First, let it be to a solid X, less than the cone EFGHN, and let the polygon EOFPGRHS be the greatest polygon possible inscribed in the circle EFGH; that is, that the excess of the circle above the inscribed polygon be less than any assigned magnitude; upon the polygon EOFPGRHS let a pyramid be described, of the same altitude

altitude of the cone, and the segments of the cone described upon the segment of the circle, greater than the polygon, be less than the excess by which the cone EFGHN exceeds the solid X; then the pyramid described on the polygon EOFPGRHS, of the same altitude as the cone, is greater than the solid X. Let the polygons ATBYCVDQ be inscribed in the circle ABCD, similar to the polygon EOFPGRHS [a], and upon it describe a pyramid of the same altitude of the cone. For, upon the polygon EOFPGRHS, suppose prisms erected, of the same altitude of the cone; then these prisms are to one another as their bases [b]. For the same reason, the prisms described on the polygon ATBYCVDQ, equiangular to those on the polygon EOPGRHS, and of the same altitude of the cone, are to one another as their base; but the bases are similar to one another; therefore the equiangular prisms are likewise similar, and likewise the pyramids; therefore the pyramids are to one another, in the triplicate ratio of their homologous sides [c]; that is, of BD to FH; but the cone ABCDL is to the solid X, in the triplicate ratio of BD to FH; therefore, as the cone ABCDL is to the solid X, so is the pyramid ATBYCVDQL to the pyramid EOFPGRHSN; but the cone is greater than the pyramid EOFPGRHSN [d]; but it is proved less; which is absurd; therefore the cone ABCDL has not to a solid less than the cone EFGHN, a triplicate ratio of BD to FH. For the same reason, the cone EFGHN has not to some solid less than the cone ABCDL a triplicate ratio of FH to BD.

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a 18. 6.

b 32. 11. and 15. 5.

c 8.

d 14. 5.

Again, the cone ABCDL has not to a solid Z, greater than EFGHN, a triplicate ratio of BD to FH; for, then, inversely, the solid Z has to the cone ABCDL a triplicate ratio of FH to BD; but the solid Z is greater than EFGHN; therefore the solid Z, to the cone ABCDL, is as the cone EFGHN to some solid less than the cone ABCDL; therefore the cone EFGHN, to some solid less than the cone ABCDL, has a triplicate ratio of FH to BD; but it is proved that it has not; therefore the cone ABCDL, to a solid greater or less than the cone EFGHN, has not a triplicate ratio of BD to FH; therefore the cones ABCDL, EFGHN, have to one another the triplicate ratio of their bases BD to FH; and, as cone is to cone, so is cylinder to cylinder [e]. Wherefore, &c.

e 15. 5.

PROP.

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PROP. XIII. THEOR.

If a cylinder be divided by a plain parallel to the opposite plains, then, as one cylinder is to the other cylinder, so is the axis of the one to the axis of the other.

Let the cylinder AD be divided by the plain GH, parallel to the opposite plains AB, CD, and meeting the axis EF in the point K; then, as the cylinder BG is to the cylinder GD, so is the axis EK to KF.

For, let the axis EF be produced both ways to L and M; let EL be taken any multiple of EK; and FM any multiple of FK; through the points L, N, X, M, draw plains parallel to AB, CD; and with the centers L, N, X, M, draw the circles OP, RS, TY, VQ, each equal to AB; and compleat the cylinders PR, RB, DT, TQ; then, because the axis LN, NE, EK, are equal, the cylinders PR, RB, BG, are equal [a]. For the same reason, the cylinders HC, DT, TQ, are equal; therefore the cylinder PG is the same multiple of the cylinder BG, that the axis LK is of EK. For the same reason, the cylinder GQ is the same multiple of GD that KM is of KF; therefore, if KL is equal to KM, PG will be equal to GQ; if greater, greater, and, if less, less. Therefore, AH is to GD as EK is to HF [b]. Wherefore, &c.

a 11.

b def. 5. 5.

PROP. XIV. THEOR.

Cones and cylinders, constituted upon equal bases, are to one another as their altitudes.

Let the cylinders EB, FD, stand upon equal bases AB, CD, then the cylinder EB is to the cylinder FD, as the altitude GH is to the altitude KL.

For, produce the axis KL to the point N, and put LN equal to GH, and let the cylinder CM be drawn about the axis LN; then the cylinders EB, CM, are to each other as their bases [a]; but their bases are equal; therefore the cylinders EB, CM, are equal; but the cylinders CM, FD, are as their axes LN, KL [b]; but the cylinders CM, EB, are equal; and their axes GH, LN, likewise equal; therefore the cylinder EB is to the cylinder FD as the axis GH to the axis KL; but, as the cylinder EB is to the cylinder FD, so is the cone ABG to the cone CDK [c]; therefore, as the axis GH is to KL, so is the cone ABG to CDK; and so the cylinder EB to FD. Wherefore, &c.

a 11.

b 13.

c 15. 5. and 10.

PROP.

PROP. XV. THEOR.

THE bases and altitudes of equal cones and cylinders are reciprocally proportional, and cones and cylinders whose base and altitudes are reciprocally proportional, are equal to one another.

Let the circles ABCD, EFGH, be the bases of the equal cones and cylinders, AC, EG, their diameters, and KL, MN, their axes; compleat the cylinders AX, EO; then, as ABCD is to EFGH, so is the altitude MN to KL.

For the altitudes KL, MN, are either equal or not. If equal, the cylinders AX, EO, are likewise equal; then the bases ABCD, EFGH, are equal [a]; therefore the bases ABCD, EFGH, are to one another as their altitudes: But, if the altitudes KL, MN, are not equal, let one of them, as MN, be the greater, and cut off PM equal to LK, and let the plain TYS, parallel to the opposite plains; cut the cylinder EO in the point P, and compleat the cylinder ES; then the cylinder AX is to the cylinder ES as the cylinder EO is to the cylinder ES [b]: but the cylinder AX is to the cylinder ES as the base ABCD to the base EFGH [a]; and, as the cylinder EO is to the cylinder ES, so is the altitude MN to the altitude MP [c]; therefore the base ABCD is to the base EFGH as the altitude MN is to the altitude KL; therefore the bases and altitudes of the cylinders AX, EO, are reciprocally proportional.

And, if the bases and altitudes of the cylinders AX, EO, are reciprocally proportional, then the cylinders are equal; for, the same construction remaining, the base ABCD is to the base EFGH, as the altitude MN is to the altitude KL; and the altitudes KL, MP, are equal; therefore the base ABCD is to the base EFGH as the cylinder AX is to the cylinder ES [a], and the altitude MN is to the altitude MP as the cylinder EO is to ES [c]; therefore the cylinder AX is to ES as EO is to ES; therefore the cylinder AX is equal to EO [e], and, because cones are one third of the cylinder of the same base [f] and altitude, and parts have the same proportions as their like multiples [g]; therefore the base and altitudes of equal cones and cylinders are reciprocally proportional. And, &c.

a 11.
b 7. 5.
c 13.
e 9. 5.
f 10.
g 15. 5.

PROP.

Book XII

PROP. XVI. PROB.

TWO circles about the same center, to inscribe in the greater a polygon of equal sides, even in number, that shall not touch the lesser circle.

Let ABCD, EFGH, be two given circles, about the same center K; it is required to inscribe a polygon in the circle ABCD, of equal sides, even in number, that shall not touch the lesser circle EFGH.

Through the center K draw the right line BD, and through the point G draw AG at right angles to BD: produce AG to
a 16. 3. C; then AC is a tangent to the circle EFGH, in the point G [a]; bisect the circumference BAD, and, again, the half thereof, and
b lem. doing this, till a circumference is found less than AD, which let be LD; draw LM perpendicular to BD, and produce it to
c 29. 3. N; join LD, ND [c]. Now, because LN is parallel to AC, the tangent of the circle EFGH, LN will not touch the circle EFGH, and, much less, the lines LD, ND; and, if right lines be applied in the circle, each equal to LD, there will be a polygon inscribed in the circle ABCD, of equal sides, and even in number, that will not touch the lesser circle EFGH; which was to be done.

PROP. XVII. PROB.

TO describe a solid polyhedron in the greater of two spheres, having the same center, which shall not touch the superficies of the lesser sphere.

Let two spheres be supposed about the center A, it is required to describe a solid polyhedron in the greater sphere, not touching the superficies of the lesser sphere.

Let the sphere be cut by some plain passing through the center,
a def. 14. 11. then the sections will be circles [a], and the circle described
b 15. 3. by the half section will be a great circle [b]; which let be BEDC; and FGH that of the lesser; and BD, CE, two of their diameters, drawn at right angles to each other; let BD meet the lesser circle in the point G; and draw GL a tangent to the lesser circle in the point G; and join AL: In the greater circle BEDC
c 16 inscribe a polygon that will not touch the lesser circle FGH [c]; let the sides of the polygon, in the quadrant BE, be the right lines

lines BK, KL, LM, ME, such that each will subtend a less arch Book XII
than a line equal to the tangent GL, then the right lines BK, KL,
LM, ME, will each be less than the tangent GL; and produce the
lines joining the points K, A, to N; and from the point A raise
AX perpendicular to the plain of the circle BEDC, meeting the
superficies of the sphere in X [d]. Let plains be drawn through d 12. 11.
AX, BD, and AX, KN, which will make circles in the
superficies of the sphere, and let BXD, KXN, be semicircles on
the diameters BD, KN; then, because XA is perpendicular to
the plain of the circle BEDC, the semicircles BXD, KXN, are
perpendicular to the same plain [e]; but the semicircles BED, e 18. 11.
BXD, KXN, are equal, for they stand upon equal diameters
BD, KN, their quadrants BE, BX, KX, shall likewise be e-
qual; therefore, as many sides of the polygon as are in the qua-
drant BE, so many equal sides may be in the quadrants BX,
KX; let these sides be BO, OP, PR, RX, KS, ST, TY, YX;
and join SO, TP, YR; from the points O, S, draw the per-
pendiculars OV, SQ, which will fall on BD, KN, the com-
mon section [f] of the plain; join VQ; then, since the equal cir- f 38. 11.
cumferences BO, SK, are taken in the equal semicircles BXD,
KXN; and, because OV, SQ, are drawn perpendiculars from
them, they are equal; as also, BV, KQ; but BA, KA, are
equal; therefore AV is to VB as AQ is to QK; therefore VQ is
parallel to BK [g], and OV is parallel to SQ [h]; but it is proved e- g 2. 6.
qual; therefore ZV, SO, are equal and parallel [i]; therefore OS, h 6. 11. i 33. 1.
BK, are parallel [k]; but, because BK is parallel to VQ, and AB k 9. 11.
equal to AK, AB is to BK as AV is to VQ [g]; and altern. AB is
to AV as BK is to VK; but AB is greater than AV; therefore
BK is greater than VK; but VK is equal to OS; therefore BK
is greater than OS; join BO, KS, then OBKS is a quadrilateral
figure in one plain [l]. For the same reason, each of the quadrila- l 7. 11.
teral figures SOPT, TPRY, and triangle YRS, are each in one
plain; therefore, if from the points O, S, P, T, K, Y, to the
point A, right lines are supposed drawn, will constitute a poly-
hedrous figure within the circumferences BX, KX, consisting of
pyramids, whose bases are KBOS, SOPT, TPRY, YRX, and
vertex the point A; and if, in the same manner, pyramids be
constructed on the sides KL, LM, ME, and on the other three
quadrants and opposite hemisphere, there will be constructed a
polyhedrous figure described in the sphere, composed of pyra-
mids whose bases are equal and similar to the foresaid quadrila-
teral figures, and triangle YRX, and vertex the point A.

But the polyhedron does not touch the superficies of the
sphere in which the circle FGH is. For, because the quadrilateral
figure KBSO is in one plain, and from the point A be drawn a
right line AZ perpendicular to the plain [m], it will be at right m 11. 11.

Book XII angles to all the right lines drawn in that plain [n]; join BZ, ZK, then AZ will be perpendicular to BZ, ZK. But the squares of AK, AB, are equal, and the squares of AZ, ZB, are equal to the square of AB [o], and the squares of AZ, ZK, are equal to the square of AK [n]; therefore the squares of AZ, ZB, are equal to the squares of AZ, ZK. Take the common square of AZ from both, then the squares of BZ, ZK, are equal; that is, BZ equal to ZK. In like manner, lines drawn from Z to the points O, S, may be proved equal to BZ, ZK; therefore a circle described about the center Z, with either of these distances, will pass through the points O, S, K, B; and, because KBSO is a quadrilateral figure inscribed in a circle, and OB, BK, KS, are equal, and OS less than BK, the angle BZK will be obtuse; therefore BK is greater than BZ; but GL is greater than KB, and therefore much greater than BZ; and the squares of AG, GL, are equal to the square of AL, AB, or AK; therefore the squares of BZ, ZA, are equal to the square of AL; and the squares of AZ, ZB, are equal to the squares of AG, GL; but the square of GL was proved greater than the square of BZ; therefore the square of AZ is greater than the square of AG; that is, the right line AZ greater than AG; but AZ is perpendicular to one of the bases of the polyhedron; and AG reaches the superficies of the lesser sphere; therefore the polyhedron does not touch the superficies of the lesser sphere. Wherefore, &c.

n def. 3. 11.
o 47. 1.

Cor. If a solid polyhedron is inscribed in another sphere, similar to that in BCDE, they shall be to one another in the triplicate ratio of the squares of their diameters; for, the solids being divided into pyramids, equal in number, and of the same order, they will be similar; and therefore to one another in the triplicate ratio of their homologous sides [p]; that is, as AB drawn from the center of the sphere BCDE, to the semidiameter of the other sphere; but the semidiameters of spheres are as their diameters [q]; and one of the antecedents is to one of the consequents as all the antecedents to all the consequents [r]; therefore, the polyhedron in the one sphere is to the similar polyhedron in the other sphere, in the triplicate ratio of their diameters.

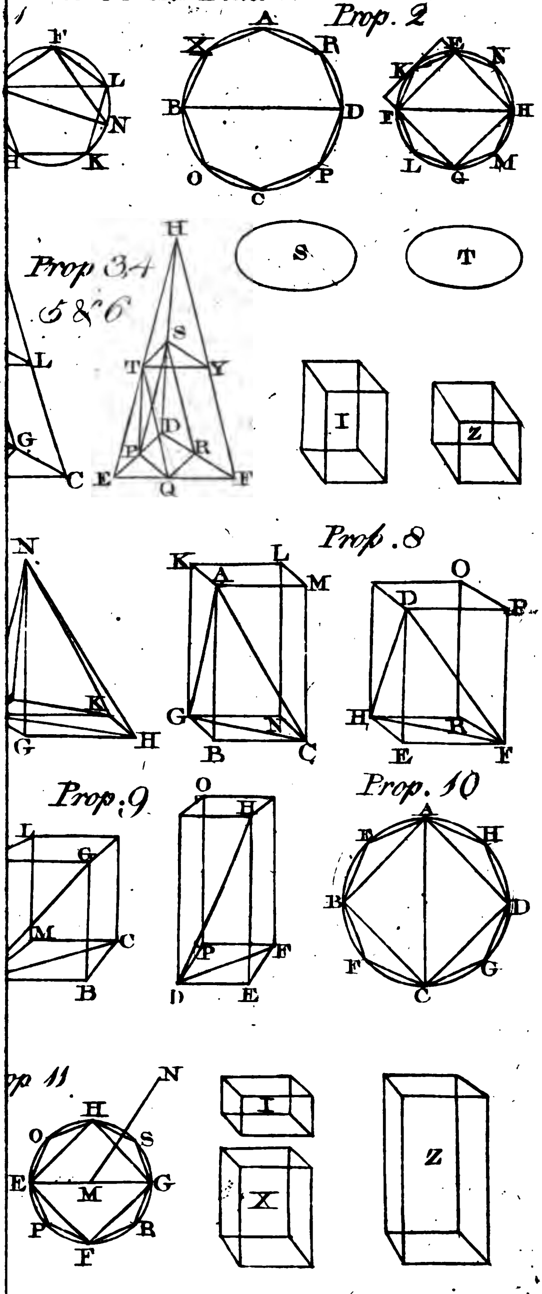
p 12.
q 15. 5.
r 12. 5.

PROP. XVIII. THEOR.

Spheres are to one another in the triplicate ratio of their diameters.

Let

Book XII Plate 1



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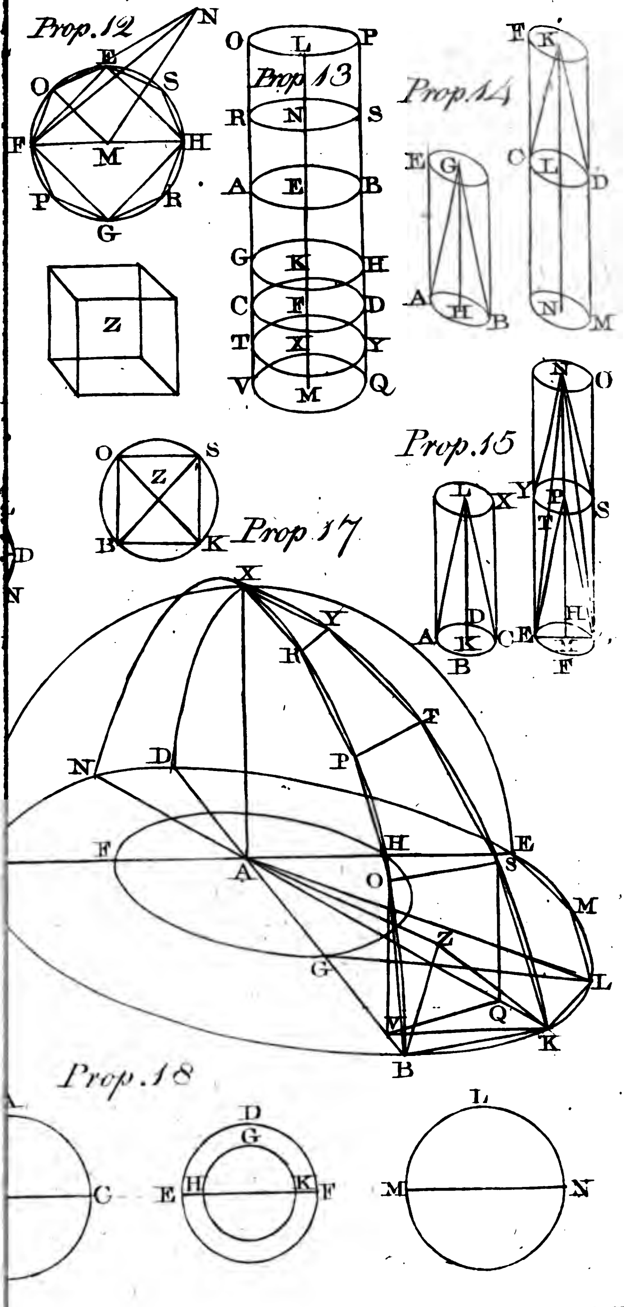
Book XII. Plate. 2



Let ABC, DEF, be two ſpheres, and BC, EF, their diameters, the ſphere ABC is to the ſphere DEF, in the triplicate ratio of BC to EF. If not, let the ſphere ABC be to a ſphere GHK leſs than the ſphere DEF, in the triplicate ratio of BC to EF. Let this ſphere GHK be inſcribed within the ſphere DEF; likewiſe, in DEF, inſcribe a polyhedron, which ſhall not touch the ſuperficies of the leſſer ſphere GHK [a]. In the ſphere ABC inſcribe a polyhedron, ſimilar and alike ſituate to that in DEF; then theſe ſimilar polyhedrons are to one another in the triplicate ratio of their diameters BC, EF [b]; but the ſphere ABC, to the ſphere GHK, hath a triplicate ratio of BC to EF [c]; therefore the ſphere ABC is to the ſphere GHK, as the polyhedron ABC to the ſimilar polyhedron in DEF; but the ſphere ABC is greater than the polyhedron in it; therefore the ſphere GHK is likewiſe greater than the polyhedron in DEF; but it is leſs, as contained in it; which is abſurd; therefore the ſphere ABC, to the ſphere leſs than DEF, has not a triplicate ratio of BC to EF. For the ſame reaſon, the ſphere DEF, to a ſphere leſs than ABC, has not a triplicate ratio of EF to BC. Again, the ſphere ABC, to a ſphere of LMN, greater than DEF, has not a triplicate ratio of BC to EF. If it can, then, by inverſ. the ſphere LMN, to the ſphere ABC, ſhall have a triplicate ratio of the diameters EF to the diameter BC; but the ſphere LMN is to the ſphere ABC as the ſphere DEF to ſome ſphere leſs than ABC, becauſe the ſphere LMN is greater than DEF; therefore the ſphere DEF, to a ſphere leſs than ABC, has a triplicate ratio of what EF has to BC; which is proved abſurd; therefore the ſphere ABC, to a ſphere greater or leſs than DEF, has not a triplicate ratio of what BC has to EF. Therefore ABC has to the ſphere DEF a triplicate ratio of what BC has to EF: Which was to be demonſtrated.

a 17.
b cor. 17.
c hyp.

THE

THE ELEMENTS OF PLAIN AND SPHERICAL TRIGONOMETRY.

PLAIN TRIGONOMETRY.

THE business of trigonometry is to find the angles when the sides are given, and the sides, or ratio of the sides, when the angles are given; and to find sides and angles, when sides and angles are given. For which, it is necessary, that, not only the periphery of the circle, but likewise certain right lines in it, be supposed divided into some determinate number of parts. The ancient geometers have supposed the periphery divided into 360 parts or degrees, and every degree into 60 minutes, and every minute into 60 seconds, &c.; and every angle is said to be of such a number of degrees and minutes as there are in that part of the periphery measuring the angle.

I.

An arch is any part of the periphery or circumference, and is the measure of the angle at the center which it subtends.

II.

The quadrant of a circle is one fourth part of the circumference; the difference of an arch from a quadrant or 90 degrees, is called the complement of that arch.

III.

A chord or subtense, is a right line drawn from one part of an arch to another.

IV. The

IV.

The right sine, or sine of any arch, is a right line drawn from the vertex of an arch perpendicular to the diameter of the circle, and is equal to half the chord of double that arch [a]. [a] 3. 3.

If the arch DB, (fig. for the def.) is an arch of 30 deg. DE is the sine of 30 deg. and twice DE, equal DO, is the subtense of 60 deg. The sine of 30 deg. is equal one half radius [b]. [b] 15. 4.

V.

Every sine, as DE, divides the radius into two parts, that part betwixt the center and sine, as CE, is called the cosine; and the part betwixt the sine and arch, as EB, is the versed sine of the arch DB. For the same reason, AE is the versed sine of the arch AD; therefore, the versed sine may be equal, greater, or less than the radius.

VI.

The arch HD is the complement of BD to a quadrant; and FD, equal CE, is the sine of that arch or cosine of BD.

VII.

If a right line, BG, is drawn from the point B, at right angles to the diameter, and meeting the right line CG, passing thro' the point D; then BG is the tangent of the arch BD, and CG is the secant of that arch.

The right line HI, drawn from the point H, at right angles to CH, and meeting CG produced in I, is the tangent of the arch HD, or cotangent of BD; and CI is the secant of HD, or cosecant of BD.

The sine totus, or greatest sine, is the radius of the circle, which is the sine of 90 deg. [c]. [c] 15. 3.

Characters used. $+$ Addition. $-$ Subtraction. $\times$ Multiplication. $=$ Equality. $::$ Proportion. $\surd$ Extraction of the square-root.

When a line is drawn over any number of quantities, these quantities are to be considered as one quantity. The marks $^{\circ}$, $'$, $''$, $'''$, put over any numbers, are to be read degr. min. second. third minutes, &c. as 23°, $17'$, $18''$, $25'''$, &c. Likewise, R. signifies Rad. S. Sine, Cos Cosine, T. Tang. Cot. Cotangent, Sec. Secant, and Cosec. Cosecant.

SCHOLIUM.

Because the triangles CED, CBG, (fig. for the definitions,) are similar, CE : ED :: CB : BG, by alter. CE : CB :: ED : BG, i. e. Cos. : R :: Sine : Tangent.

Again,

Again, CE : CD :: CB : CG, i. e Cos. : R. :: R. : Secant. And, because the triangles CDF, CED, CBG, and CHI, are similar, CE : ED :: CF : FD; but CE=FD; therefore ED=CF [d]; therefore CE is the sine of the angle CDE=DCF.

d 34. 1

Again, EC : ED :: CB : BG; altern. EC : CB : ED : BG; therefore, if EC be nearly equal to CB, ED will be nearly equal to BG; therefore, if the arch DB be a very small arch, the sine and tangent are nearly to one another in the ratio of equality.

II.

Because the chord of any arch, and its supplement to a circle, is the same, so the sine, tangent, or secant, of any arch, and its supplement to a semicircle, is likewise the same.

III.

If two sides of a right angled triangle be given, the other can be found by 47. El. 1.

PROP. I.

In a right angled triangle, if the hypothenuse be made radius, then the sides are the sines of their opposite angles; and, if either of the sides about the right angle be made radius, the other side is the tangent of its opposite angle, and the hypothenuse is the secant of that angle.

For, in the triangle ABC, if, with the center A, and distance AC, a circle be described, and AB produced till it cut that circle in D, (No 1.); then CB is the sine of the arch CD, or of the angle A [a], and AB is the cosine of CD, or sine of the angle C. Again, (No 2.) if AB is made radius, and the arch BD drawn, then BC is the tangent of the arch BD, or of the angle A; and AC is the secant of that angle. Wherefore, &c.

a def. 1.

Cor. Hence, as AC, Rad. taken in any given measure, is to BC, taken in the same measure, are so any parts into which the radius is supposed to be divided, viz. 10.000000, to a number expressing the parts in proportion to the length of the sine of the angle, that is,

AC being radius,	AC : BC :: R : S,A
And	AC : BA :: R : S,C
AB Rad.	AB : BC :: R : T,A
And	AB : AC :: R : Sec. A.
BC Rad.	BC : BA :: R : T,C
And	BC : AC :: R : Sec. C.

PROP.

PROP. II.

THE sides of plain triangles are to one another as the sines of their opposite angles.

Let ABC be the triangle, about which describe a circle ABC[a]; from the center D let fall perpendiculars upon each of the sides AB, BC, AC, which will be bisected in the points E, F, and G[b]; but the angle BDE[c] is equal to the angle CDE, and BE is the sine of the angle BDE[d], or of the angle BAC[e]. For the same reason, BF is the sine of the angle ACB, and GC the sine of the angle ABC: Therefore, BE is to BF as twice BE is to twice BF; that is, BC is to BA as the sine of the angle A is to the sine of the angle C.

a 5. 4. b 3. 3. c 8. 1. d 1. e 21. 1.

2d. If the triangle is right angled, then BD, the Rad. is the sine of the right angle; the other two angles as before.

3d. If the triangle is obtuse angled, then, if a triangle is formed upon the same base, in the opposite segment in the point I, then that angle will be acute; and BE is the sine of the angle BIC, or BAC[f]. Wherefore, &c.

f schol. 1.

PROP. III.

IN any right lined triangle, the sum of any two sides, is to their difference, as the tangent of half the sum of the angles at the base, is to the tangent of half their difference.

Let ABC be the triangle, the sum of any two of its sides, as AB, BC, is to the difference of these sides, as the tangent of half the sum of the angles BAC, ACB, at the base, is to the tangent of half their difference.

For, let AB be produced to H; make BH equal to BC; and cut off BI equal to BA; then AH is equal to the sum of the sides, and HI to the difference of the sides, and the angle HBC equal to the sum of the angles at the base[a], viz. the angles BAC, ACB. Join HC; and, from B, let BE fall perpendicular upon HC[b]; then, because HB is equal to BC, the angle BHC is equal to BCH[c], and the angles BEC, BEH, are equal[b]; therefore the angles HBE, EBC, are likewise equal[d]; and, if BE be made radius, then EC is the tangent of half the sum of the angles at the base. Draw BD parallel to AC, then the angle DBC is equal to the angle ACB[e]. Take HF equal to DC, and join FB; then

a 32. 1. b 12. 1. c 5. 1. d 32. 1. e 29. 1.

then FBD is the difference of the angles, and EBD half their difference: Through I draw IG parallel to BD or AC; then IB
f 2. 6. is to BA as GD is to DC[f]; but IB is equal to BA; therefore
g 14. 5. GD is equal to DC[g]; but AH is to HC as HI is to HG[h]; and,
h 4. 6. by altern. AH is to HI as HC is to HG, the consequents being halved, as HA is to HI, so is $\frac{1}{2}$ HC to $\frac{1}{2}$ HG; but HF, GD, are each equal to DC, and therefore equal to one another. Add, or take away, GF to or from both, then HG is equal FD; but half FD is ED; therefore AH is to HI as EC is to ED. Wherefore, &c.

PROP. IV. THEOR.

In any triangle, the rectangle under half the sum of the sides, and excess of the same, above any of the sides, taken as the base, is to the rectangle contained by the right lines, by which the half of the sum of the sides exceeds the other two sides, as the square of the rad. is to the square of the tangent of half the angle opposite to the base.

Let ABC be the triangle, BC the base; in the triangle ABC
a 4. 4. let a circle be inscribed[a], of which let G be the center, and let fall GD, GE, GF, perpendiculars to the sides AB, BC, AC; then AD is equal to AE, BD to BF, and CE to CF; and the angles at A and B bisected by the right lines AG, BG; produce AB, AC, to H, L; make BH equal to FC, and CL to BF; at the points H, L, raise the perpendiculars HK, LK, meeting the right line AG, produced in K; from the point K let fall KM perpendicular to BC; and join BK, KC; then the rectangle HAD will be to the rectangle BFC as the square of AD to the square of DG.

For, the triangles ADG, AEG, are equiangular[a], and the angle ADG equal to the angle AHK, for each are right ones;
b 28. 1. therefore DG is parallel to HK[b]; and, since the angles AHK, HAK, are equal to the angles KAL, ALK, for AK bisects
c 26. 1. them, AH is equal to AL[c]; but the angles ABC, CBH, are
d 13. 1. equal to two right angles[d]; and DGF, DBF, equal to two right
e 23. 1. angles[e]; for the angles at D and F are right ones. Take the angle DBF from both, and there remains DGF equal to HBM; but HBM, HKM, are equal to two right angles, for the angles at H and M are right ones; therefore the quadrilateral figures BDGF, BHKM, are equiangular, and BG bisects the angles DBF, DGF; therefore BK will likewise bisect the angles HBM,

HBM, HKM; therefore HB will be equal to BM, and the triangle DBG equiangular to BHK. For the ſame reaſon, MCL will be biſected by CK, and MC equal to CL.

Now, becauſe BF, FC, are equal to BH, CL; AH, AL, are equal to the ſum of the ſides AB, BC, AC, and AH equal to half the ſum of the ſides. And, becauſe the triangles DBG, BHK, are equiangular, GD is to DB as BH is to HK[f]; and the rectangle under DG, HK, equal to the rectangle DBH[g], that is, to BFC; but the triangles ADG, AHK, are equiangular; therefore AD is to DG as AH is to HK, and the rectangle under DG, HK, equal to the rectangle HAD; therefore the rectangle HAD is to the rectangle DBH, or BFC, as the ſquare of AD is to the ſquare of DG[h]; but AD is the exceſs of AH above the baſe BC, and BF, FC the right lines by which AH exceeds the ſides AB, AC; and, if AD is taken rad. then DG is the tangent of half the angle BAC. Wherefore, &c.

[f] 4. 6.
[g] 16. 6.
[h] 22. 6.

PROP. V. THEOR.

IN every plain triangle, the baſe is to the ſum of the ſides as the difference of the ſides is to the ſum or difference of the ſegments of the baſe, as the greater or leſſer ſide of the triangle is taken for the baſe.

Let ABC be a triangle; from the vertex A let fall the perpendicular AD; then the baſe is to the ſum of the ſides as the difference of the ſides is to the ſum or difference of the ſegments CD, BD, according as the baſe BC is the leſſer or greater ſide. From the vertex A, let fall the perpendicular AD upon the baſe BC; with the center A, and diſtance AC, the greater of the other two ſides, deſcribe the circle CEF, and produce AB both ways to F and E, and CB to G; then, becauſe the right lines FE, GC, cut one another in B, the rectangle FBE is equal to the rectangle GBC[a]; but CB is to BF as BE is to GB[b]; that is, when the baſe BC is the greateſt, the baſe to the ſum of the ſides, as the difference of the ſides to the difference of the ſegments of the baſe; but, when BC is the leaſt, GB is the ſum of the ſegments of the baſe. Wherefore, &c.

[a] 35. 3.
[b] 16. 6.

PROP. VI.

THE ſum and difference of any two quantities being given to find theſe quantities.

Let AB, BC, be the two quantities; place them in the same right line, as AC; and bisect AC in E; and cut off AD equal to BC; then DB is the difference of the two quantities, and EB half their difference; therefore, if to AE, half their sum, EB, half their difference, be added, the sum is equal to AB, the greater quantity; and if from AE, half the sum, ED, half their difference, be taken, gives AD equal to BC, the lesser quantity. Wherefore, &c.

THE

THE ELEMENTS OF SPHERICAL TRIGONOMETRY.

DEFINITIONS.

I.

THE poles of a ſphere are two points in the ſuperficies of the ſphere that are the extremes of the axis.

II.

The pole of a circle in a ſphere, is a point in the ſuperficies of the ſphere from which all right lines, drawn to the circumference of the circle, are equal to one another.

III.

A great circle in a ſphere is that whoſe plain paſſes through the center of the ſphere, and whoſe center is the ſame with that of the ſphere, or whoſe plain biſects the ſphere.

IV.

A ſpherical triangle, is a figure comprehended under the arches of three great circles of a ſphere.

V.

A ſpherical angle is that which is contained under two arches of greater circles in the ſuperficies of the ſphere.

PROP. I.

GREAT circles in a ſphere mutually biſect each other.

Let the two great circles be ACB, AFB, they will mutually biſect each other; for their common ſection AB is the diameter of both circles.

PROP.

PROP. II.

If from the pole of any circle, to its center, a right line be drawn, it will be perpendicular to the plain of that circle.

Let the circle be AFB, and its pole C; from which draw CD to the center, then CD will be perpendicular to the plain of that circle.

For, in it draw any diameters EF, GH, and join CG, CH, CE, CF; then, in the triangles CDF, CDE, the two sides CD, DE, are equal to the two sides CD, DF, and their bases CF, CE, are equal[a]; therefore the angle CDF is equal to the angle CDE[b]; therefore CD is perpendicular to the plain of the circle AFB[b]. Wherefore, &c.

a def. 2. b 8. 1. c 4. 11.

Cor. I. Hence, if this circle be a great circle, the distance upon the superficies of the sphere betwixt the pole and great circle is a quadrant, for the plain of it bisects the sphere.

II. Great circles, that pass through the pole of some other circle, make right angles with it; for the right line CD is the common section of such plains[d].

d 19. 11.

PROP. III.

If a great circle is described about the pole of a sphere, and from that pole two right lines be drawn to the circle, the arch of that circle contained by the two right lines is the measure of the angle at the pole.

Let A be the pole of a sphere, and ECF the great circle described about it, and let the right lines AC, AF, be drawn to the great circle; then the arch CF is the measure of the angle at A.

For, let D be the center of the sphere, then the angles ADC, ADF, are right angles[a]; and the angle CDF is the inclination of the plains ACB, AFB, and equal to the spherical triangle CAF, or CBF[b].

a cor. 2. b def. 6. 11.

Cor. I. If the arches AC, AF, are quadrants, then A is the pole of the circle passing through the points C, F; for AD is at right angles to the plain FDC[c].

c 4. 11.

II. The vertical angles are equal, for each is equal to the inclination of the circles; also, the adjacent angles are equal to two right angles.

PROP.

PROP. IV.

If two spherical triangles have two sides of the one, equal to two sides of the other, and the angle contained by the two sides of the one equal to the correspondent angles of the other, the two triangles will be equal.

For, if the two arches containing the angles are equal, their chords or subtenses are likewise equal [a], and contain equal angles; therefore their bases are equal, and remaining angles of the one, equal to the remaining angles of the other, each to each; and the right lined triangles equal [b]; but equal right lines cut off equal circumferences [c]; wherefore the spherical triangles are equal to one another.

a 29. 3
b 4. 1.
c 28. 3.

Cor. I. Hence triangles will be equal and congruous, if two angles of the one be equal to two angles of the other, each to each, and a side of the one equal to a side of the other, either the side that lies betwixt the equal angles, or subtending one of them [d].

d 29. and 24. 3. and 26. 1.

II. Equilateral triangles are likewise equiangular. [e].

III. In isosceles triangles, the angles at the bases are equal; and, if the angles at the bases are equal, the triangles are isosceles [f].

e 29. and 24. 3. and 1. 1.

IV. Any two sides of a triangle are greater than the third; for any two of their chords or subtenses, are greater than the third [g].

f 29 and 24. 3. 5. and 6, 1.
g 20, 1.

PROP. V.

Any side of a spherical triangle is less than a semicircle.

Let AC, AB, the sides of the triangle ABC, be produced till they meet in D, then the semicircle ACD is greater than the arch AC.

PROP. VI.

The three sides of a spherical triangle are less than a whole circle.

For,

For, BD, DC, two sides of the triangle BCD, are greater than the third BC [a]. Add BA, AC; then DBA, DCA, the two semicircles, are greater than the three sides of the triangle BCD [b]. Wherefore, &c.

a 1 cor. 4.
b 4.

PROP. VII.

In any triangle, the greater angle is subtended by the greater side.

Let ABC be the triangle, and A the greater angle, then BC will be the greater side. For, make the angle BAD equal to the angle B; then AD will be equal to BD [a]; therefore the side BDC is equal to AD and DC; but AD, DC, are greater than AC [b]; therefore BC is greater than AC. Wherefore, &c.

a cor. 23.
b 4.

PROP. VIII.

In any spherical triangle, if the sum of two of its sides be greater than a semicircle, then the internal angle at the base will be greater than the external and opposite angle; and the sum of the internal angles at the base will be greater than two right angles; if equal, equal, and, if less, less.

Let ABC be the spherical triangle; if the two sides, AB, BC, be greater than a semicircle, the internal angle BAC, at the base, will be greater than the external and opposite angle BCD; if equal, equal, and, if less, less; and the angles A and ACB will likewise be greater, equal, or less, than two right angles: First, let the semicircles ACD, ABD, be compleated; then, if AB, BC, be equal to ABD, the angle BCD will be equal to BDC [a], that is, to BAC. 2dly, If AB, BC, be greater than ABD, then BC will be greater than BD, and the angle BDC, that is, the angle BAC, greater than BCD [b]; if AB, BC, are less than a semicircle; then the angle A will be less than BCD. And, because the angles BCD, BCA, are equal to two right angles, if the angle A be greater than the angle BCD, then the angles A and ACB will be greater than two right angles, and, if less, less.

a 3.
b 7.

PROP.

PROP. IX.

If the poles of the sides of any spherical triangle be joined by great circles, they constitute another triangle, the sides of which are supplements of the arches that measure the angles of the given triangles; and the arches that are the measures of the angles of the supplementary triangle, are the supplements of the sides of the given triangle.

Let G, H, D, be the angular points of the given triangle GHD; and let the points G, H, D, be the poles of the great circles XCAM, TMNO, XKBN; then XN will be the supplement of BK, XM of CA, and MN of OT. Likewise, the arches KT, OC, and BA, which are the measures of the angles M, X, N, are the supplements of HD, HG, and GD.

For, because G is the pole of the circle XCAM, GM is a quadrant[a]; and, because H is the pole of the circle TMO, HM is also a quadrant: Wherefore, M is the pole of the circle GH[b]. For the same reason, N is the pole of the circle HD, and X the pole of the circle GD. Now, because NK, XB, are each quadrants, XN is the supplement of KB. For the same reason, XM is the supplement of AC, and MN of OT; which are the measures of the angles G, H, D.

a cor. 1. 2.
d cor. 1. 5

Again, because DK, HT, are each quadrants, KT is the supplement of HD. For the same reason, OC is the supplement of GH, and BA of GD; that is, the measures of the angles X, M, N, are the supplements of the sides HD, GH, and GD. Wherefore, &c.

PROP. X.

The three angles of a spherical triangle are greater than two right angles, and less than six.

Let the triangle be GHD; then the three measures of it, with the three sides of the triangle XMN, are equal to three semicircles[a]; but the three sides of the triangle XMN are less than two cemicircles[b]; therefore the measure of the three angles G, H, D, are greater than one; that is, greater than two right angles; but the outward and inward angles of any triangle are together equal to six right angles; therefore the inward angles are less than six right angles. Wherefore, &c.

a 9.
b 6.

PROP.

PROP. XI.

IF, in any great circle, a point is taken, which is not the pole of it, and from that point several arches are drawn to its circumference, the greatest of these arches is that which passes through the pole; and the remainder of it is the least; and the arch nearer to that, passing through the pole, is greater than that more remote; and they make obtuse angles with the great circle.

Let AFBE be a great circle, and any point R taken, which is not the pole of it, and from that point the arches RA, RB, RG, RV, of great circles to the circumference of AFBE, the arch RCA, which passes through the pole, is the greatest, and RB is the least; and the arch RCA is greater than RG, and RG greater than RV.

For, because C is the pole of the circle AFB, CD and RS, that is parallel to CD, are perpendicular to the plain AFB [a], from the point S draw SA, SG, SV; then SA is the greatest line, viz. greater than SG, and SG greater than SV [b]. For, in the right angled plain triangles RSA, RSG, RSV, the squares of RS, SA, that is, the square of RA, is greater than the squares of RS, SG, that is, than the square of RG, that is, RA is greater than RG. For the same reason, RG is greater than RV; therefore, the arch RA is greater than the arch RG, and RG greater than RV.

a 2. and 7. 11.
b 7. 3.

Again, the angle RGA is greater than the angle CGA, which is a right angle [c]; and the angle RVA greater than CVA, a part of it; therefore the angles RGA, RVA, are obtuse angles.

c cor. 3.

PROP. XII.

IF the sides containing the right angles of a spherical triangle be of the same affection with the opposite angles, that is, if the sides are greater or less than quadrants, the opposite angles will be greater or less than right angles.

Let AGR, AGX, be right angled spherical triangles, having the angles GAR, GAX, right ones; then, if the side AR be greater than a quadrant, the angle AGR will be greater than a right angle; and, if AX be less than a quadrant, the angle AGX is less than a right angle.

For,

For, if AC is a quadrant, C is the pole of the circle AFB, and the angles AGC, AVC, are right ones; therefore the side AR subtending the angle AGR, is greater than a right angle [a]; and, because AX is less than a quadrant, the angle AGX is less than a right angle. [a] 7.

PROP. XIII.

IF the two sides containing the angle of a spherical triangle be both less, or both greater than quadrants, then the hypothenuse is less than a quadrant.

In the triangle ARV, or BRV, let F be the pole of the circle AR; then RF is a quadrant, which is greater than RV.

PROP. XIV.

IF one of the sides is greater, and the other less than a quadrant, then the hypothenuse will be greater than a quadrant.

For, in the triangle ARG, the hypothenuse RG is greater than RF, that is, greater than a quadrant. For the same reason, if the hypothenuse is greater than a quadrant, then one of the legs is greater, and the other less, than a quadrant.

PROP. XV.

IF the angles at the base of a spherical triangle be both less, or both greater than quadrants, the perpendicular will fall within the triangle; but, if one be greater, and the other less, the perpendicular will fall without the triangle.

Let ABC be the triangle; from the point A let fall the perpendicular AP; in the first case, it will fall within the triangle; but, if not, it will fall without; then, in the triangle APB, the side AP, and angle B, are of the same affection, and likewise the side AP, and angle ACP: Therefore, since the angles ABC, and ACP, are of the same affection, the angles ACB and ABC are of different affections; but they are not [a]. Wherefore, &c. Fig. 2. [a] By Hyp.

X In

Fig. 1. In the second case, if the perpendicular does not fall without, let it fall within. Then, in the triangle ABP, the angle B, and side AP, are of the same affection; and likewise, in the triangle ACP, the angle C, and side AP, are of the same affection; therefore, the angles B and C are of the same affection; which is impossible [a]. Wherefore, &c.

a Hyp.

PROP. XVI.

IN right angled spherical triangles, having the same or equal acute angles at the base, the sines of the hypothenuse are proportional to the sines of the perpendicular arches, and the sines of the bases proportional to the tangents of the perpendicular arches.

Let the triangles be BAC, BHE, right angled at A and H, and the same acute angle B, at the base BA, the sines of the hypothenuses CP, QE, are to one another as the sines of the perpendicular arches CD, EF; and the sines of the basis AQ, HK, proportional to IA, GH, the tangents of the perpendicular arches.

For, because CD, EF, are perpendicular to the same plain, they are parallel [a]; as also, FR, DP; therefore the plains of the triangles EFR, CDP, are parallel [b]; and CP, ER, the common sections of these plains, with the plains passing through BE, EO, will be parallel [c]; therefore, the triangles CDP, EFR, are equiangular; wherefore CP, the sine of the hypothenuse BC, is to CD, the sine of the perpendicular arch CA, as ER, the sine of the hypothenuse BE, is to EF, the sine of the perpendicular arch EH [d]. For the same reason, the triangles QAI, KHG, are equiangular. Wherefore QA is to AI, as KH is to HG [d], the tangents of the perpendicular arches. Wherefore, &c.

a 9. 11.
b 18. 11.
c 16. 11.
d 4. 6.

PROP. XVII

IN any right angled spherical triangle, the cosine of the angle at the base, is to the sine of the vertical angle, as the cosine of the perpendicular is to the radius.

1st, Let the triangle be ABC, right angled at A, the cosine of B is to the sine of the angle ACB, as the cosine of CA is to radius. For, let the sides AB, BC, CA, be produced, so that BE, BF, CI, CH, be quadrants; from the poles B, C, draw the great circles EFDG, IHG; then the angles at E, F, I, H, are

are right angles; therefore D is the pole of BAE [a], and G the pole of IFCB; AE the complement of the arch AB, and FE equal GD, the measure of the angle B, and DF their complement; and likewise BC equal IF, the measure of the angle G, and CF their complement; and CA, one of the sides about the right angle, equal to HD, and DC their complement. a 2. cor.

2d, In the triangles HIC, DCF, right angled at I and F, having the same acute angle C. Since BA is less than a quadrant, DF is to DC as IH is to HC [b]; but HC is a quadrant; therefore, the sine of DF is to the sine of DC, as the sine of HI is to the sine of HC, that is, to radius [c]. Wherefore, &c. b 4. 6. c 16.

3d, For the same reason, in the triangles AED, CFD, right-angled at E, F, and having the same acute angle D, as AE is less than a quadrant, the sine of EA is to the sine of CF, as the sine of DA is to the sine of DC; that is, the cosine of the base is to the cosine of the hypothenuse, so is radius to the cosine of the perpendicular.

4th, Again, in the triangles BAC, BEF, right angled at A, E, and having the same acute angle B; as S. BA is to the S. BE, so is the T. AC to the T. EF; that is, the sine of the base is to the rad, as the tangent of the perpendicular to the tangent of the angle at the base.

5th. In the triangle GIF, GHD, right angled at I and H, and having the same acute angle G; S. GH is to the S. GI, as the T. HD to the T. IF; that is, the cosine of the vertical angle C is to rad. as the tangent of the perpendicular is to the tangent of the hypothenuse.

6th. Again, in the same triangles S. IF is to S. GF, as S. HD to sine GD; that is, the S. of the hypothenuse is to rad. as the sine of the perpendicular is to the sine of the angle at the base.

7th. In the triangles HIC, DFC, right-angled at I, F, and the same acute angle C, as S. CI is to S. CF, so is T. HI to T. DF, that is, Rad. is to Cos. BC, so is T. C to Cot. B and S. CH : S. HI. :: S. DC : S. DF; that is, Rad. is to S. ACB :: Cos. AC : Cos. B.

PROP. XVIII.

THE cosines of the angles at the base are proportional to the sines of the vertical angles.

Let BCD be the triangle, either obtuse or acute angled; let fall a perpendicular CA; then, as the cosine of B is to S. BCA, so

ſo is the coſ. CA to radius; and as coſ. CA is to radius, ſo is coſ.
b 17. D to the S. DCA[a]: therefore, by equality[b], coſ. B is to
a 25. 3. S. BCA ſo is coſ. D to S. DCA.

PROP. XIX.

In every ſpherical triangle, the coſines of the ſides are proportional to the coſines of the baſes.

Let BCD be the triangle, the ſame things ſuppoſed as in the
laſt, the coſ. BC is to the coſ. BA as coſ. CA is to rad.; and
a 17. coſ. CA is to rad. as coſ. DC is to coſ. DA[a]; wherefore the coſ.
b 23. 5. BC is to coſ. DC as coſ. BA is to the coſ. of DA[b].

PROP. XX.

In oblique ſpherical triangles, the ſines of the baſes are in the reciprocal proportion of the tangents of the angle at the baſe.

The ſame things ſuppoſed as before, S. BA is to rad. as
a 17. T. AC is to the tangent of the angle at B[a]; and, inverſely,
radius is to S. BA as T. B is to T. AC; and S. DA is to rad.
as T. D is to T. AC; wherefore, S. BA is to S. DA as T. D
b 23. 5. is to T. B[b].

PROP. XXI.

In oblique ſpherical triangles, the tangents of the ſides are in the reciprocal proportion of the coſines of the vertical angles.

The ſame things ſuppoſed as before, R. is to the coſ. ACB as
a 17. T. BC to T. AC[a]; and R. is to coſ. ACD as T. DC to T. AC[a];
and coſ. ACD × T. DC is equal to R. × T. AC; therefore coſ.
b 16. 6. ACB × T. BC is equal to R. × T. AC[b], therefore coſ.
c ax. 1. 1. ACB × T. BC is equal to coſ. ACD × T. DC[c]; therefore
d 14. 6. T. BC is to T. DC, as coſ. ACD to coſ. ACB[d].

PROP. XXII.

In every oblique ſpherical triangle, the ſines of the ſides are proportional to the ſines of their oppoſite angles.

For,

For, the same things supposed as before, S. BC is to rad. as S. CA is to S. B [a]; and S. DC is to rad. as S. AC is to S. D; [b] and, inversely, R. is to S. BC as S. D is to S. AC; wherefore, S. BC is to S. DC as S. D is to S. B [b]. [a] 17. [b] 23. 5.

PROP. XXIII.

IN every spherical triangle, the cotangent of half the sum of the angles at the base, is to the tangent of half their difference, as the tangent of half the vertical angle to the tangent of the angle that the perpendicular makes with the line bisecting the vertical angle.

Let BDC be the triangle, and let CF bisect the vertical angle C; then, as rad. : S. ACB :: cos. AC : cos. of the angle B [a], [a] 17 and rad. : S. ACD :: cos. AC : cos. D; therefore, by eq. and permutation, cos. B : cos. D :: S. ACB : S. ACD; therefore, cos. B + cos. D : cos. B — cos. D :: S. ACB + S. ACD : S. ACB — S. ACD :: cot. $\frac{B+D}{2}$: T. $\frac{B-D}{2}$ :: T. BCF : T. ACF = T. $\frac{B-D}{2}$.

PROP. XXIV.

IN any spherical triangle, the rectangle contained under the sines of two sides, is to the square of the radius, as the difference of the versed sines of the base, and difference of the sides, to the versed sine of the angle opposite to the base.

Let ABC be the triangle, and CF, AE, the sines of the sides AB and CB, or MB = CB; then CF = MF × AE : AO × ON :: IL, the difference of the versed sines of the base AC, and the difference of the sides BC and BA, to the versed sine of the angle B.

For, describe a great circle PN about the pole B; let BP, BN, be quadrants; then the arch PN is the measure of the angle B. From the same pole B describe a lesser circle CFM through C; the plains of these circles will be perpendicular to the plain BON [a]; and, let PG, CH, be perpendicular to the same plain, they will fall on the common sections ON, FM [a], [a] 38. 11. suppose in G and H; and through H draw HI perpendicular to AO; then the plain passing through CH, HI, will be perpendicular to the plain AOB; and AI, which is perpendicular to HI, is likewise perpendicular to CI [b]; and AI is the versed sine [b] 4. 11.

of

of the arch AC, and AL the verſed ſine of the arch AM, which is equal to BM — BA = BC — BA; and, becauſe MF, NO,
c 16. 11. are parallel to CF, PO [c], the iſoſceles triangles are equiangular; therefore, if perpendiculars CH, PG, be drawn to the ſides FM, ON, the triangles will be divided ſimilarly; therefore FM : ON :: MH : GN; and, becauſe the triangles AOE, DIH, DLM, are equiangular AE : AO :: IL : MH; for IL = ID, DL, and MH = MD, DH; but FM : ON :: MH :
d 23, 6. GN; therefore [d], AE × FM, or CF :: A[illegible] × ON : IL × MH; MH × GN; the two laſt divide by M[illegible] will be AE × FM : AO × ON :: IL : GN; that is, the rectangle under the ſines of the legs is to the ſquare of the rad. as the difference of the verſed ſine of the baſe, and the difference of the legs, is to the verſed ſine of the angle B. Wherefore, &c.

PROP. XXV.

IN any ſpherical triangle, the difference of the verſed ſines of two arches, multiplied into half the radius, is equal to the rectangle contained by the ſine of half the ſum, and the ſine of half the difference of theſe arches.

Let there be two arches AC, BA, whoſe difference BC let be biſected in D; then AD is half the ſum, and DB half the difference of theſe arches; and BT = GH the difference of their verſed ſines, and CE the ſine of half their difference. Then, becauſe the triangles ODV, CBT, are equiangular, as DV : BT :: OD : CB :: $\frac{1}{2}$ DO : $\frac{1}{2}$ CB; wherefore DV × $\frac{1}{2}$ BC; or, DV × CE = BT × $\frac{1}{2}$ DO = HG × $\frac{1}{2}$ DO. Wherefore, &c.

PROP. XXVI.

THE verſed ſine of any arch, multiplied by half the radius, is equal to the ſquare of the ſine of half of that arch.

Let the arch be DB, and C its center; draw the right lines DB, BC, and DE, perpendicular to BC; then DE is the ſine of that arch; which let be biſected by the right line CM; then are the triangles CMB, DEB, equiangular; for the angles at M and E are right ones; and the angle at B is common; therefore EB : BD :: BM : BC; therefore EB × BC = BM × BD, and EB × $\frac{1}{2}$ BC = BM × $\frac{1}{2}$ BD = BM ſquare. Wherefore, &c.

PROP. XXVII.

In any spherical triangle, the rectangle contained under the sines of any two sides, is to the square of the radius, as the rectangle contained by the sines of these arches, which are the base and the difference of the sides and the sine of that arch, which is half the difference of the same, to the square of the sine of half the angle opposite to the base.

Let the triangle be ABC, and BC, BA, the sides containing the angle B, and AC the base subtending that angle; and let the arch AM be taken equal to the difference of the legs; then, the rectangle under the sines of the sides BC, BA, will be to the square of the radius, as the rectangle under the sine of the arch $\frac{AC + AM}{2}$; and the sine of the arch $\frac{AC - AM}{2}$ is to the sine of one half the angle B.

For, because the rectangle under the sines of the legs AB, BC, is to the square of the radius, as IL is to the versed sine of the angle B[a]; and, since $\frac{1}{2} R \times IL$ equal to the rectangle under the a 24.
sines of the arches $\frac{AC + AM}{2}$, and $\frac{AC - AM}{2}$[b]; and half b 25.
radius multiplied by the versed sine of the angle B, is equal to the square of the sine of one half the angle B[c]; therefore the c 26.
rectangle under the sine of the sides, is to the square of the rad. as the rectangle under the sines of the arches $\frac{AC + AM}{2}$ and $\frac{AC - AM}{2}$ is to the square of the sine of one half the angle B.

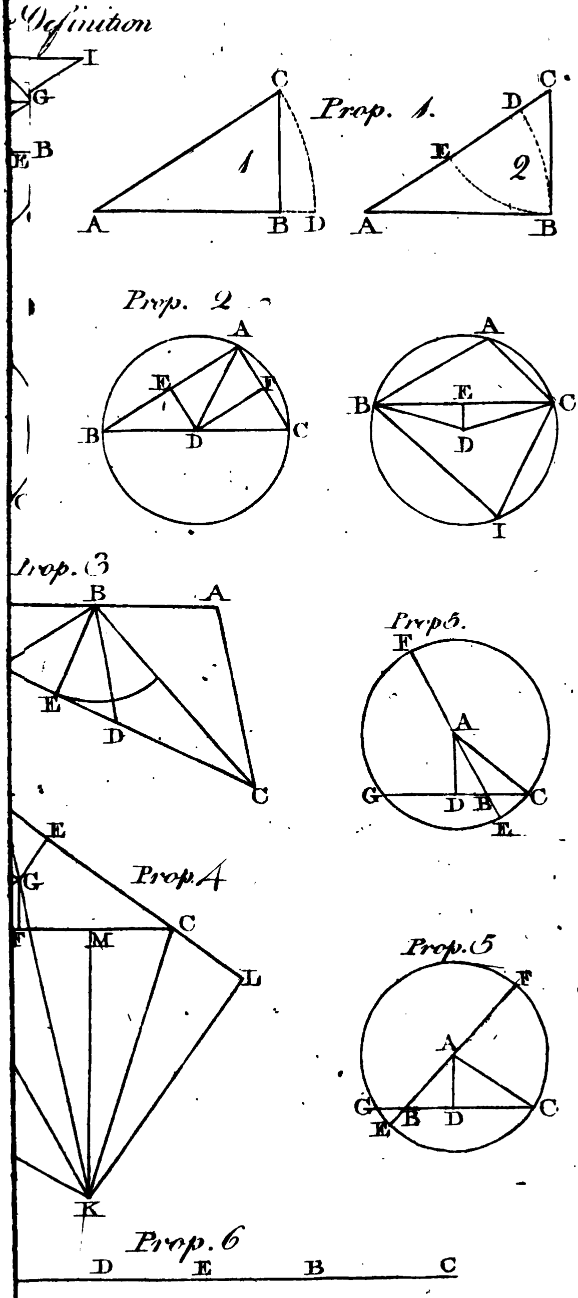
Wherefore, &c.

A short EXPLANATION of the TABLES of LOGARITHMS, and SINES and TANGENTS.

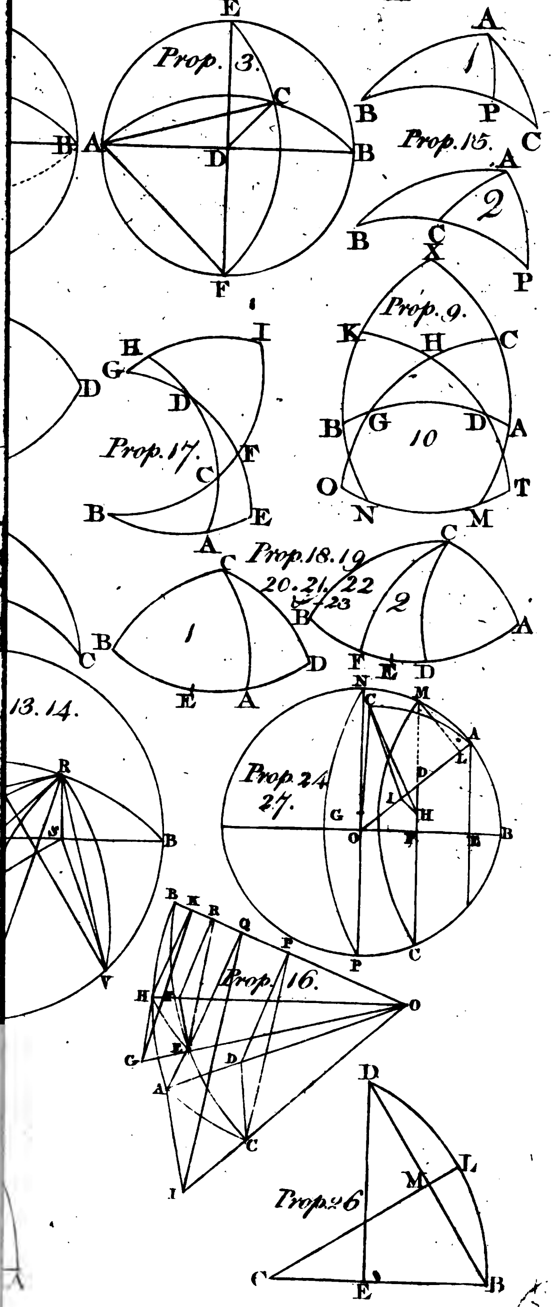
SUPPOSING now the young geometrician acquainted with the Elements of Euclid, that he may likewise be acquainted with the Practice, as well as the Elements of Trigonometry, for which the Tables of Logarithms of Numbers, of Sines, Tangents, &c. are necessary; but, not being so far advanced in mathematics as to understand the proper method of constructing these tables, that depending in a great measure on infinite series; although, at the same time, as much is contained in the Elements of Trigonometry as is sufficient to explain the nature of these, I shall only here give a short definition of Logarithms, and show their different uses in the practice of Trigonometry, &c.

Logarithms, the invention of Lord Napier, published in the year 1614, improved by himself and Mr Brigs of Oxford College, are such an arrangement of artificial numbers, with natural ones, that the addition of the artificial numbers answer to the multiplication of the natural ones, in such a series as is judged most convenient by the calculator. That series which at present is judged most commodious is that which increases in a tenfold proportion, as 1; 10, 100, 1000, 10000, &c. the artificial numbers answering to these, are, 0, 1, 2, 3, 4, &c. which are a series of numbers, in arithmetical progression, beginning with 0, as above, which are the exponents or the indices of the former; the addition or subtraction of which indices answer to the multiplication or division of the numbers, as $04=+1+3=2+2$, &c. which are likewise the indices of $1\times10000=10\times1000=100\times100$, &c. any number of arithmetical means taken betwixt 1 and 10, 10 and 100, &c. answering to the intermediate natural numbers, in such manner as the addition of the one answers to the multiplication of the other, taken to any number of places, constitute a Table of Logarithms, which, taken instead of their correspondent numbers, many tedious operations are evited; e. g. if two numbers are to be multiplied together, add their log.; if to be divided, subtract the log. of the divisor from the log. of the dividend; if a number is to be squared, double its log.; if to be cubed, multiply its log. by 3; if the square-root is to be extracted, half its log.; if the cube-root of any number is to be extracted, take $\frac{1}{3}$ of its log. the number answering to which is the root required; the same of any other power. In finding the log. answering to any number, if the number consist of one place, the exponent is 0; if of 2 places, is 1; if of 3 places, 2, &c.; and, if the log. of any number have its expo-

AIN TRIGONOMETRY



ERICAL TRIGONOMETRY



ERRAT

Pl. Trig. pr. 4. l. 1

Sph. Trig. pr. 3. f

cor. 4·4—pr. 7.

—pr. 8. for a 3.

23.5. r. b 22.5; t

exponent 0, the number is from 1 to 10; if the exponent is 2, the number is from 10 to 100; if 3, from 100 to 1000, &c. But, as the following table consists only of 4 places, or from 1 to 10,000, which is for the most part sufficient; but, if not, by the following easy proportion, it may be extended to any number of places thought necessary. If of 5 places, find the first four places, three of which are in the column below N, and their log. in the column next, on the right hand, in the column below 0; and the others below the fourth figure, which will be found along the top of the column, except such figures as are in common with those in the first columns; but, if the fourth figure to the right hand is a cypher, its log. is the same as the three first figures, but the exponent 3; the same, if all the places to the right hand of the significant figures are cyphers, only putting their exponent according the number of places, as before directed; but, if the figures to the right hand are some significant figures, the first four being found, find the log. of the next greater; then, as that difference is to 1, and cyphers to the number of places, so is the figure given to the log. of these figures.

Example 1. What is the log. of 7675? The log. of 47670 is is 4.6782452, the next greater is 47680=4.6783362; then, as 10, the diff. of the numbers, is to 910, the diff. of their log. so is 5, the number wanted, to 455, the log. to be added to the log. 4.6782452 = 4.6782907.

Ex. 2. What is the log. of 47675478? The log. of 47670000 is 7.6782452, next greater, viz. 47680000, is 7.6783362; then, as 10000, the diff. of the number, is to 910, the diff. of the log. so is 5478 to 498, the log. to be added to 7.6782452 = 7.6782950.

Any Logarithm being given, to find the number answering to it.

Ex. What is the number answering to the log. of 7.5571689? which number, from the exponent, must consist of 8 places. The nearest log. in the table is .5571461, their difference is 228; the difference between the log. found and next greater, is 1204. Then, as 1204 is to 10000, the diff. in places from the number found in the table, and that wanted; so is 228 to 1893.6; which, placed to the right hand of the number already found, is the number answering to the given log. thus 36071893.6; and so of any other.

If the log. of a fraction is wanted, subtract the log. of the denominator from the log. of the numerator; the remainder is the log sought; but its exponent negative the same of a decimal fraction.

If the log. of a mixed number is wanted, find the log. of the whole number, and the fraction, as above, whose log. add to the log. of the whole number, or reduce the mixed number to an improper fraction, and find its log. as above. If the log. of a whole number, and decimal fraction is wanted, find the log. as if all were whole numbers; but prefix the exponents only for the whole numbers. If, at any time, the hyperbolic, or Neper's log. is wanted, the modulus betwixt which and Brigg's log. is the decimal .434294481903, &c. Suppose the hyperbolic log. 10 is wanted, divide Brigg's log. of 10=1,0000000 by the above modulus, gives 2,302585092994; the same of any other. If the hyperbolic log. is given, being multiplied by the modulus, gives Brigg's log.

Of SINES, TANGENTS, &c.

THE radius of the circle being supposed divided into any number of parts, the sines of such a number of these parts as the arches, of which they are the sines, are of a quadrant; and, as the tangent of 45° is equal radius, the parts of the tangent above 45° will be proportionally greater than radius: The same of secants, as they are always greater than radius; but the sines being given, the tangents and secants can be found from the proportion given page 150; the versed sine may be found thus, because the cosine of any arch + the versed sine, is equal to rad. Therefore, from rad. subtract the cosine, gives the versed sine required. The above are natural sines, tangents, &c.

The log. sines, tangents, &c. are only the log. of the natural sines, tangents, &c. so that from the log. tables of numbers, find the log. answering the number of the log. sine, tangent, secants, versed sines, gives the log. sine, tangent, &c. of the arch required.

Ex. If the log. sine of 50° is required, its natural sine is .7660,444, of such parts as the rad. is 1.000,000, the log. of which sine is 9.8842540; and as many places as the natural sine wants of 1. so many units will the log. sine want of index 9; and as many places as the natural tangent or secant exceeds units, so many units will the log. tangent, or secant, exceed index 10; but the log. sine, tangent, &c. may be found without the log. tables, by an infinite series, the rad. of the log. being 10.0000000; the same of any other sine. If the tangent of any arch, as of 50° is wanted, then, because the tangent of any arch is a fourth proportional to the cos. sine, and rad. [a], therefore adding the log sine to rad. and, subtracting its cosine, gives the tangent of that arch.

[a] schol. 1. pl. trig.

Ex.

Ex. Log. S. of 50° + rad. = 19.8842540 — log. of 40° = 9.8080-675 = 10.0761865 = tang. 50°; and, becauſe the ſecant is a third proportional to the coſine and rad. therefore, from twice rad. ſubtract the coſine of 50°, gives 10.1919325, the ſecant of 50°. If the verſed ſine is wanted, ſuppoſe of 50°, ſubtract its coſine from rad. gives 3572.124, its natural ſine; its log. ſine, found, as before directed, is 9.5529265: The ſame thing of any other. But, becauſe the verſed ſine of an arch above 90° is frequently wanted, the exceſs of the arch above rad. + rad. = the verſed ſine above 90°. Suppoſe the verſed ſine of 130° is wanted, its exceſs above rad. is 40°, the natural ſine of which is 6427.876 + R. = 16427.876, its log. is 10.2155814; or, without the natural ſines, thus, the log. of 2 = 30103000 + twice the log. ſine of half the arch, = 20.2155814 — rad. = 10.2155-814 = log. verſed ſine of 130°.

In the table of ſines, tangents, &c. begin, as uſual, in all tables of the like nature with one minute, increaſing to 45°, the degrees below 45° are placed on the left ſide of the page; the minutes anſwering to them increaſe downwards; thoſe above 45° are placed on the right ſide of the page; the minutes anſwering to them increaſe from the bottom of the page upwards, as uſual.

To find the nat. or log. ſine, tangent, &c. of ſecond and third Minutes, for which a Table is calculated, for every ſecond; and each 6‴ to 5 places, at the end of Table of ſines, tangents, &c.

TO find which, find the degrees and minutes in the table of ſines, tangents, &c. and take the difference betwixt that found and next greater; then, as 1′ is to that difference, ſo is ſecond and third minutes, required in the denomination of a minute, which is done in decimals, in the table, at the end of the tables of ſines, &c. to a fourth proportional; which add to the ſine or tangent found in the table of ſines, &c.

Ex. If the nat. ſine of 58°, 19′, 22″, 36‴, is wanted, find the ſine of 58°.19′, the difference betwixt which and 58°.20′, the next greater is 1,528; which, × the decimal of 22″, 36‴, as found in the table, gives 576.056 + 8509, 639 = 8510,215, the decimal .056 being rejected. If the log. ſine of 58° 19′, 22″, 36‴, is wanted, find log. ſine 58° 19′, the difference betwixt which and 58° 20′ is 779 × 377, found, as before, is 293.683 + log. ſine of 58° 19′ = 9.9299112 = 9.9299405, the decimal 683 being rejected, as before: The ſame of any other ſine, tangent, ſecant, whether above or below 45°, or of any verſed ſine, whether above or below 90°.

If

If an arch is given, to find the degrees, minutes, seconds, and thirds answering to that arch.

Find in the table of sines, &c. the nearest, and take the difference betwixt that found and arch given; divide that difference by the difference betwixt that found and next greater, gives a quotient; which, in the table at the end of the sines, &c. shows the second and third minutes answering to it.

Ex. Let the arch given be a sine of 9.9876543, the nearest less is 9.9876488, the sine of 76°.24′, the difference is 55; which divide by the difference betwixt the sine of 76°, 24′, and 76° 25′ = 306, gives 17073, which look for in the table of seconds and thirds, gives 10″, 45‴ Observe the same of any other arch, whether tangent, secant, or versed sine.

A
TABLE
OF
LOGARITHMS

For NUMBERS increasing in their Natural Order, from an Unite to 10,000.

With a Table of Artificial SINES, TANGENTS, and SECANTS, the Radius 10,0000000; and to every Degree and Minute of the Quadrant.

Num.	Log.	Num.	Log.	Num.	Log.
1	0.0000000	34	1.5314789	67	1.8260748
2	0.3010300	35	1.5440680	68	1.8325089
3	0.4771213	36	1.5563025	69	1.8388491
4	0.6020600	37	1.5682017	70	1.8450980
5	0.6989700	38	1.5797836	71	1.8512583
6	0.7781513	39	1.5910646	72	1.8573325
7	0.8450980	40	1.6020600	73	1.8633229
8	0.9030900	41	1.6127839	74	1.8692317
9	0.9542425	42	1.6232493	75	1.8750613
10	1.0000000	43	1.6334685	76	1.8808136
11	1.0413927	44	1.6434527	77	1.8864907
12	1.0791812	45	1.6532125	78	1.8920946
13	1.1139434	46	1.6627578	79	1.8976271
14	1.1461280	47	1.6720979	80	1.9030900
15	1.1760913	48	1.6812412	81	1.9084850
16	1.2041200	49	1.6901961	82	1.9138139
17	1.2304489	50	1.6989700	83	1.9190781
18	1.2552725	51	1.7075702	84	1.9242793
19	1.2787536	52	1.7160033	85	1.9294189
20	1.3010300	53	1.7242759	86	1.9344985
21	1.3222193	54	1.7323938	87	1.9395193
22	1.3424227	55	1.7403627	88	1.9444827
23	1.3617278	56	1.7481880	89	1.9493900
24	1.3802112	57	1.7558749	90	1.9542425
25	1.3979400	58	1.7634280	91	1.9590414
26	1.4149733	59	1.7708520	92	1.9637878
27	1.4313638	60	1.7781513	93	1.9684829
28	1.4471580	61	1.7853298	94	1.9731279
29	1.4623980	62	1.7923917	95	1.9777236
30	1.4771213	63	1.7993405	96	1.9822712
31	1.4913617	64	1.8061800	97	1.9867717
32	1.5051500	65	1.8129134	98	1.9912261
33	1.5185139	66	1.8195439	99	1.9956352

Nm	0	1	2	3	4	5	6	7	8	9
100	0000000	04441	08677	13009	17337	21661	25980	30295	34605	38912
101	0043214	47512	51805	56094	60380	64660	68937	73210	77478	81742
102	0086002	90257	94509	98756	03000	07239	11474	15704	19931	24154
103	0128372	32587	36797	41003	45205	49403	53598	57788	61974	66155
104	0170333	74507	78677	82843	87005	91163	95317	99467	03613	07755
105	0211893	16027	20157	24284	28406	32525	36639	40750	44857	48960
106	0253059	57154	61245	65333	69416	73496	77572	81644	85713	89777
107	0293838	97895	01948	05997	10043	14085	18123	22157	26188	30214
108	0334238	38257	42273	46285	50293	54297	58298	62295	66289	70279
109	0374265	78248	82226	86202	90173	94141	98106	02066	06023	09977
110	0413927	17873	21816	25755	29691	33623	37551	41476	45398	49315
111	0453230	57141	61048	64952	68852	72749	76642	80532	84418	88301
112	0492180	96056	99929	03798	07663	11525	15384	19239	23091	26939
113	0530784	34626	38464	42299	46131	49959	53783	57605	61423	65237
114	0569049	72856	76661	80462	84260	88055	91846	95634	99419	03200
115	0606978	10753	14525	18293	22058	25820	29578	33334	37086	40834
116	0644580	48322	52061	55797	59530	63259	66986	70709	74428	78145
117	0681859	85569	89276	92980	96681	00379	04073	07765	11453	15138
118	0718820	22499	26175	29847	33517	37184	40847	44507	48164	51819
119	0755470	59118	62763	66404	70043	73679	77312	80942	84568	88192
120	0791812	95430	69045	02656	06265	09870	13473	17073	20669	24263
121	0827854	31441	35026	38608	42187	45763	49336	52906	56473	60037
122	0863598	67157	70712	74265	77814	81361	84905	88446	91984	95519
123	0899051	02581	06107	09631	13152	16670	20185	23697	27206	30713
124	0934217	37718	41216	44711	48204	51694	55180	58665	62146	65624
125	0969100	72573	76043	79511	82975	86437	89896	93353	96806	00257
126	1003705	07151	10594	14034	17471	20905	24337	27766	31193	34616
127	1038037	41456	44871	48284	51694	55102	58507	61909	65309	68705
128	1072100	75491	78880	82267	85650	89031	92410	95785	99159	02529
129	1105897	09262	12625	15985	19343	22698	26050	29400	32747	36092
130	1139434	42773	46110	49444	52776	56105	59432	62756	66077	69396
131	1172713	76027	79338	82647	85954	89258	92559	95858	99154	02448
132	1205739	09028	12315	15598	18880	22159	25435	28709	31981	35250
133	1238516	41781	45042	48301	51558	54813	58065	61314	64561	67806
134	1271048	74288	77525	80760	83993	87223	90451	93676	96899	00119
135	1303338	06553	09767	12978	16187	19393	22597	25798	28998	32195
136	1335389	38581	41771	44959	48144	51327	54507	57685	60861	64034
137	1367206	70375	73541	76705	79867	83027	86184	89339	92492	95643
138	1398791	01937	05080	08222	11361	14498	17632	20765	23895	27022
139	1430148	33271	36392	39511	42628	45742	48854	51964	55072	58177
140	1461280	64381	67480	70577	73671	76763	79853	82941	86027	89110
141	1492191	95270	98347	01422	04494	07564	10633	13699	16762	19824
142	1522883	25941	28996	32049	35100	38149	41195	44240	47282	50322
143	1553360	56396	59430	62462	65492	68519	71544	74568	77589	80608
144	1583625	86640	89653	92663	95672	98678	01683	04685	07686	10684
145	1613680	16674	19666	22656	25644	28630	31614	34596	37575	40553
146	1643529	46502	49474	52443	55411	58376	61340	64301	67261	70218
147	1673173	76127	79078	82027	84975	87920	90864	93805	96744	99682
148	1702617	05551	08482	11412	14339	17265	20188	23110	26029	28947
149	1731863	34776	37688	40598	43506	46412	49316	52218	55118	58016
150	1760913	63807	66699	69590	72478	75365	78250	81133	84013	86892
151	1789769	92645	95518	98389	01259	04126	06992	09856	12718	15578
152	1818436	21292	24147	26999	29850	32698	35545	38390	41234	44075
153	1846914	49752	52588	55422	58254	61084	63912	66739	69563	72386
154	1875207	78026	80844	83659	86473	89285	92095	94903	97710	00514
155	1903317	06118	08917	11715	14510	17304	20096	22886	25675	28461
156	1931246	34029	36810	39590	42367	45143	47918	50690	53461	56229
157	1958997	61762	64525	67287	70047	72806	75562	78317	81070	83821
158	1986571	89319	92065	94809	97552	00293	03032	05769	08505	11239
159	2013971	16702	19431	22158	24883	27607	30329	33049	35768	38485

No	0	1	2	3	4	5	6	7	8	9
160	2041200	43913	46625	49335	52044	54750	57455	60159	62860	65560
161	2068259	70955	73650	76344	79035	81725	84414	87100	89785	92468
162	2095150	97830	00508	03185	05860	08534	11205	13876	16544	19211
163	2121876	24540	27202	29862	32521	35178	37833	40487	43139	45790
164	2148438	51086	53732	56376	59018	61659	64298	66936	69572	72207
165	2174839	77471	80100	82729	85355	87980	90603	93225	95845	98464
166	2201081	03696	06310	08922	11533	14942	16750	19356	21960	24563
167	2227165	29764	32363	34959	37555	40148	42740	45331	47920	50507
168	2253093	55677	58260	60841	63421	65999	68576	71151	73724	76296
169	2278867	81436	84004	86570	89134	91697	94258	96818	99377	01934
170	2304489	07043	09596	12146	14696	17244	19790	22335	24879	27421
171	2329961	32500	35038	37574	40108	42641	45173	47703	50232	52759
172	2355284	57809	60331	62853	65373	67891	70408	72923	75437	77950
173	2380461	82971	85479	87986	90491	92995	95497	97998	00498	02996
174	2405492	07988	10482	12974	15465	17954	20442	22929	25414	27898
175	2430380	32861	35341	37819	40296	42771	45245	47718	50189	52658
176	2455127	57594	60059	62523	64986	67447	69907	72365	74823	77278
177	2479733	82186	84637	87087	89536	91984	94430	96874	99318	01759
178	2504200	06639	09077	11513	13949	16382	18815	21246	23675	26103
179	2528530	30956	33380	35803	38224	40645	43063	45481	47897	50312
180	2552725	55137	57548	59957	62365	64772	67177	69582	71984	74386
181	2576786	79185	81582	83978	86373	88766	91158	93549	95939	98327
182	2600714	03099	05484	07867	10248	12629	15008	17385	19762	22137
183	2624511	26883	29255	31625	33993	36361	38727	41092	43455	45817
184	2648178	50538	52896	55253	57609	59964	62317	64669	67020	69369
185	2671717	74064	76410	78754	81097	83439	85780	88119	90457	92794
186	2695129	97464	99797	02129	04459	06788	09116	11443	13769	16093
187	2718416	20738	23058	25378	27696	30013	32328	34643	36956	39268
188	2741578	43888	46196	48503	50809	53114	55417	57719	60020	62320
189	2764618	66915	69211	71506	73800	76092	78383	80673	82962	85250
190	2787536	89821	92105	94388	96669	98950	01229	03507	05784	08059
191	2810334	12607	14879	17150	19419	21688	23955	26221	28486	30750
192	2833012	35274	37534	39793	42051	44307	46563	48817	51070	53322
193	2855573	57823	60071	62319	64565	66810	69054	71296	73538	75778
194	2878017	80255	82492	84728	86963	89196	91428	93660	95890	98118
195	2900346	02573	04798	07022	09246	11468	13689	15908	18127	20344
196	2922561	24776	26990	29203	31415	33626	35835	38044	40251	42457
197	2944662	46866	49069	51271	53471	55671	57869	60067	62263	64458
198	2966652	68845	71037	73227	75417	77605	79792	81979	84164	86348
199	2988531	90713	92893	95073	97252	99429	01605	03781	05955	08128
200	3010300	12471	14641	16809	18977	21144	23309	25474	27637	29799
201	3031961	34121	36280	38438	40595	42751	44905	47059	49212	51363
202	3053514	55663	57812	59959	62105	64250	66394	68537	70680	72820
203	3074960	77099	79237	81374	83509	85644	87778	89910	92042	94172
204	3096302	98430	00557	02684	04809	06933	09056	11178	13300	15420
205	3117539	19657	21774	23889	26004	28118	30231	32343	34454	36563
206	3138672	40780	42887	44992	47097	49201	51303	53405	55505	57605
207	3159703	61801	63898	65993	68088	70181	72273	74365	76455	78545
208	3180633	82721	84807	86893	88977	91061	93143	95224	97305	99384
209	3201463	03540	05617	07692	09767	11840	13913	15984	18055	20124
210	3222193	24261	26327	28393	30457	32521	34584	36645	38706	40766
211	3242825	44882	46939	48995	51050	53104	55157	57209	59260	61310
212	3263359	65407	67454	69500	71545	73589	75633	77675	79716	81757
213	3283796	85834	87872	89909	91944	93979	96012	98045	00077	02108
214	3304138	06167	08195	10222	12248	14273	16297	18320	20343	22364
215	3324385	26404	28423	30440	32457	34473	36488	38501	40514	42526
216	3344538	46548	48557	50565	52573	54579	56585	58589	60593	62596
217	3364597	66598	68598	70597	72595	74593	76589	78584	80579	82572
218	3384565	86557	88547	90537	92526	94514	96502	98488	00473	02458
219	3404441	06424	08405	10386	12366	14345	16323	18301	20277	22252

Nᵒ	0	1	2	3	4	5	6	7	8	9
220	3424227	26200	28173	30145	32116	34086	36055	38023	39991	41957
221	3443923	45887	47851	49814	51776	53737	55698	57657	59615	61573
222	3463530	65486	67441	69395	71348	73300	75252	77202	79152	81101
223	3483049	84996	86942	88887	90832	92775	94718	96660	98601	00541
224	3502480	04419	06356	08293	10229	12163	14098	16031	17963	19895
225	3521825	23755	25684	27612	29539	31465	33391	35316	37239	39162
226	3541084	43006	44926	46846	48764	50682	52599	54515	56431	58345
227	3560259	62171	64038	65994	67905	69814	71723	73630	75537	77443
228	3579348	81253	83156	85059	86961	88862	90762	92662	94560	96458
229	3598355	00251	02146	04041	05934	07827	09719	11610	13500	15390
230	3617278	19166	21053	22939	24825	26709	28593	30476	32358	34239
231	3636120	37999	39878	41756	43633	45510	47386	49260	51134	53007
232	3654880	56751	58622	60492	62361	64230	66097	67964	69830	71695
233	3673559	75423	77285	79147	81008	82869	84728	86587	88445	90302
234	3692159	94014	95869	97723	99576	01428	03280	05131	06981	08830
235	3710679	12526	14373	16219	18065	19909	21753	23596	25438	27279
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697	8422328	32951	33574	34197	34819	35442	36065	36687	37310	37932
698	8438554	39176	39798	40420	41042	41664	42286	42907	43529	44150
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701	8457180	57800	58419	59038	59658	60277	60896	61515	62134	[illegible]
702	8463371	63990	64608	65227	65845	66463	67081	67700	68318	[illegible]
703	8469553	70171	70789	71406	72024	72641	73258	73876	74493	[illegible]
704	8475727	76343	76960	77577	78193	78810	79426	80043	80659	[illegible]
705	8481891	82507	83123	83739	84355	84970	85586	86201	86817	[illegible]
706	8488047	88662	89277	89892	90507	91122	91736	92351	92965	[illegible]
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712	8524800	25410	26020	26629	27239	27849	28458	29068	29677	30286
713	8530895	31504	32113	32722	33331	33940	34548	35157	35765	36374
714	8536982	37590	38198	38807	39414	40022	40630	41238	41845	42453
715	8543060	43668	44275	44882	45489	46096	46703	47310	47917	48524
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717	8555192	55797	56403	57008	57614	58219	58824	59429	60035	60640
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727	8615344	15941	16539	17136	17733	18330	18927	19524	20121	20717
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782	8932068	32623	33178	33733	34288	34843	35398	35953	36508	37063
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788	8965262	65813	66364	66915	67466	68017	68568	69118	69669	70220
789	8970770	71320	71871	72421	72971	73521	74071	74621	75171	75721
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791	8981765	82314	82863	83412	83960	84509	85058	85606	86155	86703
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813	9100905	01440	01974	02508	03042	03576	04109	04643	05177	05710
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818	9127533	28064	28595	29126	29656	30187	30717	31248	31778	32309
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833	9206450	06971	07493	08014	08535	09056	09577	10098	10619	11140
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842	9253121	53637	54152	54668	55184	55699	56215	56730	57245	57761
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894	9513375	13861	14347	14832	15318	15803	16289	16774	17260	17745
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907	9576073	76552	77030	77509	77988	78466	78945	79423	79902	80380
908	9580858	81337	81815	82293	82771	83249	83727	84205	84683	85161
909	9585639	86117	86594	87072	87549	88027	88505	88982	89459	89937
910	9590414	90891	91368	91845	92322	92800	93276	93753	94230	94707
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914	9609462	09937	10412	10887	11362	11837	12312	12787	13262	13736
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921	9642596	43068	43539	44011	44482	44953	45425	45896	46367	46838
922	9647309	47780	48251	48722	49193	49664	50135	50605	51076	51546
923	9652017	52488	52958	53428	53899	54369	54839	55309	55780	56250
924	9656720	57190	57660	58130	58599	59069	59539	60009	60478	60948
925	9661417	61887	62356	62826	63295	63764	64233	64703	65172	65641
926	9666110	66579	67048	67517	67985	68454	68923	69392	69860	70329
927	9670797	71266	71734	72203	72671	73139	73607	74076	74544	75012
928	9675480	75948	76416	76884	77351	77819	78287	78754	79222	79690
929	9680157	80625	81092	81559	82027	82494	82961	83428	83895	84362
930	9684829	85296	85763	86230	86697	87164	87630	88097	88564	89030
931	9689497	89963	90430	90896	91362	91829	92295	92761	93227	93693
932	9694159	94625	95091	95557	96023	96488	96954	97420	97885	98351
933	9698816	99282	99747	00213	00678	01143	01608	02074	02539	03004
934	9703469	03934	04399	04863	05328	05793	06258	06722	07187	07652
935	9708116	08581	09045	09509	09974	10438	10902	11366	11830	12294
936	9712758	13222	13686	14150	14614	15078	15542	16005	16469	16932
937	9717396	17859	18333	18786	19249	19713	20176	20639	21102	21565
938	9722028	22491	22954	23417	23880	24343	24805	25268	25731	26193
939	9726656	27118	27581	28043	28506	28968	29430	29892	30354	30816

Nm	0	1	2	3	4	5	6	7	8	9
940	9731279	31741	32202	32664	33126	33588	34050	34511	34973	35435
941	9733896	36358	36819	37281	37742	38208	38664	39126	39587	40048
942	9740509	40970	41431	41892	42353	42814	43274	43735	44196	44656
943	9745117	45577	46038	46498	46959	47419	47879	48340	48800	49260
944	9749720	50180	50640	51100	51560	52020	52479	52939	53399	53858
945	9754318	54778	55237	55697	56156	56615	57075	57534	57993	58452
946	9758911	59370	59829	60288	60747	61206	61665	62124	62582	63041
947	9763500	63958	64417	64875	65334	65792	66251	66709	67167	67625
948	9768083	68541	69000	69458	69915	70373	70831	71289	71747	72204
949	9772662	73120	73577	74035	74492	74950	75407	75864	76322	76779
950	9777236	77693	78150	78607	79064	79521	79978	80435	80892	81348
951	9781805	82262	82718	83175	83631	84088	84544	85001	85457	85913
952	9786369	86826	87282	87738	88194	88650	89106	89562	90017	90473
953	9790929	91385	91840	92296	92751	93207	93662	94118	94573	95028
954	9795484	95939	96394	96849	97304	97759	98214	98669	99124	99579
955	9800034	00488	00943	01398	01852	02307	02761	03216	03670	04125
956	9804579	05033	05487	05942	06396	06850	07304	07758	08212	08666
957	9809119	09573	10027	10481	10934	11388	11841	12295	12748	13202
958	9813655	14108	14562	15015	15468	15921	16374	16827	17280	17733
959	9818186	18639	19092	19544	19997	20450	20902	21355	21807	22260
960	9822712	23165	23617	24069	24522	24974	25426	25878	26330	26782
961	9827234	27686	28138	28589	29041	29493	29945	30396	30848	31299
962	9831751	32202	32654	33105	33556	34007	34459	34910	35361	35812
963	9836263	36714	37165	37616	38066	38517	38968	39419	39869	40320
964	9840770	41221	41671	42122	42572	43022	43473	43923	44373	44823
965	9845273	45723	46173	46623	47073	47523	47973	48422	48872	49322
966	9849771	50221	50670	51120	51569	52019	52468	52917	53366	53816
967	9854265	54714	55163	55612	56061	56510	56959	57407	57856	58305
968	9858754	59202	59651	60099	60548	60996	61445	61893	62341	62790
969	9863238	63686	64134	64582	65030	65478	65926	66374	66822	67270
970	9867717	68165	68613	69060	69508	69955	70403	70850	71298	71745
971	9872192	72640	73087	73534	73981	74428	74875	75322	75769	76216
972	9876663	77109	77556	78003	78450	78896	79343	79789	80236	80682
973	9881128	81575	82021	82467	82913	83360	83806	84252	84698	85144
974	9885590	86035	86481	86927	87373	87818	88264	88710	89155	89601
975	9890046	90492	90937	91382	91828	92273	92718	93163	93608	94053
976	9894498	94943	95388	95833	96278	96722	97167	97612	98057	98501
977	9898946	99390	99835	00279	00723	01168	00612	02056	02500	02944
978	9903389	03833	04277	04721	05164	05608	06052	06496	06940	07383
979	9907827	08271	08714	09158	09601	10044	10488	10931	11374	11818
980	9912261	12704	13147	13590	14033	14476	14919	15362	15805	16247
981	9916690	17133	17575	18018	18461	18903	19345	19788	20230	20673
982	9921115	21557	21999	22441	22884	23326	23768	24210	24651	25093
983	9925535	25977	26419	26860	27302	27744	28185	28627	29068	29510
984	9929951	30392	30834	31275	31716	32157	32598	33039	33480	33921
985	9934362	34803	35244	35685	36125	36566	37007	37447	37888	38329
986	9938769	39209	39650	40090	40531	40971	41411	41851	42291	42731
987	9943171	43611	44051	44491	44931	45371	45811	46250	46690	47130
988	9947569	48009	48448	48888	49327	49767	50206	50645	51085	51524
989	9951963	52402	52841	53280	53719	54158	54597	55036	55474	55913
990	9956352	56791	57229	57668	58106	58545	58983	59422	59860	60298
991	9960736	61175	61613	62051	62489	62927	63365	63803	64241	64679
992	9965117	65554	65992	66430	66867	67305	67743	68180	68618	69055
993	9969492	64930	70367	70804	71242	71679	72116	72553	72990	73427
994	9973864	74301	74738	75174	75611	76048	76484	76921	77358	77794
995	9978231	78667	79104	79540	79976	80413	80849	81285	81721	82157
996	9982593	83029	83465	83901	84337	84773	85209	85645	86080	86516
997	9986952	87387	87823	88258	88694	89129	89564	90000	90435	90870
998	9991305	91740	92176	92611	93046	93481	93916	94350	94785	95220
999	9995655	96089	96524	96959	97393	97828	98262	98697	99131	99566

A
TABLE
OF
Artificial SINES, TANGENTS, and SECANTS.

o Degree.

M	Sine.		Tang.		Secant.		
0	0.0000000	10.0000000	0.0000000	Infinite.	10.0000000	Infinite.	60
1	4637261	9.9999999	6.4637261	13.5362739	0000000	13.5362739	59
2	7647561	9999999	7647562	2352438	0000001	2352439	58
3	9408473	9999998	9408475	0591525	0000002	0591527	57
4	7.0657860	9999997	7.0657863	12.9342137	0000003	12.9342140	56
5	1626960	9999995	1626964	8373036	0000005	8373040	55
6	2418771	9999993	2418778	7581222	0000007	7581229	54
7	3088239	9999991	3088248	6911752	0000009	6911761	53
8	3668157	9999988	3668169	6331831	0000012	6331843	52
9	4179681	9999985	4179696	5820304	0000015	5820319	51
10	4637255	9999982	4637273	5362727	0000018	5362745	50
11	5051181	9999978	5051203	4948797	0000022	4948819	49
12	5429065	9999974	5429091	4570909	0000026	4570935	48
13	5776684	9999969	5776715	4223285	0000031	4223316	47
14	6098530	9999964	6098566	3901434	0000036	3901470	46
15	6398160	9999959	6398201	3601799	0000041	3601840	45
16	6678445	9999953	6678492	3321508	0000047	3321555	44
17	6941733	9999947	6941786	3058214	0000053	3058267	43
18	7189966	9999940	7190026	2809974	0000060	2810034	42
19	7424775	9999934	7424841	2575159	0000066	2575225	41
20	7647537	9999927	7647610	2352390	0000073	2352463	40
21	7859427	9999919	7859508	2140492	0000081	2140573	39
22	8061458	9999911	8061547	1938453	0000089	1938542	38
23	8254507	9999903	8254604	1745396	0000097	1745493	37
24	8439338	9999894	8439444	1560556	0000106	1560662	36
25	8616623	9999885	8616738	1383262	0000115	1383377	35
26	8786953	9999876	8787077	1212923	0000124	1213047	34
27	8950854	9999866	8950988	1049012	0000134	1049146	33
28	9108793	9999856	9108938	0891062	0000144	0891207	32
29	9261190	9999845	9261344	0738656	0000155	0738810	31
30	9408419	9999835	9408584	0591416	0000165	0591581	30
31	9550819	9999823	9550996	0449004	0000177	0449181	29
32	9688698	9999812	9688886	0311114	0000188	0311302	28
33	9822334	9999800	9822534	0177466	0000200	0177666	27
34	9951980	9999788	9952192	0047808	0000212	0048020	26
35	8.0077867	9999775	8.0078092	11.9921908	0000225	11.9922133	25
36	0200207	9999762	0200445	9799555	0000238	9799793	24
37	0319195	9999748	0319446	9680554	0000252	9680805	23
38	0435009	9999735	0435274	9564726	0000265	9564991	22
39	0547814	9999721	0548094	9451906	0000279	9452186	21
40	0657763	9999706	0658057	9341943	0000294	9342237	20
41	0764997	9999691	0765306	9234694	0000309	9235003	19
42	0869646	9999676	0869970	9130030	0000324	9130354	18
43	0971832	9999660	0972172	9027828	0000340	9028168	17
44	1071669	9999644	1072025	8927975	0000356	8928331	16
45	1169262	9999628	1169634	8830366	0000372	8830738	15
46	1264710	9999611	1265099	8734901	0000389	8735290	14
47	1358104	9999594	1358510	8641490	0000406	8641896	13
48	1449532	9999577	1449956	8550044	0000423	8550468	12
49	1539075	9999559	1539516	8460484	0000441	8460925	11
50	1626808	9999541	1627267	8372733	0000459	8373192	10
51	1712804	9999522	1713282	8286718	0000478	8287196	9
52	1797129	9999503	1797626	8202374	0000497	8202871	8
53	1879848	9999484	1880364	8119636	0000516	8120152	7
54	1961020	9999464	1961556	8038444	0000536	8038980	6
55	2040703	9999444	2041259	7958741	0000556	7959297	5
56	2118949	9999424	2119526	7880474	0000576	7881051	4
57	2195811	9999403	2196408	7803592	0000597	7804189	3
58	2271335	9999382	2271953	7728047	0000618	7728665	2
59	2345568	9999360	2346208	7653792	0000640	7654432	1
60	2418553	9999338	2419215	7580785	0000662	7581447	0

1 Degree.

M	Sine.		Tang.		Secant.		
0	8.2418553	9.9999338	8.2419215	11.7580785	10.0000662	11.7581447	60
1	2490332	9999316	2491015	7508985	0000684	7509668	59
2	2560943	9999294	2561649	7438351	0000706	7439057	58
3	2630424	9999271	2631153	7368847	0000729	7369576	57
4	2698810	9999247	2699563	7300437	0000753	7301190	56
5	2766136	9999224	2766912	7233088	0000776	7233864	55
6	2832434	9999200	2833234	7166766	0000800	7167566	54
7	2897734	9999175	2898559	7101441	0000825	7102266	53
8	2962067	9999150	2962917	7037083	0000850	7037933	52
9	3025460	9999125	3026335	6973665	0000875	6974540	51
10	3087941	9999100	3088842	6911158	0000900	6912059	50
11	3149536	9999074	3150462	6849538	0000926	6850464	49
12	3210269	9999047	3211221	6788779	0000953	6789731	48
13	3270163	9999021	3271143	6728857	0000979	6729837	47
14	3329243	9998994	3330249	6669751	0001006	6670757	46
15	3387529	9998966	3388563	6611437	0001034	6612471	45
16	3445043	9998939	3446105	6553895	0001061	6554957	44
17	3501805	9998911	3502895	6497105	0001089	6498195	43
18	3557835	9998882	3558953	6441047	0001118	6442165	42
19	3613150	9998853	3614297	6385703	0001147	6386850	41
20	3667769	9998824	3668945	6331055	0001176	6332231	40
21	3721210	9998794	3722915	6277085	0001206	6278290	39
22	3774988	9998764	3776223	6233777	0001236	6225012	38
23	3827620	9998734	3828886	6171114	0001266	6172380	37
24	3879622	9998703	3880918	6119082	0001297	6120378	36
25	3931008	9998672	3932336	6067664	0001328	6068992	35
26	3981793	9998641	3983152	6016848	0001359	6018207	34
27	4031990	9998609	4033381	5966619	0001391	5968010	33
28	4081614	9998577	4083037	5916963	0001423	5918386	32
29	4130676	9998544	4132132	5867868	0001456	5869324	31
30	4179190	9998512	4180679	5819321	0001488	5820810	30
31	4227168	9998478	4228690	5771310	0001522	5772832	29
32	4274621	9998445	4276176	5723824	0001555	5725379	28
33	4321561	9998411	4323150	5676850	0001589	5678439	27
34	4367999	9998376	4369622	5630378	0001624	5632001	26
35	4413944	9998342	4415603	5584397	0001658	5586056	25
36	4459409	9998306	4461103	5538897	0001694	5540591	24
37	4504402	9998271	4506131	5493869	0001729	5495598	23
38	4548934	9998235	4550699	5449301	0001765	5451066	22
39	4593013	9998199	4594814	5405186	0001801	5406987	21
40	4636649	9998162	4638486	5361514	0001838	5363351	20
41	4679850	9998125	4681725	5318275	0001875	5320150	19
42	4722626	9998088	4724538	5275462	0001912	5277374	18
43	4764984	9998050	4766933	5233067	0001950	5235016	17
44	4806932	9998012	4808920	5191080	0001988	5193068	16
45	4848479	9997974	4850505	5149495	0002026	5151521	15
46	4889632	9997935	4891696	5108304	0002065	5110368	14
47	4930398	9997896	4932502	5067498	0002104	5069602	13
48	4970784	9997856	4972928	5027072	0002144	5029216	12
49	5010798	9997817	5012982	4987018	0002183	4989202	11
50	5050447	9997776	5052671	4947329	0002224	4949553	10
51	5089736	9997736	5092001	4987999	0002264	4910264	9
52	5128673	9997695	5130978	4869022	0002305	4871327	8
53	5167264	9997653	5169610	4830390	0002347	4832736	7
54	5205514	9997612	5207902	4792098	0002388	4794486	6
55	5243430	9997570	5245860	4754140	0002430	4756570	5
56	5281017	9997527	5283490	4716510	0002473	4718983	4
57	5318281	9997484	5320797	4679203	0002516	4681719	3
58	5355228	9997441	5357787	4642213	0002559	4644772	2
59	5391863	9997398	5394466	4605534	0002602	4608137	1
60	5428192	9997354	5430838	4569162	0002646	4571808	0

2 Degrees.

M	Sine.		Tang.		Secant.		
0	8.5428192	9.9997354	8.5430838	11.4569162	10.0002646	11.4571808	60
1	5464218	9997309	5466909	4533091	0002691	4535782	59
2	5499948	9997265	5502683	4497317	0002735	4500052	58
3	5535386	9997220	5538166	4461834	0002780	4464614	57
4	5570536	9997174	5573362	4426638	0002826	4429464	56
5	5605404	9997128	5608276	4391724	0002872	4394596	55
6	5639994	9997082	5642912	4357088	0002918	4360006	54
7	5674310	9997036	5677275	4322725	0002964	4325690	53
8	5708357	9996989	5711368	4288632	0003011	4291643	52
9	5742129	9996942	5745197	4254803	0003058	4257861	51
10	5775660	9996894	5778766	4221234	0003106	4224340	50
11	5808923	9996846	5812077	4187923	0003154	4191077	49
12	5841933	9996798	5845136	4154864	0003202	4158067	48
13	5874694	9996749	5877945	4122055	0003251	4125306	47
14	5907209	9996700	5910509	4089491	0003300	4092791	46
15	5939483	9996650	5942832	4057168	0003350	4060517	45
16	5971517	9996601	5974917	4025083	0003399	4028483	44
17	6003317	9996550	6006767	3993233	0003450	3996683	43
18	6034886	9996500	6038386	3961614	0003500	3965114	42
19	6066226	9996449	6069777	3930223	0003551	3933774	41
20	6097341	9996398	6100943	3899057	0003602	3902659	40
21	6128235	9996346	6131889	3868111	0003654	3871765	39
22	6158910	9996294	6162616	3837384	0003706	3841090	38
23	6189369	9996242	6193127	3806873	0003758	3810631	37
24	6219616	9996189	6223427	3776573	0003811	3780384	36
25	6249653	9996136	6253518	3746482	0003864	3750347	35
26	6279484	9996082	6283402	3716598	0003918	3720516	34
27	6309111	9996028	6313083	3686917	0003972	3690889	33
28	6338537	9995974	6342563	3657437	0004026	3661463	32
29	6367764	9995919	6371845	3628155	0004081	3632236	31
30	6396796	9995865	6400931	3599069	0004135	3603204	30
31	6425634	9995809	6429825	3570175	0004191	3574366	29
32	6454282	9995753	6458528	3541472	0004247	3545718	28
33	6482742	9995697	6487044	3512956	0004303	3517258	27
34	6511016	9995641	6515375	3484625	0004359	3488984	26
35	6539107	9995584	6543522	3456478	0004416	3460893	25
36	6567017	9995527	6571490	3428510	0004473	3432983	24
37	6594748	9995469	6599279	3400721	0004531	3405252	23
38	6622303	9995411	6626891	3373109	0004589	3377697	22
39	6649684	9995353	6654331	3345669	0004647	3350316	21
40	6676893	9995295	6681598	3318402	0004705	3323107	20
41	6703932	9995236	6708697	3291303	0004764	3296068	19
42	6730804	9995176	6735628	3264372	0004824	3269196	18
43	6757510	9995116	6762393	3237607	0004884	3242490	17
44	6784052	9995056	6788996	3211004	0004944	3215948	16
45	6810433	9994996	6815437	3184563	0005004	3189567	15
46	6836654	9994935	6841719	3158281	0005065	3163346	14
47	6862718	9994874	6867844	3132156	0005126	3137282	13
48	6888625	9994812	6893813	3106187	0005188	3111375	12
49	6914379	9994750	6919629	3080371	0005250	3085621	11
50	6939986	9994688	6945292	3054708	0005312	3060020	10
51	6965431	9994625	6970806	3029194	0005375	3034569	9
52	6990734	9994562	6996172	3003828	0005438	3009266	8
53	7015889	9994498	7021390	2978610	0005502	2984111	7
54	7040899	9994435	7046465	2953535	0005565	2959101	6
55	7065766	9994370	7071395	2928605	0005630	2934234	5
56	7090490	9994306	7096185	2903815	0005694	2909510	4
57	7115075	9994241	7120834	2879166	0005759	2884925	3
58	7139520	9994176	7145345	2854655	0005824	2860480	2
59	7163829	9994110	7169719	2830281	0005890	2836171	1
60	7188002	9994044	7193958	2806042	0005956	2811998	0

3 Degrees.

M	Sine.		Tang.		Secant.		
0	8.7188002	9.9994044	8.7193958	11.2806042	10.0005956	11.2811998	60
1	7212040	9993978	7218063	2781937	0006022	2787960	59
2	7235946	9993911	7242035	2757965	0006089	2764054	58
3	7259721	9993844	7265877	2734123	0006156	2740279	57
4	7283366	9993776	7289589	2710411	0006224	2716634	56
5	7306882	9993708	7313174	2686826	0006292	2693118	55
6	7330272	9993640	7336631	2663369	0006360	2669728	54
7	7353535	9993572	7359964	2640036	0006428	2646465	53
8	7376675	9993503	7383172	2616828	0006497	2623325	52
9	7399691	9993433	7406258	2593742	0006567	2600309	51
10	7422586	9993364	7429222	2570778	0006636	2577414	50
11	7445360	9993293	7452067	2547933	0006707	2554640	49
12	7468015	9993223	7474792	2525208	0006777	2531985	48
13	7490553	9993152	7497400	2502600	0006848	2509447	47
14	7512973	9993081	7519892	2480108	0006919	2487027	46
15	7535278	9993009	7542269	2457731	0006991	2464722	45
16	7557469	9992938	7564531	2435469	0007062	2442531	44
17	7579546	9992865	7586681	2413319	0007135	2420454	43
18	7601512	9992793	7608719	2391281	0007207	2398488	42
19	7623366	9992720	7630647	2369353	0007280	2376634	41
20	7645111	9992646	7652465	2347535	0007354	2354889	40
21	7666747	9992572	7674175	2325825	0007428	2333253	39
22	7688275	9992498	7695777	2304223	0007502	2311725	38
23	7709697	9992424	7717274	2282726	0007576	2290303	37
24	7731014	9992349	7738665	2261335	0007651	2268986	36
25	7752226	9992274	7759952	2240048	0007726	2247774	35
26	7773334	9992198	7781136	2218864	0007802	2226666	34
27	7794340	9992122	7902218	2197782	0007878	2205660	33
28	7815244	9992046	7823199	2176801	0007954	2184756	32
29	7836048	9991969	7844079	2155921	0008031	2163952	31
30	7856753	9991892	7864861	2135139	0008108	2143247	30
31	7877359	9991815	7885544	2114456	0008185	2122641	29
32	7897867	9991737	7906130	2093870	0008263	2102133	28
33	7918278	9991659	7926620	2073380	0008341	2081722	27
34	7938594	9991580	7947014	2052986	0008420	2061406	26
35	7958814	9991501	7967313	2032687	0008499	2041186	25
36	7978941	9991422	7987519	2012481	0008578	2021059	24
37	7998974	9991342	8007632	1992368	0008658	2001026	23
38	8018915	9991262	8027653	1972347	0008738	1981085	22
39	8038764	9991182	8047583	1952417	0008818	1961236	21
40	8058523	9991101	8067422	1932578	0008899	1941477	20
41	8078192	9991020	8087172	1912828	0008980	1921808	19
42	8097772	9990938	8106834	1893166	0009062	1902228	18
43	8117264	9990856	8126407	1873593	0009144	1882736	17
44	8136668	9990774	8145894	1854106	0009226	1863332	16
45	8155985	9990691	8165294	1834706	0009309	1844015	15
46	8175217	9990608	8184608	1815392	0009392	1824783	14
47	8194363	9990525	8203838	1796162	0009475	1805637	13
48	8213425	9990441	8222984	1777016	0009559	1786575	12
49	8232404	9990357	8242046	1757954	0009643	1767596	11
50	8251299	9990273	8261026	1738974	0009727	1748701	10
51	8270112	9990188	8279924	1720076	0009812	1729888	9
52	8288844	9990103	8298741	1701259	0009897	1711156	8
53	8307495	9990017	8317478	1682522	0009983	1692505	7
54	8326066	9989931	8336134	1663866	0010069	1673934	6
55	8344557	9989845	8354712	1645288	0010155	1655443	5
56	8362969	9989758	8373211	1626789	0010242	1637031	4
57	8381304	9989671	8391633	1608367	0010329	1618696	3
58	8399561	9989584	8409977	1590023	0010416	1600439	2
59	8417741	9989496	8428245	1571755	0010504	1582259	1
60	8435845	9989408	8446437	1553563	0010592	1564155	0

4 Degrees.

M	Sine.		Tang.		Secant.		
0	8.8435845	9.9989408	8.8446437	11.1553563	10.0010592	11.1564155	[illegible]
1	8453874	9989319	8464554	1535446	0010681	1546126	[illegible]
2	8471827	9989230	8482597	1517403	0010770	1528173	[illegible]
3	8489707	9989141	8500566	1499434	0010859	1510293	[illegible]
4	8507512	9989052	8518461	1481539	0010948	1492488	[illegible]
5	8525245	9988962	8536283	1463717	0011038	1474755	55
6	8542905	9988871	8554034	1445966	0011129	1457095	54
7	8560493	9988780	8571713	1428287	0011220	1439507	53
8	8578010	9988689	8589321	1410679	0011311	1421990	52
9	8595457	9988598	8606859	1393141	0011402	1404543	51
10	8612833	9988506	8624327	1375673	0011494	1387167	50
11	8630139	9988414	8641725	1358275	0011586	1369861	49
12	8647376	9988321	8659055	1340945	0011679	1352624	48
13	8664545	9988228	8676317	1323683	0011772	1335455	47
14	8681646	9988135	8693511	1306489	0011865	1318354	46
15	8698680	9988041	8710638	1289362	0011959	1301320	45
16	8715646	9987947	8727699	1272301	0012053	1284354	44
17	8732546	9987853	8744694	1255306	0012147	1267454	43
18	8749381	9987758	8761623	1238377	0012242	1250619	42
19	8766150	9987663	8778487	1221513	0012337	1233850	41
20	8782854	9987567	8795286	1204714	0012433	1217146	40
21	8799493	9987471	8812022	1187978	0012529	1200507	39
22	8816069	9987375	8828694	1171306	0012625	1183931	38
23	8832581	9987278	8845303	1154697	0012722	1167419	37
24	8849031	9987181	8861850	1138150	0012819	1150969	36
25	8865418	9987084	8878334	1121666	0012916	1134582	35
26	8881743	9986986	8894757	1105243	0013014	1118257	34
27	8898007	9986888	8911119	1088881	0013112	1101993	33
28	8914209	9986790	8927420	1072580	0013210	1085791	32
29	8930351	9986691	8943660	1056340	0013309	1069649	31
30	8946433	9986591	8959842	1040158	0013409	1053567	30
31	8962455	9986492	8975963	1024037	0013508	1037545	29
32	8978418	9986392	8992026	1007974	0013608	1021582	28
33	8994322	9986292	9008030	0991970	0013708	1005678	27
34	9010168	9986191	9023977	0976023	0013809	0989832	26
35	9025955	9986090	9039866	0960134	0013910	0974045	25
36	9041685	9985938	9055697	0944303	0014012	0958315	24
37	9057358	9985886	9071472	0928528	0014114	0942642	23
38	9072975	9985784	9087190	0912810	0014216	0927025	22
39	9088535	9985682	9102853	0897147	0014318	0911465	21
40	9104039	9985579	9118460	0881540	0014421	0895961	20
41	9119487	9985475	9134012	0865988	0014525	0880513	19
42	9134881	9985372	9149509	0850491	0014628	0865119	18
43	9150219	9985268	9164952	0835048	0014732	0849781	17
44	9165504	9985163	9180340	0819660	0014837	0834496	16
45	9180734	9985058	9195675	0804325	0014942	0819266	15
46	9195911	9984953	9210957	0789043	0015047	0804089	14
47	9211034	9984848	9226186	0773814	0015152	0788966	13
48	9226105	9984742	9241363	0758637	0015258	0773895	12
49	9241123	9984636	9256487	0743513	0015364	0758877	11
50	9256089	9984529	9271560	0728440	0015471	0743911	10
51	9271003	9984422	9286581	0713419	0015578	0728997	9
52	9285866	9984315	9301552	0698448	0015685	0714134	8
53	9300678	9984207	9316471	0683529	0015793	0699322	7
54	9315439	9984099	9331340	0668660	0015901	0684561	6
55	9330150	9983990	9346160	0653840	0016010	0669850	5
56	9344811	9983881	9360929	0639071	0016119	0655189	4
57	9359422	9983772	9375650	0624350	0016228	0640578	3
58	9373983	9983663	9390321	0609679	0016337	0626017	2
59	9388496	9983553	9404944	0595056	0016447	0611504	1
60	9402960	9983442	9419518	0580482	0016558	0597040	0

5 Degrees.

M	Sine.		Tang.		Secant.		
0	8.9402960	9.9983442	8.9419518	11.0580482	10.0016558	11.0597040	60
1	9417376	9983332	9434044	0565956	0016668	0582624	59
2	9431743	9983220	9448523	0551477	0016780	0568257	58
3	9446063	9983109	9462954	0537046	0016891	0553937	57
4	9460335	9982997	9477338	0522662	0017003	0539665	56
5	9474561	9982885	9491676	0508324	0017115	0525439	55
6	9488739	9982772	9505967	0494033	0017228	0511261	54
7	9502871	9982660	9520211	0479789	0017340	0497129	53
8	9516957	9982546	9534410	0465590	0017454	0483043	52
9	9530996	9982433	9548564	0451436	0017567	0469004	51
10	9544991	9982318	9562672	0437328	0017682	0455009	50
11	9558940	9982204	9576735	0423265	0017796	0441060	49
12	9572843	9982089	9590754	0409246	0017911	0427157	48
13	9586703	9981974	9604728	0395272	0018026	0413297	47
14	9600517	9981859	9618659	0381341	0018141	0399483	46
15	9614288	9981743	9632545	0367455	0018257	0385712	45
16	9628014	9981626	9646388	0353612	0018374	0371986	44
17	9641697	9981510	9660188	0339812	0018490	0358303	43
18	9655337	9981393	9673944	0326056	0018607	0344663	42
19	9668934	9981275	9687658	0312342	0018725	0331066	41
20	9682487	9981158	9701330	0298670	0018842	0317513	40
21	9695999	9981040	9714959	0285041	0018960	0304001	39
22	9709468	9980921	9728547	0271453	0019079	0290532	38
23	9722895	9980802	9742092	0257908	0019198	0277105	37
24	9736280	9980683	9755597	0244403	0019317	0263720	36
25	9749624	9980563	9769060	0230940	0019437	0250376	35
26	9762926	9980443	9782483	0217517	0019557	0237074	34
27	9776188	9980323	9795865	0204135	0019677	0223812	33
28	9789408	9980202	9809206	0190794	0019798	0210592	32
29	9802589	9980081	9822507	0177493	0019919	0197411	31
30	9815729	9979960	9835769	0164231	0020040	0184271	30
31	9828829	9979838	9848991	0151009	0020162	0171171	29
32	9841889	9979716	9862173	0137827	0020284	0158111	28
33	9854910	9979593	9875317	0124683	0020407	0145090	27
34	9867891	9979470	9888421	0111579	0020530	0132109	26
35	9880834	9979347	9901487	0098513	0020653	0119166	25
36	9893737	9979223	9914514	0085486	0020777	0066263	24
37	9906602	9979099	9927503	0072497	0020901	0093398	23
38	9919429	9978975	9940454	0059546	0021025	0080571	22
39	9932217	9978850	9953367	0046633	0021150	0067783	21
40	9944968	9978725	9966243	0033757	0021275	0055032	20
41	9957681	9978599	9979081	0020919	0021401	0042319	19
42	9970356	9978473	9991883	0008117	0021527	0029644	18
43	9982994	9978347	9.0004647	10.9995353	0021653	0017006	17
44	9995595	9978220	0017375	9982625	0021780	0004405	16
45	9.0008160	9978093	0030066	9969934	0021907	10.9991840	15
46	0020687	9977966	0042721	9957279	0022034	9979313	14
47	0033179	9977838	0055340	9944660	0022162	9966821	13
48	0045634	9977710	0067924	9932076	0022290	9954366	12
49	0058053	9977582	0080471	9919529	0022418	9941947	11
50	0070436	9977453	0092984	9907016	0022547	9929564	10
51	0082784	9977323	0105461	9894539	0022677	9917216	9
52	0095096	9977194	0117903	9882097	0022806	9904904	8
53	0107374	9977064	0130310	9869690	0022936	9892626	7
54	0119616	9976933	0142682	9857318	0023067	9880384	6
55	0131823	9976803	0155021	9844979	0023197	9868177	5
56	0143996	9976672	0167325	9832675	0023328	9856004	4
57	0156135	9976540	0179594	9820406	0023460	9843865	3
58	0168239	9976408	0191831	9808169	0023592	9831761	2
59	0180309	9976276	0204033	9795967	0023724	9819691	1
60	0192346	9976143	0216202	9783798	0023857	9807654	0

6 Degrees.

M	Sine.		Tang.		Secant.		
0	9.0192346	9.9976143	9.0216202	10.9783798	10.0023857	10.9807654	60
1	0204348	9976011	0228338	9771662	0023989	9795652	59
2	0216318	9975877	0240441	9759559	0024123	9783682	58
3	0228254	9975743	0252510	9747490	0024257	9771746	57
4	0240157	9975609	0264548	9735452	0024391	9759843	56
5	0252027	9975475	0276552	9723448	0024525	9747973	55
6	0263865	9975340	0288524	9711476	0024660	9736135	54
7	0275669	9975205	0300464	9699536	0024795	9724331	53
8	0287442	9975069	0312373	9687627	0024931	9712558	52
9	0299182	9974933	0324249	9675751	0025067	9700818	51
10	0310890	9974797	0336093	9663907	0025203	9689110	50
11	0322567	9974660	0347906	9652094	0025340	9677433	49
12	0334212	9974523	0359688	9640312	0025477	9665788	48
13	0345825	9974386	0371439	9628561	0025614	9654175	47
14	0357407	9974248	0383159	9616841	0025752	9642593	46
15	0368958	9974110	0394848	9605152	0025890	9631042	45
16	0380477	9973971	0406506	9593494	0026029	9619523	44
17	0391966	9973833	0418134	9581866	0026167	9608034	43
18	0403424	9973693	0429731	9570269	0026307	9596576	42
19	0414852	9973554	0441299	9558701	0026446	9585148	41
20	0426249	9973414	0452836	9547164	0026586	9573751	40
21	0437617	9973273	0464343	9535657	0026727	9562383	39
22	0448954	9973132	0475821	9524179	0026868	9551046	38
23	0460261	9972991	0487270	9512730	0027009	9539739	37
24	0471538	9972850	0498689	9501311	0027150	9528462	36
25	0482786	9972708	0510078	9489922	0027292	9517214	35
26	0494005	9972566	0521439	9478561	0027434	9505995	34
27	0505194	9972423	0532771	9467229	0027577	9494806	33
28	0516354	9972280	0544074	9455926	0027720	9483646	32
29	0527485	9972137	0555349	9444651	0027863	9472515	31
30	0538588	9971993	0566595	9433405	0028007	9461412	30
31	0549661	9971849	0577813	9422187	0028151	9450339	29
32	0560706	9971704	0589002	9410998	0028296	9439294	28
33	0571723	9971559	0600164	9399836	0028441	9428277	27
34	0582711	9971414	0611297	9388703	0028586	9417289	26
35	0593672	9971268	0622403	9377597	0028732	9406328	25
36	0604604	9971122	0633482	9366518	0028878	9395396	24
37	0615509	9970976	0644533	9355467	0029024	9384491	23
38	0626386	9970829	0655556	9344444	0029171	9373614	22
39	0637235	9970682	0666553	9333447	0029318	9362765	21
40	0648057	9970535	0677522	9322478	0029465	9351943	20
41	0658852	9970387	0688465	9311535	0029613	9341148	19
42	0669619	9970239	0699381	9300619	0029761	9330381	18
43	0680360	9970090	0710270	9289730	0029910	9319640	17
44	0691074	9969941	0721133	9278867	0030059	9308926	16
45	0701761	9969792	0731969	9268031	0030208	9298239	15
46	0712421	9969642	0742779	9257221	0030358	9287579	14
47	0723055	9969492	0753563	9246437	0030508	9276945	13
48	0733663	9969342	0764321	9235679	0030658	9266337	12
49	0744244	9969191	0775053	9224947	0030809	9255756	11
50	0754799	9969040	0785760	9214240	0030960	9245201	10
51	0765329	9968888	0796441	9203559	0031112	9234671	9
52	0775832	9968736	0807096	9192904	0031264	9224168	8
53	0786310	9968584	0817726	9182274	0031416	9213690	7
54	0796762	9968431	0828331	9171669	0031569	9203238	6
55	0807189	9968278	0838911	9161089	0031722	9192811	5
56	0817590	9968125	0849466	9150534	0031875	9182410	4
57	0827966	9967971	0859996	9140004	0032029	9172034	3
58	0838317	9967817	0870501	9129499	0032183	9161683	2
59	0848643	9967662	0880981	9119019	0032338	9151357	1
60	0858945	9967507	0891438	9108562	0032493	9141055	0

7 Degrees. 82 Degrees.

M	Sine.		Tang.		Secant.		
0	9.0858945	9.9967507	9.0891438	10.9108562	10.0032493	10.9141055	60
1	0869221	9967352	0901869	9098131	0032648	9130779	59
2	0879473	9967196	0912277	9087723	0032804	9120527	58
3	0889700	9967040	0922660	9077340	0032960	9110300	57
4	0899903	9966884	0933020	9066980	0033116	9100097	56
5	0910082	9966727	0943355	9056645	0033273	9089918	55
6	0920237	9966570	0953667	9046333	0033430	9079763	54
7	0930367	9966412	0963955	9036045	0033588	9069633	53
8	0940474	9966254	0974219	9025781	0033746	9059526	52
9	0950556	9966096	0984460	9015540	0033904	9049444	51
10	0960615	9965937	0994678	9005322	0034063	9039385	50
11	0970651	9965778	1004872	8995128	0034222	9029349	49
12	0980662	9965619	1015044	8984956	0034381	9019338	48
13	0990651	9965459	1025192	8974808	0034541	9009349	47
14	1000616	9965299	1035317	8964683	0034701	8999384	46
15	1010558	9965138	1045420	8954580	0034862	8989442	45
16	1020477	9964977	1055500	8944500	0035023	8979523	44
17	1030373	9964816	1065557	8934443	0035184	8969627	43
18	1040246	9964655	1075591	8924409	0035345	8959754	42
19	1050096	9964493	1085604	8914396	0035507	8949904	41
20	1059924	9964330	1095594	8904406	0035670	8940076	40
21	1069729	9964167	1105562	8894438	0035833	8930271	39
22	1079512	9964004	1115508	8884492	0035996	8920488	38
23	1089272	9963841	1125431	8874569	0036159	8910728	37
24	1099010	9963677	1135333	8864667	0036323	8900990	36
25	1108726	9963513	1145213	8854787	0036487	8891274	35
26	1118420	9963348	1155072	8844928	0036652	8881580	34
27	1128092	9963183	1164909	8835091	0036817	8871908	33
28	1137742	9963018	1174724	8825276	0036982	8862258	32
29	1147370	9962852	1184518	8815482	9037148	8852630	31
30	1156977	9962686	1194291	8805709	0037314	8843023	30
31	1166562	9962519	1204043	8795957	0037481	8833438	29
32	1176125	9962352	1213773	8786227	0037648	8823875	28
33	1185667	9962185	1223482	8776518	0037815	8814333	27
34	1195188	9962017	1233171	8766829	0037983	8804812	26
35	1204688	9961849	1242839	8757161	0038151	8795312	25
36	1214167	9961681	1252486	8747514	0038319	8785833	24
37	1223624	9961512	1262112	8737888	0038488	8776376	23
38	1233061	9961343	1271718	8728282	0038657	8766939	22
39	1242477	9961174	1281303	8718697	0038826	8757523	21
40	1251872	9961004	1290868	8709132	0038996	8748128	20
41	1261246	9960834	1300413	8699587	0039166	8738754	19
42	1270600	9960663	1309937	8690063	0039337	8729400	18
43	1279934	9960492	1319442	8680558	0039508	8720066	17
44	1289247	9960321	1328926	8671074	0039679	8710753	16
45	1298539	9960149	1338391	8661609	0039851	8701461	15
46	1307812	9959977	1347835	8652165	0040023	8692188	14
47	1317064	9959804	1357260	8642740	0040196	8682936	13
48	1326297	9959631	1366665	8633335	0040369	8673703	12
49	1335509	9959458	1376051	8623949	0040542	8664491	11
50	1344702	9959284	1385417	8614583	0040716	8655298	10
51	1353875	9959111	1394764	8605236	0040889	8646125	9
52	1363028	9958936	1404092	8595908	0041064	8636972	8
53	1372161	9958761	1413400	8586600	0041239	8627839	7
54	1381275	9958586	1422689	8577311	0041414	8618725	6
55	1390370	9958411	1431959	8568041	0041589	8609630	5
56	1399445	9958235	1441210	8558790	0041765	8600555	4
57	1408501	9958059	1450442	8549558	0041941	8591499	3
58	1417537	9957882	1459655	8540345	0042118	8582463	2
59	1426555	9957705	1468849	8531151	0042295	8573445	1
60	1435553	9957528	1478025	8521975	0042472	8564447	0

8 Degrees. 81 Degrees.

M	Sine.		Tang.		Secant.		
0	9.1435553	9.9957528	9.1478025	10.8521975	10.0042472	10.8564447	60
1	1444532	9957350	1487182	8512818	0042650	8555468	59
2	1453493	9957172	1496321	8503679	0042828	8546507	58
3	1462435	9956993	1505441	8494559	0043007	8537565	57
4	1471358	9956815	1514543	8485457	0043185	8528642	56
5	1480262	9956635	1523627	8476373	0043365	8519738	55
6	1489148	9956456	1532692	8467308	0043544	8510852	54
7	1498015	9956276	1541739	8458261	0043724	8501985	53
8	1506864	9956095	1550769	8449231	0043905	8493136	52
9	1515694	9955915	1559780	8440220	0044085	8484306	51
10	1524507	9955734	1568773	8431227	0044266	8475493	50
11	1533301	9955552	1577748	8422252	0044448	8466699	49
12	1542076	9955370	1586706	8413294	0044630	8457924	48
13	1550834	9955188	1595646	8404354	0044812	8449166	47
14	1559574	9955005	1604569	8395431	0044995	8440426	46
15	1568296	9954822	1613473	8386527	0045178	8431704	45
16	1577000	9954639	1622361	8377639	0045361	8423000	44
17	1585686	9954455	1631231	8368769	0045545	8414314	43
18	1594354	9954271	1640083	8359917	0045729	8405646	42
19	1603005	9954087	1648919	8351081	0045913	8396995	41
20	1611639	9953902	1657737	8342263	0046098	8388361	40
21	1620254	9953717	1666538	8333462	0046283	8379746	39
22	1628853	9953531	1675322	8324678	0046469	8371147	38
23	1637434	9953345	1684089	8315911	0046655	8362566	37
24	1645998	9953159	1692839	8307161	0046841	8354002	36
25	1654544	9952972	1701572	8298428	0047028	8345456	35
26	1663074	9952785	1710289	8289711	0047215	8336926	34
27	1671586	9952597	1718989	8281011	0047403	8328414	33
28	1680081	9952409	1727672	8272328	0047591	8319919	32
29	1688559	9952221	1736338	8263662	0047779	8311441	31
30	1697021	9952033	1744988	8255012	0047967	8302979	30
31	1705465	9951844	1753622	8246378	0048156	8294535	29
32	1713893	9951654	1762239	8237761	0048346	8286107	28
33	1722305	9951464	1770840	8229160	0048536	8277695	27
34	1730699	9951274	1779425	8220575	0048726	8269301	26
35	1739077	9951084	1787993	8212007	0048916	8260923	25
36	1747439	9950893	1796546	8203454	0049107	8252561	24
37	1755784	9950702	1805082	8194918	0049298	8244216	23
38	1764112	9950510	1813602	8186398	0049490	8235888	22
39	1772425	9950318	1822106	8177894	0049682	8227575	21
40	1780721	9950126	1830595	8169405	0049874	8219279	20
41	1789001	9949933	1839068	8160932	0050067	8210999	19
42	1797265	9949740	1847525	8152475	0050260	8202735	18
43	1805512	9949546	1855966	8144034	0050454	8194488	17
44	1813744	9949352	1864392	8135608	0050648	8186256	16
45	1821960	9949158	1872802	8127198	0050842	8178040	15
46	1830160	9948964	1881196	8118804	0051036	8169840	14
47	1838344	9948769	1889575	8110425	0051231	8161656	13
48	1846512	9948573	1897939	8102061	0051427	8153488	12
49	1854665	9948377	1906287	8093713	0051623	8145335	11
50	1862802	9948181	1914621	8085379	0051819	8137198	10
51	1870923	9947985	1922939	8077061	0052015	8129077	9
52	1879029	9947788	1931241	8068759	0052212	8120971	8
53	1887120	9947591	1939529	8060471	0052409	8112880	7
54	1895195	9947393	1947802	8052198	0052607	8104805	6
55	1903254	9947195	1956059	8043941	0052805	8096746	5
56	1911299	9946997	1964302	8035698	0053003	8088701	4
57	1919328	9946798	1972530	8027470	0053202	8080672	3
58	1927342	9946599	1980743	8019257	0053401	8072658	2
59	1935341	9946399	1988941	8011059	0053601	8064659	1
60	1943324	9946199	1997125	8002875	0053801	8056676	0

9 Degrees. 80 Degrees.

M	Sine.		Tang.		Secant.		
0	9.1943324	9.9946199	9.1997125	10.8002875	10.0053801	10.8056676	60
1	1951293	9945999	2005294	7994706	0054001	8048707	59
2	1959247	9945798	2013449	7986551	0054202	8040753	58
3	1967186	9945597	2021588	7978411	0054403	8032814	57
4	1975110	9945396	2029714	7970286	0054604	8024890	56
5	1983019	9945194	2037825	7962175	0054806	8016981	55
6	1990913	9944992	2045922	7954078	0055008	8009087	54
7	1998793	9944789	2054004	7945996	0055211	8001207	53
8	2006658	9944587	2062072	7937928	0055413	7993342	52
9	2014509	9944383	2070126	7929874	0055617	7985491	51
10	2022345	9944180	2078165	7921835	0055820	7977655	50
11	2030167	9943975	2086191	7913809	0056025	7969833	49
12	2037974	9943771	2094203	7905797	0056229	7962026	48
13	2045766	9943566	2102200	7897800	0056434	7954234	47
14	2053545	9943361	2110184	7889816	0056639	7946455	46
15	2061309	9943156	2118153	7881847	0056844	7938691	45
16	2069059	9942950	2126109	7873891	0057050	7930941	44
17	2076795	9942743	2134051	7865949	0057257	7923205	43
18	2084516	9942537	2141980	7858020	0057463	7915484	42
19	2092224	9942330	2149894	7850106	0057670	7907776	41
20	2099917	9942122	2157795	7842205	0057878	7900083	40
21	2107597	9941914	2165683	7834317	0058086	7892403	39
22	2115263	9941706	2173556	7826444	0058294	7884737	38
23	2122914	9941498	2181417	7818583	0058502	7877086	37
24	2130552	9941289	2189264	7810736	0058711	7869448	36
25	2138176	9941079	2197097	7802903	0058921	7861824	35
26	2145787	9940870	2204917	7795083	0059130	7854213	34
27	2153384	9940659	2212724	7787276	0059341	7846616	33
28	2160967	9940449	2220518	7779482	0059551	7839033	32
29	2168536	9940238	2228298	7771702	0059762	7831464	31
30	2176092	9940027	2236065	7763935	0059973	7823908	30
31	2183635	9939815	2243819	7756181	0060185	7816365	29
32	2191164	9939603	2251561	7748439	0060397	7808836	28
33	2198680	9939391	2259289	7740711	0060609	7801320	27
34	2206182	9939178	2267004	7732996	0060822	7793818	26
35	2213671	9938965	2274706	7725294	0061035	7786329	25
36	2221147	9938752	2282395	7717605	0061248	7778853	24
37	2228609	9938538	2290071	7709929	0061462	7771391	23
38	2236059	9938324	2297735	7702265	0061676	7763941	22
39	2243495	9938109	2305386	7694614	0061891	7756505	21
40	2250918	9937894	2313024	7686976	0062106	7749082	20
41	2258328	9937679	2320650	7679350	0062321	7741672	19
42	2265725	9937463	2328262	7671738	0062537	7734275	18
43	2273110	9937247	2335863	7664137	0062753	7726890	17
44	2280481	9937030	2343451	7656549	0062970	7719519	16
45	2287839	9936813	2351026	7648974	0063187	7712161	15
46	2295185	9936596	2358589	7641411	0063404	7704815	14
47	2302518	9936378	2366139	7633861	0063622	7697482	13
48	2309838	9936160	2373678	7626322	0063840	7690162	12
49	2317145	9935942	2381203	7618797	0064058	7682855	11
50	2324440	9935723	2388717	7611283	0064277	7675560	10
51	2331722	9935504	2396218	7603782	0064496	7668278	9
52	2338992	9935285	2403708	7596293	0064715	7661008	8
53	2346249	9935065	2411185	7588815	0064935	7653751	7
54	2353494	9934844	2418650	7581350	0065156	7646506	6
55	2360726	9934624	2426103	7573897	0065376	7639274	5
56	2367946	9934403	2433543	7566457	0065597	7632054	4
57	2375153	9934181	2440972	7559028	0065819	7624847	3
58	2382349	9933959	2448389	7551611	0066041	7617651	2
59	2389532	9933737	2455794	7544206	0066263	7610468	1
60	2396702	9933515	2463188	7536812	0066485	7603298	0

10 Degrees. 79 Degrees.

M	Sine.		Tang.		Secant.		
0	9.2396702	9.9933515	9 2463188	10.7536812	10 0066485	10.7603298	60
1	2403861	9933292	2470569	7529431	0066708	7596139	59
2	2411007	9933068	2477939	7522061	0066932	7588993	58
3	2418141	9932845	2485297	7514703	0067155	7581859	57
4	2425264	9932621	2492643	7507357	0067379	7574736	56
5	2432374	9932396	2499978	7500022	0067604	7567626	55
6	2439472	9932171	2507301	7492699	0067829	7560528	54
7	2446558	9931946	2514612	7485388	0068054	7553442	53
8	2453632	9931720	2521912	7478088	0068280	7546368	52
9	2460695	9931494	2529200	7470800	0068506	7539305	51
10	2467746	9931268	2536477	7463523	0068732	7532254	50
11	2474784	9931041	2543743	7456257	0068959	7525216	49
12	2481811	9930814	2550997	7449003	0069186	7518189	48
13	2488827	9930587	2558240	7441760	0069413	7511173	47
14	2495830	9930359	2565472	7434528	0069641	7504170	46
15	2502822	9930131	2572692	7427308	0069869	7497178	45
16	2509803	9929902	2579901	7420099	0070098	7490197	44
17	2516772	9929673	2587099	7412901	0070327	7483228	43
18	2523729	9929444	2594285	7405715	0070556	7476271	42
19	2530675	9929214	2601461	7398539	0070786	7469325	41
20	2537609	9928984	2608625	7391375	0071016	7462391	40
21	2544532	9928753	2615779	7384221	0071247	7455468	39
22	2551444	9928522	2622921	7377079	0071478	7448556	38
23	2558344	9928291	2630053	7369947	0071709	7441656	37
24	2565233	9928059	2637173	7362827	0071941	7434767	36
25	2572110	9927827	2644283	7355717	0072173	7427890	35
26	2578977	9927595	2651382	7348618	0072405	7421023	34
27	2585832	9927362	2658470	7341530	0072638	7414168	33
28	2592676	9927129	2665547	7334453	0072871	7407324	32
29	2599509	9926895	2672613	7327387	0073105	7400491	31
30	2606330	9926661	2679669	7320331	0073339	7393670	30
31	2613141	9926427	2686714	7313286	0073573	7386859	29
32	2619941	9926192	2693749	7306251	0073808	7380059	28
33	2626729	9925957	2700772	7299228	0074043	7373271	27
34	2633507	9925722	2707786	7292214	0074278	7366493	26
35	2640274	9925486	2714788	7285212	0074514	7359726	25
36	2647030	9925250	2721780	7278220	0074750	7352970	24
37	2653775	9925013	2728762	7271238	0074987	7346225	23
38	2660509	9924776	2735733	7264267	0075224	7339491	22
39	2667232	9924539	2742694	7257306	0075461	7332768	21
40	2673945	9924301	2749644	7250356	0075699	7326055	20
41	2680647	9924063	2756584	7243416	0075937	7319353	19
42	2687338	9923824	2763514	7236486	0076176	7312662	18
43	2694019	9923585	2770434	7229566	0076415	7305981	17
44	2700689	9923346	2777343	7222657	0076654	7299311	16
45	2707348	9923106	2784242	7215758	0076894	7292652	15
46	2713997	9922866	2791131	7208869	0077134	7286003	14
47	2720635	9922626	2798009	7201991	0077374	7279365	13
48	2727263	9922385	2804878	7195122	0077615	7272737	12
49	2733880	9922144	2811736	7188264	0077856	7266120	11
50	2740487	9921902	2818585	7181415	0078098	7259513	10
51	2747083	9921660	2825423	7174577	0078340	7252917	9
52	2753669	9921418	2832251	7167749	0078582	7246331	8
53	2760 45	9921175	2839070	7160930	0078825	7239755	7
54	2766811	9920932	2845878	7154122	0079068	7233189	6
55	2773366	9920689	2852677	7147323	0079311	7226634	5
56	2779911	9920445	2859466	7140534	0079555	7220089	4
57	2786445	9920201	2866245	7133755	0079799	7213555	3
58	2792970	9919956	2873014	7126986	0080044	7207030	2
59	2799484	9919711	2879773	7120227	0080289	7200516	1
60	2805988	9919466	2886523	7113477	0080534	7194012	0

11 Degrees. 78 Degrees.

M	Sine.		Tang.		Secant.		
0	9.2805988	9.9919466	9.2886523	10.7113477	10.0080534	10.7194012	60
1	2812483	9919220	2893263	7106737	0080780	7187517	59
2	2818967	9918974	2899993	7100007	0081026	7181033	58
3	2825441	9918727	2906713	7093287	0081273	7174559	57
4	2831905	9918480	2913424	7086576	0081520	7168095	56
5	2838359	9918233	2920126	7079874	0081767	7161641	55
6	2844803	9917986	2926817	7073183	0082014	7155197	54
7	2851237	9917737	2933500	7066500	0082263	7148763	53
8	2857661	9917489	2940172	7059828	0082511	7142339	52
9	2864076	9917240	2946836	7053164	0082760	7135924	51
10	2870480	9916991	2953489	7046511	0083009	7129520	50
11	2876875	9916741	2960134	7039866	0083259	7123125	49
12	2883260	9916492	2966769	7033231	0083508	7116740	48
13	2889636	9916241	2973395	7026605	0083759	7110364	47
14	2896001	9915990	2980011	7019989	0084010	7103999	46
15	2902357	9915739	2986618	7013382	0084261	7097643	45
16	2908704	9915488	2993216	7006784	0084512	7091296	44
17	2915040	9915236	2999804	7000196	0084764	7084960	43
18	2921367	9914984	3006383	6993617	0085016	7078633	42
19	2927685	9914731	3012954	6987046	0085269	7072315	41
20	2933993	9914478	3019514	6980486	0085522	7066007	40
21	2940291	9914225	3026066	6973934	0085775	7059709	39
22	2946580	9913971	3032609	6967391	0086029	7053420	38
23	2952859	9913717	3039143	6960857	0086283	7047141	37
24	2959129	9913462	3045667	6954333	0086538	7040871	36
25	2965390	9913207	3052183	6947817	0086793	7034610	35
26	2971641	9912952	3058689	6941311	0087048	7028359	34
27	2977883	9912696	3065187	6934813	0087304	7022117	33
28	2984116	9912440	3071675	6928325	0087560	7015884	32
29	2990339	9912184	3078155	6921845	0087816	7009661	31
30	2996553	9911927	3084626	6915374	0088073	7003447	30
31	3002758	9911670	3091088	6908912	0088330	6997242	29
32	3008953	9911412	3097541	6902459	0088588	6991047	28
33	3015140	9911154	3103985	6896015	0088846	6984860	27
34	3021317	9910896	3110421	6889579	0089104	6978683	26
35	3027485	9910637	3116848	6883152	0089363	6972515	25
36	3033644	9910378	3123266	6876734	0089622	6966356	24
37	3039794	9910119	3129675	6870325	0089881	6960206	23
38	3045934	9909859	3136076	6863924	0090141	6954066	22
39	3052066	9909598	3142468	6857532	0090402	6947934	21
40	3058189	9909338	3148851	6851149	0090662	6941811	20
41	3064303	9909077	3155226	6844774	0090923	6935697	19
42	3070407	9908815	3161592	6838408	0091185	6929593	18
43	3076503	9908553	3167950	6832050	0091447	6923497	17
44	3082590	9908291	3174299	6825701	0091709	6917410	16
45	3088668	9908029	3180640	6819360	0091971	6911332	15
46	3094737	9907766	3186972	6813028	0092234	6905263	14
47	3100798	9907502	3193295	6806705	0092498	6899202	13
48	3106849	9907239	3199611	6800389	0092761	6893151	12
49	3112892	9906974	3205918	6794082	0093026	6887108	11
50	3118926	9906710	3212216	6787784	0093290	6881074	10
51	3124951	9906445	3218506	6781494	0093555	6875049	9
52	3130968	9906180	3224788	6775212	0093820	6869032	8
53	3136976	9905914	3231061	6768939	0094086	6863024	7
54	3142975	9905648	3237327	6762673	0094352	6857025	6
55	3148965	9905382	3243584	6756416	0094618	6851035	5
56	3154947	9905115	3249832	6750168	0094885	6845053	4
57	3160921	9904848	3256073	6743927	0095152	6839079	3
58	3166885	9904580	3262305	6737695	0095420	6833115	2
59	3172841	9904312	3268529	6731471	0095688	6827159	1
[illegible]	3178789	9904044	3274745	6725255	0095956	6821211	0

12 Degrees. 77 Degrees.

M	Sine.		Tang.		Secant.		
0	9 3178789	9·9904044	9·3274745	10 6725255	10.0095956	10.6821211	60
1	3184728	9903775	3280953	6719047	0096225	6815272	59
2	3190659	9903506	3287153	6712847	0096494	6809341	58
3	3196581	9903237	3293345	6706655	0096763	6803419	57
4	3202495	9902967	3299528	6700472	0097033	6797505	56
5	3208400	9902697	3305704	6694296	0097303	6791600	55
6	3214297	9902426	3311872	6688128	0097574	6785703	54
7	3220186	9902155	3318031	6681969	0097845	6779814	53
8	3226066	9901883	3324183	6675817	0098117	6773934	52
9	3231938	9901612	3330327	6669673	0098388	6768062	51
10	3237802	9901339	3336463	6663537	0098661	6762198	50
11	3243657	9901067	3342591	6657409	0098933	6756343	49
12	3249505	9900794	3348711	6651289	0099206	6750495	48
13	3255344	9900521	3354823	6645177	0099479	6744656	47
14	3261174	9900247	3360927	6639073	0099753	6738826	46
15	3266997	9899973	3367024	6632976	0100027	6733003	45
16	3272811	9899698	3373113	6626887	0100302	6727189	44
17	3278617	9899423	3379194	6620806	0100577	6721383	43
18	3284416	9899148	3385267	6614733	0100852	6715584	42
19	3290206	9898873	3391333	6608667	0101127	6709794	41
20	3295988	9898597	3397391	6602609	0101403	6704012	40
21	3301761	9898320	3403441	6596559	0101680	6698239	39
22	3307527	9898043	3409484	6590516	0101957	6692473	38
23	3313285	9897766	3415519	6584481	0102234	6686715	37
24	3319035	9897489	3421546	6578454	0102511	6680965	36
25	3324777	9897211	3427566	6572434	0102789	6675223	35
26	3330511	9896932	3433578	6566422	0103068	6669489	34
27	3336237	9896654	3439583	6560417	0103346	6663763	33
28	3341955	9896374	3445580	6554420	0103626	6658045	32
29	3347665	9896095	3451570	6548430	0103905	6652335	31
30	3353368	9895815	3457552	6542448	0104185	6646632	30
31	3359062	9895535	3463527	6536473	0104465	6640938	29
32	3364749	9895254	3469494	6530506	0104746	6635251	28
33	3370428	9894973	3475454	6524546	0105027	6629572	27
34	3376099	9894692	3481407	6518593	0105308	6623901	26
35	3381762	9894410	3487352	6512648	0105590	6618238	25
36	3387418	9894128	3493290	6506710	0105872	6612582	24
37	3393065	9893845	3499220	6500780	0106155	6606935	23
38	3398706	9893562	3505143	6494857	0106438	6601294	22
39	3404338	9893279	3511059	6488941	0106721	6595662	21
40	3409963	9892995	3516968	6483032	0107005	6590037	20
41	3415580	9892711	3522869	6477131	0107289	6584420	19
42	3421190	9892427	3528763	6471237	0107573	6578810	18
43	3426792	9892142	3534650	6465350	0107858	6573208	17
44	3432386	9891856	3540530	6459470	0108144	6567614	16
45	3437973	9891571	3546402	6453598	0108429	6562027	15
46	3443552	9891285	3552267	6447733	0108715	6556448	14
47	3449124	9890998	3558126	6441874	0109002	6550876	13
48	3454688	9890711	3563977	6436023	0109289	6545312	12
49	3460245	9890424	3569821	6430179	0109576	6539755	11
50	3465794	9890137	3575658	6424342	0109863	6534206	10
51	3471336	9889849	3581487	6418513	0110151	6528664	9
52	3476870	9889560	3587310	6412690	0110440	6523130	8
53	3482397	9889271	3593126	6406874	0110729	6517603	7
54	3487917	9888982	3598935	6401065	0111018	6512083	6
55	3493429	9888693	3604736	6395264	0111307	6506571	5
56	3498934	9888403	3610531	6389469	0111597	6501066	4
57	3504432	9888113	3616319	6383681	0111887	6495568	3
58	3509922	9887822	3622100	6377900	0112178	6490078	2
59	3515405	9887531	3627874	6372126	0112469	6484595	1
60	3520880	9887239	3633641	6366359	0112761	6479120	0

13 Degrees. 76 Degrees.

M	Sine.		Tang.		Secant.		
0	9.3520880	9.9887239	9.3633641	10.6366359	10.0112761	10.6479120	60
1	3526349	9886947	3639401	6360599	0113053	6473651	59
2	3531810	9886655	3645155	6354845	0113345	6468190	58
3	3537264	9886363	3650901	6349099	0113637	6462736	57
4	3542710	9886070	3656641	6343359	0113930	6457290	56
5	3548150	9885776	3662374	6337626	0114224	6451850	55
6	3553582	9885482	3668100	6331900	0114518	6446418	54
7	3559007	9885188	3673819	6326181	0114812	6440993	53
8	3564426	9884894	3679532	6310468	0115106	6435574	52
9	3569836	9884599	3685238	6314762	0115401	6430164	51
10	3575240	9884303	3690937	6309063	0115697	6424760	50
11	3580637	9884008	3696629	6303371	0115992	6419363	49
12	3586027	9883712	3702315	6297685	0116288	6413973	48
13	3591409	9883415	3707994	6292006	0116585	6408591	47
14	3596785	9883118	3713667	6286333	0116882	6403215	46
15	3602154	9882821	3719333	6280667	0117179	6397846	45
16	3607515	9882523	3724992	6275008	0117477	6392485	44
17	3612870	9882225	3730645	6269355	0117775	6387130	43
18	3618217	9881927	3736291	6263709	0118073	6381783	42
19	3623558	9881628	3741930	6258070	0118372	6376442	41
20	3628892	9881329	3747563	6252437	0118671	6371108	40
21	3634219	9881029	3753190	6246810	0118971	6365781	39
22	3639539	9880729	3758810	6241190	0119271	6360461	38
23	3644852	9880429	3764423	6235577	0119571	6355148	37
24	3650158	9880128	3770030	6229970	0119872	6349842	36
25	3655458	9879827	3775631	6224369	0120173	6344542	35
26	3660750	9879525	3781225	6218775	0120475	6339250	34
27	3666036	9879223	3786813	6213187	0120777	6333964	33
28	3671315	9878921	3792394	6207606	0121079	6328685	32
29	3676587	9878618	3797969	6202031	0121382	6323413	31
30	3681853	9878315	3803537	6196463	0121685	6318147	30
31	3687111	9878012	3809100	6190900	0121988	6312889	29
32	3692363	9877708	3814655	6185345	0122292	6307637	28
33	3697608	9877404	3820205	6179795	0122596	6302392	27
34	3702847	9877099	3825748	6174252	0122901	6297153	26
35	3708079	9876794	3831285	6168715	0123206	6291921	25
36	3713304	9876488	3836816	6163184	0123512	6286696	24
37	3718523	9876183	3842340	6157660	0123817	6281477	23
38	3723735	9875876	3847858	6152142	0124124	6276265	22
39	3728940	9875570	3853370	6146630	0124430	6271060	21
40	3734139	9875263	3858876	6141124	0124737	6265861	20
41	3739331	9874955	3864376	6135624	0125045	6260669	19
42	3744517	9874648	3869869	6130131	0125352	6255483	18
43	3749696	9874339	3875356	6124644	0125661	6250304	17
44	3754868	9874031	3880837	6119163	0125969	6245132	16
45	3760034	9873722	3886312	6113688	0126278	6239966	15
46	3765194	9873413	3891781	6108219	0126587	6234806	14
47	3770347	9873103	3897244	6102756	0126897	6229653	13
48	3775493	9872793	3902700	6097300	0127207	6224507	12
49	3780633	9872482	3908151	6091849	0127518	6219367	11
50	3785767	9872171	3913595	6086405	0127829	6214233	10
51	3790894	9871860	3919034	6080966	0128140	6209106	9
52	3796015	9871549	3924466	6075534	0128451	6203985	8
53	3801129	9871236	3929893	6070107	0128764	6198871	7
54	3806237	9870924	3935313	6064687	0129076	6193763	6
55	3811339	9870611	3940727	6059273	0129389	6188661	5
56	3816434	9870298	3946136	6053864	0129702	6183566	4
57	3821523	9869984	3951538	6048462	0130016	6178477	3
58	3826605	9869670	3956935	6043065	0130330	6173395	2
59	3831682	9869356	3962326	6037674	0130644	6168318	1
60	3836752	9869041	3967711	6032289	0130959	6163248	0

14 Degrees. M	Sine		Tang.		Secant.		75 Degrees.
0	9.3836752	9.9869041	9.3967711	10.6032289	10.0130959	10.6163248	60
1	3841815	9868726	3973089	6026911	0131274	6158185	59
2	3846873	9868410	3978463	6021537	0131590	6153127	58
3	3851924	9868094	3983830	6016170	0131906	6148076	57
4	3856969	9867778	3989191	6010809	0132222	6143031	56
5	3862008	9867461	3994547	6005453	0132539	6137992	55
6	3867040	9867144	3999896	6000104	0132856	6132960	54
7	3872067	9866827	4005240	5994760	0133173	6127933	53
8	3877087	9866509	4010578	5989422	0133491	6122913	52
9	3882101	9866191	4015910	5984090	0133809	6117899	51
10	3887109	9865872	4021237	5978763	0134128	6112891	50
11	3892111	9865553	4026558	5973442	0134447	6107889	49
12	3897106	9865233	4031873	5968137	0134767	6102894	48
13	3920096	9864913	4037182	5962818	0135087	6097904	47
14	3907079	9864593	4042486	5957514	0135407	6092921	46
15	3912057	9864273	4047784	5952216	0135727	6087943	45
16	3917028	9863952	4053076	5946924	0136048	6082972	44
17	3921993	9863630	4058363	5941637	0136370	6078007	43
18	3926952	9863308	4063644	5936356	0136692	6073048	42
19	3931905	9862986	4068919	5931081	0137014	6068095	41
20	3936852	9862663	4074189	5925811	0137337	6063148	40
21	3941794	9862340	4079453	5920547	0137660	6058206	39
22	3946729	9862017	4084712	5915288	0137983	6053271	38
23	3951658	9861993	4089965	5910035	0138307	6048342	37
24	3956581	9861369	4095212	5904788	0138631	6043419	36
25	3961499	9861045	4100454	5899546	0138955	6038501	35
26	3966410	9860720	4105690	5894310	0139280	6033590	34
27	3971315	9860394	4110921	5889079	0139606	6028685	33
28	3976215	9860069	4116146	5883854	0139931	6023787	32
29	3981109	9859742	4121366	5878634	0140258	6018891	31
30	3985996	9859416	4126581	5873419	0140584	6014004	30
31	3990878	9859089	4131789	5868211	0140911	6009122	29
32	3995754	9858762	4136993	5863007	0141238	6004246	28
33	4000625	9858434	4142191	5857809	0141566	5999375	27
34	4005489	9858106	4147383	5852617	0141894	5994511	26
35	4010348	9857777	4152570	5847430	0142223	5989652	25
36	4015201	9857449	4157752	5842248	0142551	5984799	24
37	4020048	9857119	4162928	5837072	0142881	5979952	23
38	4024889	9856790	4168099	5831901	0143210	5975111	22
39	4029724	9856460	4173265	5826735	0143540	5970276	21
40	4034554	9856129	4178425	5821575	0143871	5965446	20
41	4039378	9855798	4183580	5816420	0144202	5960622	19
42	4044196	9855467	4188729	5811271	0144533	5955804	18
43	4049009	9855135	4193874	5806126	0144865	5950991	17
44	4053816	9854803	4199013	5800987	0145197	5946184	16
45	4058617	9854471	4204146	5795854	0145529	5941383	15
46	4063413	9854138	4209275	5790725	0145862	5936587	14
47	4068203	9853805	4214398	5785602	0146195	5931797	13
48	4072987	9853471	4219515	5780485	0146529	5927013	12
49	4077766	9853138	4224628	5775372	0146862	5922234	11
50	4082539	9852803	4229735	5770265	0147197	5917461	10
51	4087306	9852468	4234838	5765162	0147532	5912694	9
52	4092068	9852133	4239935	5760065	0147867	5907932	8
53	4096824	9851798	4245026	5754974	0148202	5903176	7
54	4101575	9851462	4250113	5749887	0148538	5898425	6
55	4106320	9851125	4255194	5744806	0148875	5893680	5
56	4111059	9850789	4260271	5739729	0149211	5888941	4
57	4115793	9850452	4265342	5734658	0149548	5882407	3
58	4120522	9850114	4270408	5729592	0149886	5879478	2
59	4125245	9849776	4275469	5724531	0150224	5874755	1
60	4129962	9849438	4280525	5719475	0150562	5870038	0

15 Degrees. 74 Degrees.

M	Sine.		Tang.		Secant.		
0	9.4129962	9.9849438	9.4280525	10.5719475	10.0150562	10.5870038	60
1	4134674	9849099	4285575	5714425	0150901	5865326	59
2	4139381	9848760	4290621	5709379	0151240	5860619	58
3	4144082	9848420	4295661	5704339	0151580	5855918	57
4	4148778	9848081	4300697	5699303	0151919	5851222	56
5	4153468	9847740	4305727	5694273	0152260	5846532	55
6	4158152	9847400	4310753	5689247	0152600	5841848	54
7	4162832	9847059	4315773	5684227	0152941	5837168	53
8	4167506	9846717	4320789	5679211	0153283	5832494	52
9	4172174	9846375	4325799	5674201	0153625	5827826	51
10	4176837	9846033	4330804	5669196	0153967	5823163	50
11	4181495	9845690	4335805	5664195	0154310	5818505	49
12	4186148	9845347	4340800	5659200	0154653	5813852	48
13	4190795	9845004	4345791	5654209	0154996	5809205	47
14	4195436	9844660	4350776	5649224	0155340	5804564	46
15	4200073	9844316	4355757	5644243	0155684	5799927	45
16	4204704	9843971	4360733	5639267	0156029	5795296	44
17	4209330	9843626	4365704	5634296	0156374	5790670	43
18	4213950	9843281	4370670	5629330	0156719	5786050	42
19	4218566	9842935	4375631	5624369	0157065	5781434	41
20	4223176	9842589	4380587	5619413	0157411	5776824	40
21	4227780	9842242	4385538	5614462	0157758	5772220	39
22	4232380	9841895	4390485	5609515	0158105	5767620	38
23	4236974	9841548	4395426	5604574	0158452	5763026	37
24	4241563	9841200	4400363	5599637	0158800	5758437	36
25	4246147	9840852	4405295	5594705	0159148	5753853	35
26	4250726	9840503	4410222	5589778	0159497	5749274	34
27	4255299	9840154	4415145	5584855	0159846	5744701	33
28	4259867	9839805	4420062	5579938	0160195	5740133	32
29	4264430	9839455	4424975	5575025	0160545	5735570	31
30	4268988	9839105	4429883	5570117	0160895	5731012	30
31	4273541	9838755	4434786	5565214	0161245	5726459	29
32	4278089	9838404	4439685	5560315	0161596	5721911	28
33	4282631	9838052	4444579	5555421	0161948	5717369	27
34	4287169	9837701	4449468	5550532	0162299	5712831	26
35	4291701	9837348	4454352	5545648	0162652	5708299	25
36	4296228	9836996	4459232	5540768	0163004	5703772	24
37	4300750	9836643	4464107	5535893	0163357	5699250	23
38	4305267	9836290	4468978	5531022	0163710	5694733	22
39	4309779	9835936	4473843	5526157	0164064	5690221	21
40	4314286	9835582	4478704	5521296	0164418	5685714	20
41	4318788	9835227	4483561	5516439	0164773	5681212	19
42	4323285	9834872	4488413	5511587	0165128	5676715	18
43	4327777	9834517	4493260	5506740	0165483	5672223	17
44	4332264	9834161	4498102	5501898	0165839	5667736	16
45	4336746	9833805	4502940	5497060	0166195	5663254	15
46	4341223	9833449	4507774	5492226	0166551	5658777	14
47	4345694	9833092	4512602	5487398	0166908	5654306	13
48	4350161	9832735	4517427	5482573	0167265	5649839	12
49	4354623	9832377	4522246	5477754	0167623	5645377	11
50	4359080	9832019	4527061	5472939	0167981	5640920	10
51	4363532	9831661	4531872	5468128	0168339	5636468	9
52	4367980	9831302	4536678	5463322	0168698	5632020	8
53	4372422	9830942	4541479	5458521	0169058	5627578	7
54	4376859	9830583	4546276	5453724	0169417	5623141	6
55	4381292	9830223	4551069	5448931	0169777	5618708	5
56	4385719	9829862	4555857	5444143	0170138	5614281	4
57	4390142	9829501	4560641	5439359	0170499	5609858	3
58	4394560	9829140	4565420	5434580	0170860	5605440	2
	4398973	9828778	4570194	5429806	0171222	5601027	1
	4403381	9828416	4574964	5425036	0171584	559[illegible]19	0

16 Degrees, 73 Degrees.

M	Sine.		Tang.		Secant.		
0	9.4403381	9.9828416	9.4574964	10.5425036	10.0171584	10.5596619	60
1	4407784	9828054	4579730	5420270	0171946	5592216	59
2	4412182	9827691	4584491	5415509	0172309	5587818	58
3	4416576	9827328	4589248	5410752	0172672	5583424	57
4	4420965	9826964	4594001	5405999	0173036	5579035	56
5	4425349	9826600	4598749	5401251	0173400	5574651	55
6	4429728	9826236	4603492	5396508	0173764	5570272	54
7	4434103	9825871	4608232	5391768	0174129	5565897	53
8	4438472	9825506	4612967	5387033	0174494	5561528	52
9	4442837	9825140	4617697	5382303	0174860	5557163	51
10	4447197	9824774	4622423	5377577	0175226	5552803	50
11	4451553	9824408	4627145	5372855	0175592	5548447	49
12	4455904	9824041	4631863	5368137	0175959	5544096	48
13	4460250	9823674	4636576	5363424	0176326	5539750	47
14	4464591	9823306	4641285	5358715	0176694	5535409	46
15	4468927	9822938	4645990	5354010	0177062	5531073	45
16	4473259	9822569	4650690	5349310	0177431	5526741	44
17	4477586	9822201	4655386	5344614	0177799	5522414	43
18	4481909	9821831	4660078	5339922	0178169	5518091	42
19	4486227	9821462	4664765	5335235	0178538	5513773	41
20	4490540	9821092	4669448	5330552	0178908	5509460	40
21	4494849	9820721	4674127	5325873	0179279	5505151	39
22	4499153	9820351	4678802	5321198	0179649	5500847	38
23	4503452	9819979	4683473	5316527	0180021	5496548	37
24	4507747	9819608	4688139	5311861	0180392	5492253	36
25	4512037	9819236	4692801	5307199	0180764	5487963	35
26	4516322	9818863	4697459	5302541	0181137	5483678	34
27	4520603	9818490	4702112	5297888	0181510	5479397	33
28	4524879	9818117	4706762	5293238	0181883	5475121	32
29	4529151	9817744	4711407	5288593	0182256	5470849	31
30	4533418	9817370	4716048	5283952	0182630	5466582	30
31	4537681	9816995	4720685	5279315	0183005	5462319	29
32	4541939	9816620	4725318	5274682	0183380	5458061	28
33	4546192	9816245	4729947	5270053	0183755	5453808	27
34	4550441	9815870	4734572	5265428	0184130	5449559	26
35	4554686	9815494	4739192	5260808	0184506	5445314	25
36	4558926	9815117	4743808	5256192	0184883	5441074	24
37	4563161	9814740	4748421	5251579	0185260	5436839	23
38	4567392	9814363	4753029	5246971	0185637	5432608	22
39	4571618	9813986	4757633	5242367	0186014	5428382	21
40	4575840	9813608	4762233	5237767	0186392	5424160	20
41	4580058	9813229	4766829	5233171	0186771	5419942	19
42	4584271	9812850	4771421	5228579	0187150	5415729	18
43	4588480	9812471	4776009	5223991	0187529	5411520	17
44	4592684	9812091	4780592	5219408	0187909	5407316	16
45	4596884	9811711	4785172	5214828	0188289	5403116	15
46	4601079	9811331	4789748	5210252	0188669	5398921	14
47	4605270	9810950	4794319	5205681	0189050	5394730	13
48	4609456	9810569	4798887	5201113	0189431	5390544	12
49	4613638	9810187	4803451	5196549	0189813	5386362	11
50	4617816	9809805	4808011	5191989	0190195	5382184	10
51	4621989	9809423	4812566	5187434	0190577	5378011	9
52	4626158	9809040	4817118	5182882	0190960	5373842	8
53	4630323	9808657	4821666	5178334	0191343	5369677	7
54	4634483	9808273	4826210	5173790	0191727	5365517	6
55	4638639	9807889	4830750	5169250	0192111	5361361	5
56	4642790	9807505	4835286	5164714	0192495	5357210	4
57	4646938	9807120	4839818	5160182	0192880	5353062	3
58	4651081	9806735	4844346	5155654	0193265	5348919	2
59	4655219	9806349	4848870	5151130	0193651	5344781	1
60	4659353	9805963	4853390	5146610	0194037	5340647	0

Degrees. 72 Degrees.

	Sine.		Tang.		Secant.		
	9.4659353	9.9805963	9.4853390	10.5146610	10.0194037	10.5340647	60
	4663483	9805577	4857907	5142093	0194423	5336517	59
	4667609	9805190	4862419	5137581	0194810	5332391	58
	4671730	9804803	4866928	5133072	0195197	5328270	57
	4675848	9804415	4871433	5128567	0195585	5324152	56
	4679960	9804027	4875933	5124067	0195973	5320040	55
	4684069	9803639	4880430	5119570	0196361	5315931	54
	4688173	9803250	4884924	5115076	0196750	5311827	53
	4692273	9802860	4889413	5110587	0197140	5307727	52
	4696369	9802471	4893898	5106102	0197529	5303631	51
	4700461	9802081	4898380	5101620	0197919	5299539	50
	4704548	9801690	4902858	5097142	0198310	5295452	49
	4708631	9801299	4907332	5092668	0198701	5291369	48
	4712710	9800908	4911802	5088198	0199092	5287290	47
	4716785	9800516	4916269	5083731	0199484	5283215	46
	4720856	9800124	4920731	5079269	0199876	5279144	45
	4724922	9799732	4925190	5074810	0200268	5275078	44
	4728985	9799339	4929646	5070354	0200661	5271015	43
	4733043	9798946	4934097	5065903	0201054	5266957	42
	4737097	9798552	4938545	5061455	0201448	5262903	41
	4741146	9798158	4942988	5057012	0201842	5258854	40
	4745192	9797764	4947429	5052571	0202236	5254808	39
	4749234	9797369	4951865	5048135	0202631	5250766	38
	4753271	9796973	4956298	5043702	0203027	5246729	37
	4757304	9796578	4960727	5039273	0203422	5242696	36
	4761334	9796182	4965152	5034848	0203818	5238666	35
	4765359	9795785	4969574	5030426	0204215	5234641	34
	4769380	9795388	4973991	5026009	0204612	5230620	33
	4773396	9794991	4978406	5021594	0205009	5226604	32
	4777409	9794593	4982816	5017184	0205407	5222591	31
30	4781418	9794195	4987223	5012777	0205805	5218582	30
31	4785423	9793796	4991626	5008374	0206204	5214577	29
32	4789423	9793398	4996026	5003974	0206602	5210577	28
33	4793420	9792998	5000422	4999578	0207002	5206580	27
34	4797412	9792599	5004814	4995186	0207401	5202588	26
35	4801401	9792198	5009203	4990797	0207802	5198599	25
36	4805385	9791798	5013588	4986412	0208202	5194615	24
37	4809366	9791397	5017969	4982031	0208603	5190634	23
38	4813342	9790996	5022347	4977653	0209004	5186658	22
39	4817315	9790594	5026721	4973279	0209406	5182685	21
40	4821283	9790192	5031092	4968908	0209808	5178717	20
41	4825248	9789789	5035459	4964541	0210211	5174752	19
42	4829208	9789386	5039822	4960178	0210614	5170792	18
43	4833165	9788983	5044182	4955818	0211017	5166835	17
44	4837117	9788579	5048538	4951462	0211421	5162883	16
45	4841066	9788175	5052891	4947109	0211825	5158934	15
46	4845010	9787770	5057240	4942760	0212230	5154990	14
47	4848951	9787365	5061586	4938414	0212635	5151049	13
48	4852888	9786960	5065928	4934072	0213040	5147112	12
49	4856820	9786554	5070267	4929733	0213446	5143180	11
50	4860749	9786148	5074602	4925398	0213852	5139251	10
51	4864674	9785741	5078933	4921067	0214259	5135326	9
52	4868595	9785334	5083261	4916739	0214666	5131405	8
53	4872512	9784927	5087586	4912414	0215073	5127488	7
54	4876426	9784519	5091907	4908093	0215481	5123574	6
55	4880335	9784111	5096224	4903776	0215889	5119665	5
56	4884240	9783702	5100539	4899461	0216298	5115760	4
57	4888142	9783293	5104849	4895151	0216707	5111858	3
58	4892040	9782883	5109156	4890844	0217117	5107960	2
59	4895934	9782474	5113460	4886540	0217526	5104066	1
60	4899824	9782063	5117760	4882240	0217937	5100176	0

18 Degrees. 71 Degrees.

M	Sine.		Tang.		Secant.		
0	9.4899824	9.9782063	9.5117760	10.4882240	10.0217937	10.5100176	60
1	4903710	9781653	5122057	4877943	0218347	5096290	59
2	4907592	9781241	5126351	4873649	0218759	5092408	58
3	4911471	9780830	5130641	4869359	0219170	5088529	57
4	4915345	9780418	5134927	4865073	0219582	5084655	56
5	4919216	9780006	5139210	4860790	0219994	5080784	55
6	4923083	9779593	5143490	4856510	0220407	5076917	54
7	4926946	9779180	5147766	4852234	0220820	5073054	53
8	4930806	9778766	5152039	4847961	0221234	5069194	52
9	4934661	9778353	5156309	4843691	0221647	5065339	51
10	4938513	9777938	5160575	4839425	0222062	5061487	50
11	4942361	9777523	5164838	4835162	0222477	5057639	49
12	4946205	9777108	5169097	4830903	0222892	5053795	48
13	4950046	9776693	5173353	4826647	0223307	5049954	47
14	4953883	9776277	5177606	4822394	0223723	5046117	46
15	4957716	9775860	5181855	4818145	0224140	5042284	45
16	4961545	9775444	5186101	4813899	0224556	5038455	44
17	4965370	9775026	5190344	4809656	0224974	5034630	43
18	4969192	9774609	5194583	4805417	0225391	5030808	42
19	4973010	9774191	5198819	4801181	0225809	5026990	41
20	4976824	9773772	5203052	4796948	0226228	5023176	40
21	4980635	9773354	5207282	4792718	0226646	5019365	39
22	4984442	9772934	5211508	4788492	0227066	5015558	38
23	4988245	9772515	5215730	4784270	0227485	5011755	37
24	4992045	9772095	5219950	4780050	0227905	5007955	36
25	4995840	9771674	5224166	4775834	0228326	5004160	35
26	4999633	9771253	5228379	4771621	0228747	5000367	34
27	5003421	9770832	5232589	4767411	0229168	4996579	33
28	5007206	9770410	5236795	4763205	0229590	4992794	32
29	5010987	9769988	5240999	4759001	0230012	4989013	31
30	5014764	9769566	5245199	4754801	0230434	4985236	30
31	5018538	9769143	5249395	4750605	0230857	4981462	29
32	5022308	9768720	5253589	4746411	0231280	4977692	28
33	5026075	9768296	5257779	4742221	0231704	4973925	27
34	5029838	9767872	5261966	4738034	0232128	4970162	26
35	5033597	9767447	5266150	4733850	0232553	4966403	25
36	5037353	9767022	5270331	4729669	0232978	4962647	24
37	5041105	9766597	5274508	4725492	0233403	4958895	23
38	5044853	9766171	5278682	4721318	0233829	4955147	22
39	5048598	9765745	5282853	4717147	0234255	4951402	21
40	5052339	9765318	5287021	4712979	0234682	4947661	20
41	5056077	9764891	5291186	4708814	0235109	4943923	19
42	5059811	9764464	5295347	4704653	0235536	4940189	18
43	5063542	9764036	5299505	4700495	0235964	4936458	17
44	5067269	9763608	5303661	4696339	0236392	4932731	16
45	5070992	9763179	5307813	4692187	0236821	4929008	15
46	5074712	9762750	5311961	4688039	0237250	4925288	14
47	5078428	9762321	5316107	4683893	0237679	4921572	13
48	5082141	9761891	5320250	4679750	0238109	4917859	12
49	5085850	9761461	5324389	4675611	0238539	4914150	11
50	5089556	9761030	5328526	4671474	0238970	4910444	10
51	5093258	9760599	5332659	4667341	0239401	4906742	9
52	5096956	9760167	5336789	4663211	0239833	4903044	8
53	5100651	9759736	5340916	4659084	0240264	4899349	7
54	5104343	9759303	5345040	4654960	0240697	4895657	6
55	5108031	9758870	5349161	4650839	0241130	4891969	5
56	5111716	9758437	5353278	4646722	0241563	4888284	4
57	5115397	9758004	3357393	4642607	0241996	4884603	3
58	5119074	9757570	5361505	4638495	0242430	4880926	2
59	5122749	9757135	5365613	4634387	0242865	4877251	1
60	5126419	9756701	5369719	4630281	0243299	4873581	0

19 Degrees. 70 Degrees.

M	Sine.		Tang.		Secant.		
0	9.5126419	9.9756701	9.5369719	10.4630281	10.0243299	10.4873581	60
1	5130086	9756265	5373821	4626179	0243735	4869914	59
2	5133750	9755830	5377920	4622080	0244170	4866250	58
3	5137410	9755394	5382017	4617983	0244606	4862590	57
4	5141067	9754957	5386110	4613890	0245043	4858933	56
5	5144721	9754521	5390200	4609800	0245479	4855279	55
6	5148371	9754083	5394287	4605713	0245917	4851629	54
7	5152017	9753646	5398371	4601629	0246354	4847983	53
8	5155660	9753208	5402453	4597547	0246792	4844340	52
9	5159300	9752769	5406531	4593469	0247231	4840700	51
10	5162936	9752330	5410606	4589394	0247670	4837064	50
11	5166569	9751891	5414678	4585322	0248109	4833431	49
12	5170198	9751451	5418747	4581253	0248549	4829802	48
13	5173824	9751011	5422813	4577187	0248989	4826176	47
14	5177447	9750570	5426877	4573123	0249430	4822553	46
15	5181066	9750129	5430937	4569063	0249871	4818934	45
16	5184682	9749688	5434994	4565006	0250312	4815318	44
17	5188295	9749246	5439048	4560952	0250754	4811705	43
18	5191904	9748804	5443100	4556900	0251196	4808096	42
19	5195510	9748361	5447148	4552852	0251639	4804490	41
20	5199112	9747918	5451193	4548807	0252082	4800888	40
21	5202711	9747475	5455236	4544764	0252525	4797289	39
22	5206307	9747031	5459276	4540724	0252969	4793693	38
23	5209899	9746587	5463312	4536688	0253413	4790101	37
24	5213488	9746142	5467346	4532654	0253858	4786512	36
25	5217074	9745697	5471377	4528623	0254303	4782926	35
26	5220656	9745252	5475405	4524595	0254748	4779344	34
27	5224235	9744806	5479430	4520570	0255194	4775765	33
28	5227811	9744359	5483452	4516548	0255641	4772189	32
29	5231383	9743913	5487471	4512529	0256087	4768617	31
30	5234953	9743466	5491487	4508513	0256534	4765047	30
31	5238518	9743018	5495500	4504500	0256982	4761482	29
32	5242081	9742570	5499511	4500489	0257430	4757919	28
33	5245640	9742122	5503519	4496481	0257878	4754360	27
34	5249196	9741673	5507523	4492477	0258327	4750804	26
35	5252749	9741224	5511525	4488475	0258776	4747251	25
36	5256298	9740774	5515524	4484476	0259226	4743702	24
37	5259844	9740324	5519521	4480479	0259676	4740156	23
38	5263387	9739873	5523514	4476486	0260127	4736613	22
39	5266927	9739422	5527504	4472496	0260578	4733073	21
40	5270463	9738971	5531492	4468508	0261029	4729537	20
41	5273997	9738519	5535477	4464523	0261481	4726003	19
42	5277526	9738067	5539459	4460541	0261933	4722474	18
43	5281053	9737615	5543438	4456562	0262385	4718947	17
44	5284577	9737162	5547415	4452585	0262838	4715423	16
45	5288097	9736709	5551388	4448612	0263291	4711903	15
46	5291614	9736255	5555359	4444641	0263745	4708386	14
47	5295128	9735801	5559327	4440673	0264199	4704872	13
48	5298638	9735346	5563292	4436708	0264654	4701362	12
49	5302146	9734891	5567255	4432745	0265109	4697854	11
50	5305650	9734435	5571214	4428786	0265565	4694350	10
51	5309151	9733980	5575171	4424829	0266020	4690849	9
52	5312649	9733523	5579125	4420875	0266477	4687351	8
53	5316143	9733067	5583077	4416923	0266933	4683857	7
54	5319635	9732610	5587025	4412975	0267390	4680365	6
55	5323123	9732152	3590971	4409029	0267848	4676877	5
56	5326608	9731694	5594914	4405086	0268306	4673392	4
57	5330090	9731236	5598854	4401146	0268764	4669910	3
58	5333569	9730777	5602792	4397208	0269223	4666431	2
59	5337044	9730318	5606727	4393273	0269682	4662956	1
60	5340517	9729858	5610659	4389341	0270142	4659483	0

20 Degrees. 69 Degrees.

M	Sine		Tang.		Secant.		
0	9.5340517	9.9729858	9.5610659	10.4389341	10.0270142	10.4659483	60
1	5343986	9729398	5614588	4385412	0270602	4656014	59
2	5347452	9728938	5618515	4381485	0271062	4652548	58
3	5350915	9728477	5622439	4377561	0271523	4649080	57
4	5354375	9728016	5626360	4373640	0271984	4645625	56
5	5357832	9727554	5630278	4369722	0272446	4642168	55
6	5361286	9727092	5634194	4365806	0272908	4638714	54
7	5364737	9726629	5638107	4361893	0273371	4635263	53
8	5368184	9726166	5642018	4357982	0273834	4631816	52
9	5371629	9725703	5645925	4354075	0274297	4628371	51
10	5375070	9725239	5649831	4350169	0274761	4624930	50
11	5378508	9724775	5653733	4346267	0275225	4621492	49
12	5381943	9724310	5657633	4342367	0275690	4618057	48
13	5385375	9723845	5661530	4338470	0276155	4614625	47
14	5388804	9723380	5665424	4334576	0276620	4611196	46
15	5392230	9722914	5669316	4330684	0277086	4607770	45
16	5395653	9722448	5673205	4326795	0277552	4604347	44
17	5399073	9721981	5677091	4322909	0278019	4600927	43
18	5402489	9721514	5680975	4319025	0278486	4597511	42
19	5405903	9721047	5684856	4315144	0278953	4594097	41
20	5409314	9720579	5688735	4311265	0279421	4590686	40
21	5412721	9720110	5692611	4307389	0279890	4587279	39
22	5416126	9719642	5696484	4303516	0280358	4583874	38
23	5419527	9719172	5700355	4299645	0280828	4580473	37
24	5422926	9718703	5704223	4295777	0281297	4577074	36
25	5426321	9718233	5708088	4291912	0281767	4573679	35
26	5429713	9717762	5711951	4288049	0282238	4570287	34
27	5433103	9717291	5715811	4284189	0282709	4566897	33
28	5436489	9716820	5719669	4280331	0283180	4563511	32
29	5439873	9716348	5723524	4276476	0283652	4560127	31
30	5443253	9715876	5727377	4272623	0284124	4556747	30
31	5446630	9715404	5731227	4268773	0284596	4553370	29
32	5450005	9714931	5735074	4264926	0285069	4549995	28
33	5453376	9714457	5738919	4261081	0285543	4546624	27
34	5456745	9713984	5742761	4257239	0286016	4543255	26
35	5460110	9713509	5746601	4253399	0286491	4539890	25
36	5463472	9713035	5750438	4249562	0286965	4536528	24
37	5466832	9712560	5754272	4245728	0287440	4533168	23
38	5470189	9712084	5758104	4241896	0287916	4529811	22
39	5473542	9711608	5761934	4238066	0288392	4526458	21
40	5476893	9711132	5765761	4234239	0288868	4523107	20
41	5480240	9710655	5769585	4230415	0289345	4519760	19
42	5483585	9710178	5773407	4226593	0289822	4516415	18
43	5486927	9709701	5777226	4222774	0290299	4513073	17
44	5490266	9709223	5781043	4218957	0290777	4509734	16
45	5493602	9708744	5784858	4215142	0291256	4506398	15
46	5496935	9708265	5788669	4211331	0291735	4503065	14
47	5500265	9707786	5792479	4207521	0292214	4499735	13
48	5503592	9707306	5796286	4203714	0292694	4496408	12
49	5506916	9706826	5800090	4199910	0293174	4493084	11
50	5510237	9706346	5803892	4196108	0293654	4489763	10
51	5513556	9705865	5807691	4197309	0294135	4486444	9
52	5516871	9705383	5811488	4188512	0294617	4483129	8
53	5520184	9704902	5815282	4184718	0295098	4479816	7
54	5523494	9704419	5819074	4180926	0295581	4476506	6
55	5526801	9703937	5822864	4177136	0296063	4473199	5
56	5530105	9703454	5826651	4173349	0296546	4469895	4
57	5533406	9702970	5830435	4169565	0297030	4466594	3
58	5536704	9702486	5834217	4165783	0297514	4463296	2
59	5539999	0702002	5837997	4162003	0297998	4460001	1
60	5543292	9701517	5841774	4158226	0298483	4456708	0

21 Degrees. 68 Degrees.

M	Sine.		Tang.		Secant.		
0	9.5543292	9.9701517	9.5841774	10 4158226	10.0298483	10.4456708	60
1	5546581	9701032	5845549	4154451	0298968	4453419	59
2	5549868	9700547	5849321	4150679	0299453	4450132	58
3	5553152	9700061	5853091	4146909	0299939	4446848	57
4	5556433	9699574	5856859	4143141	0300426	4443567	56
5	5559711	9699087	5860624	4139376	0300913	4440289	55
6	5562987	9698600	5864386	4135614	0301400	4437013	54
7	5566259	9698112	5868147	4131853	0301888	4433741	53
8	5569529	9697624	5871904	4128096	0302376	4430471	52
9	5572796	9697136	5875660	4124340	0302864	4427204	51
10	5576060	9696647	5879413	4120587	0303353	4423940	50
11	5579321	9696158	5883163	4116837	0303842	4420679	49
12	5582579	9695668	5886912	4113088	0304332	4417421	48
13	5585835	9695177	5890657	4109343	0304823	4414165	47
14	5589088	9694687	5894401	4105599	0305313	4410912	46
15	5592338	9694196	5898142	4101858	0305804	4407662	45
16	5595585	9693704	5901881	4098119	0306296	4404415	44
17	5598829	9693212	5905617	4094383	0306788	4401171	43
18	5602071	9692720	5909351	4090649	0307280	4397929	42
19	5605310	9692227	5913082	4086918	0307773	4394690	41
20	5608546	9691734	5916812	4083188	0308266	4391454	40
21	5611779	9691241	5920539	4079461	0308759	4388221	39
22	5615010	9690746	5924263	4075737	0309254	4384990	38
23	5618237	9690252	5927985	4072015	0309748	4381763	37
24	5621462	9689757	5931705	4068295	0310243	4378538	36
25	5624685	9689262	5935423	4064577	0310738	4375315	35
26	5627904	9688766	5939138	4060862	0311234	4372096	34
27	5631121	9688270	5942851	4057149	0311730	4368879	33
28	5634335	9687773	5946561	4053489	0312227	4365665	32
29	5637546	9687276	5950269	4049731	0312724	4362454	31
30	5640754	9686779	5953975	4046025	0313221	4359246	30
31	5643960	9686281	5957679	4042321	0313719	4356040	29
32	5647163	9685783	5961380	4038620	0314217	4352837	28
33	5650363	9685284	5965079	4034921	0314716	4349637	27
34	5653561	9684785	5968776	4031224	0315215	4346439	26
35	5656756	9684286	5972470	4027530	0315714	4343244	25
36	5659948	9683786	5976162	4023838	0316214	4340052	24
37	5663137	9683285	5979852	4020148	0316715	4336863	23
38	5666324	9682784	5983540	4016460	0317216	4333676	22
39	5669508	9682283	5987225	4012775	0317717	4330492	21
40	5672689	9681781	5990908	4009092	0318219	4327311	20
41	5675868	9681279	5994588	4005412	0318721	4324132	19
42	5679044	9680777	5998267	4001733	0319223	4320956	18
43	5682217	9680274	6001943	3998057	0319726	4317783	17
44	5685387	9679771	6005617	3994383	0320229	4314613	16
45	5688555	9679267	6009289	3990711	0320733	4311445	15
46	5691721	9678763	6012958	3987042	0321237	4308279	14
47	5694883	9678258	6016625	3983375	0321742	4305117	13
48	5698043	9677753	6020290	3979710	0322247	4301957	12
49	5701200	9677247	6023953	3976047	0322753	4298800	11
50	5704355	9676741	6027613	3972387	0323259	4295645	10
51	5707506	9676235	6031271	3968729	0323765	4292494	9
52	5710656	9675728	6034927	3965073	0324272	4289344	8
53	5713802	9675221	6038581	3961419	0324779	4286198	7
54	5716946	9674713	6042233	3957767	0325287	4283054	6
55	5720087	9674205	6045882	3954118	0325795	4279913	5
56	5723226	9673697	6049529	3950471	0326303	4276774	4
57	5726362	9673188	6053174	3946826	0326812	4273638	3
58	5729495	9672679	6056817	3943183	0327321	4270505	2
59	5732626	9672169	6060457	3939543	0327831	4267374	1
60	5735754	9671659	6064096	3935904	0328341	4264246	0

22 Degrees. 67 Degrees.

M	Sines.		Tang.		Secants.		
0	9.5735754	9.9671659	9.6064096	10.3935904	10.0328341	10.4264246	60
1	5738880	9671148	6067732	3932268	0328852	4261120	59
2	5742003	9670637	6071366	3928634	0329363	4257997	58
3	5745123	9670125	6074997	3925003	0329875	4254877	57
4	5748240	9669614	6078627	3921373	0330386	4251760	56
5	5751356	9669101	6082254	3917746	0330899	4248644	55
6	5754468	9668588	6085880	3914120	0331412	4245532	54
7	5757578	9668075	6089503	3910497	0331925	4242422	53
8	5760685	9667562	6093124	3906876	0332438	4239315	52
9	5763790	9667048	6096742	3903258	0332952	4236210	51
10	5766892	9666533	6100359	3899641	0333467	4233108	50
11	5769991	9666018	6103973	3896027	0333982	4230009	49
12	5773088	9665503	6107586	3892414	0334497	4226912	48
13	5776183	9664987	6111196	3888804	0335013	4223817	47
14	5779275	9664471	6114804	3885196	0335529	4220725	46
15	5782364	9663954	6118409	3881591	0336046	4217636	45
16	5785450	9663437	6122013	3877987	0336563	4214550	44
17	5788535	9662920	6125615	3874385	0337080	4211465	43
18	5791616	9662402	6129214	3870786	0337598	4208384	42
19	5794695	9661884	6132812	3867188	0338116	4205305	41
20	5797772	9661365	6136407	3863593	0338635	4202228	40
21	5800845	9660846	6140000	3860000	0339154	4199155	39
22	5803917	9660326	6143591	3856409	0339674	4196083	38
23	5806986	9659806	6147180	3852820	0340194	4193014	37
24	5810052	9659285	6150766	3849234	0340715	4189948	36
25	5813116	9658764	6154351	3845649	0341236	4186884	35
26	5816177	9658243	6157934	3842066	0341757	4183823	34
27	5819236	9657721	6161514	3838486	0342279	4180764	33
28	5822292	9657199	6165093	3834907	0342801	4177708	32
29	5825345	9656677	6168669	3831331	0343323	4174655	31
30	5828397	9656153	6172243	3827757	0343847	4171603	30
31	5831445	9655630	6175815	3824185	0344370	4168555	29
32	5834491	9655106	6179385	3820615	0344894	4165509	28
33	5837535	9654582	6182953	3817047	0345418	4162465	27
34	5840576	9654057	6186519	3813481	0345943	4159424	26
35	5843615	9653532	6190083	3809917	0346468	4156385	25
36	5846651	9653006	6193645	3806355	0346994	4153349	24
37	5849685	9652480	6197205	3802795	0347520	4150315	23
38	5852716	9651953	6200762	3799238	0348047	4147284	22
39	5855745	9651426	6204318	3795682	0348574	4144255	21
40	5858771	9650899	6207872	3792128	0349101	4141229	20
41	5861795	9650371	6211423	3788577	0349629	4138205	19
42	5864816	9649843	6214973	3785027	0350157	4135184	18
43	5867835	9649314	6218520	3781480	0350686	4132165	17
44	5870851	9648785	6222066	3777934	0351215	4129149	16
45	5873865	9648256	6225609	3774391	0351744	4126135	15
46	5876876	9647726	6229150	3770850	0352274	4123124	14
47	5879885	9647195	6232690	3767310	0352805	4120115	13
48	5882892	9646665	6236227	3763773	0353335	4117108	12
49	5885896	9646133	6239763	3760237	0353867	4114104	11
50	5888897	9645602	6243296	3756704	0354398	4111103	10
51	5891897	9645069	6246827	3753173	0354931	4108103	9
52	5894893	9644537	6250356	3749644	0355463	4105107	8
53	5897888	9644004	6253884	3746116	0355996	4102112	7
54	5900880	9643470	6257409	3742591	0356530	4099120	6
55	5903869	9642937	6260932	3739068	0357063	4096131	5
56	5906856	9642402	6264454	3735546	0357598	4093144	4
57	5909841	9641868	6267973	3732027	0358132	4090159	3
58	5912823	9641332	6271491	3728509	0358668	4087177	2
59	5915803	9640797	6275006	3724994	0359203	4084197	1
60	5918780	9640261	6278519	3721481	0359739	4081220	0

23 Degrees. 66 Degrees.

M	Sine.		Tang.		Secant.		
0	9.5918780	9.9640261	9.6278519	10.3721481	10.0359739	10.4081220	60
1	5921755	9639724	6282031	3717969	0360276	4078245	59
2	5924728	9639187	6285540	3714460	0360813	4075272	58
3	5927698	9638650	6289048	3710952	0361350	4072302	57
4	5930666	9638112	6292553	3707447	0361888	4069334	56
5	5933631	9637574	6296057	3703943	0362426	4066369	55
6	5936594	9637036	6299558	3700442	0362964	4063406	54
7	5939555	9636496	6303058	3696942	0363504	4060445	53
8	5942513	9635957	6306556	3693444	0364043	4057487	52
9	5945469	9635417	6310052	3689948	0364583	4054531	51
10	5948422	9634877	6313545	3686455	0365123	4051578	50
11	5951373	9634336	6317037	3682963	0365664	4048627	49
12	5954322	9633795	6320527	3079473	0366205	4045678	48
13	5957268	9633253	6324015	3675985	0366747	4042732	47
14	5960212	9632711	6327501	3672499	0367289	4039788	46
15	5963154	9632168	6330985	3669015	0367832	4036846	45
16	5966093	9631625	6334468	3665532	0368375	4033907	44
17	5969030	9631082	6337948	3662052	0368918	4030970	43
18	5971965	9630538	6341426	3658574	0369462	4028035	42
19	5974897	9629994	6344903	3655097	0370006	4025103	41
20	5977827	9629449	6348378	3651622	0370551	4022173	40
21	5980754	9628904	6351850	3648150	0371096	4019246	39
22	5983679	9628358	6355321	3644679	0371642	4016321	38
23	5986602	9627812	6358790	3641210	0372188	4013398	37
24	5989523	9627266	6362257	3637743	0372734	4010477	36
25	5992441	9626719	6365722	3634278	0373281	4007559	35
26	5995357	96 6172	6369185	3630815	0373828	4004643	34
27	5998270	9625624	6372646	3627354	0374376	4001730	33
28	6001181	9625076	6376106	3623894	0374924	3998819	32
29	6004090	9624527	6379563	3620437	0375473	3995910	31
30	6006997	9623978	6383019	3616981	0376022	3993003	30
31	6009901	9623428	6386473	3613527	0376572	3990099	29
32	6012803	9622878	6389925	3610075	0377122	3987197	28
33	6015703	9622328	6393375	3606625	0377672	3984297	27
34	6018600	9621777	6396823	3603177	0378223	3981400	26
35	6021495	9621226	6400269	3599731	0378774	3978505	25
36	6024388	9620674	6403714	3596286	0379326	3975612	24
37	6027278	9620122	6407156	3592844	0379878	3972722	23
38	6030166	9619569	6410597	3589403	0380431	3969834	22
39	6033052	9619016	6414036	3585964	0380984	3966948	21
40	6035936	9618463	6417473	3582527	0381537	3964064	20
41	6038817	9617909	6420908	3579092	0382091	3961183	19
42	6041696	9617355	6424342	3575658	0382645	3958304	18
43	6044573	9616800	6427773	3572227	0383200	3955427	17
44	6047448	9616245	6431203	3568797	0383755	3952552	16
45	6050320	9615689	6434631	3565369	0384311	3949680	15
46	6053190	9615133	6438057	3561943	0384867	3946810	14
47	6056057	9614576	6441481	3558519	0385424	3943943	13
48	6058923	9614020	6444903	3555097	0385980	3941077	12
49	6061786	9613462	6448324	3551676	0386538	3938214	11
50	6064647	9612904	6451743	3548257	0387096	3935353	10
51	6067506	9612346	6455160	3544840	0387654	3932494	9
52	6070362	9611787	6458575	3541425	0388213	3929638	8
53	6073216	9611228	6461988	3538012	0388772	3926784	7
54	6076068	9610668	6465400	3534600	0389332	3923932	6
55	6078918	9610108	6468810	3531190	0389892	3921082	5
56	6081765	9609548	6472217	3527783	0390452	3918235	4
57	6084611	9608987	6475624	3524376	0391013	3915389	3
58	6087454	9608426	6479028	3520972	0391574	3912546	2
59	6090294	9607864	6482431	3517569	0392136	3909706	1
60	6093133	9607302	6485831	3514169	0392698	3906867	0

D

M	24 Degrees. Sine.		Tang.		Secant.	65 Degrees.	
0	9.6093133	9.9607302	9.6485831	10.3514169	10.0392698	10.3906867	60
1	6095969	9606739	6489230	3510770	0393261	3904031	59
2	6098803	9606176	6492628	3507372	0393824	3901197	58
3	6101635	9605612	6496023	3503977	0394388	3898365	57
4	6104465	9605048	6499417	3500583	0394952	3895535	56
5	6107293	9604484	6502809	3497191	0395516	3892707	55
6	6110118	9603919	6506199	3493801	0396081	3889882	54
7	6112941	9603354	6509587	3490413	0396646	3887059	53
8	6115762	9602788	6512974	3487026	0397212	3884238	52
9	6118580	9602222	6516359	3483641	0397778	3881420	51
10	6121397	9601655	6519742	3480258	0398345	3878603	50
11	6124211	9601088	6523123	3476877	0398912	3875789	49
12	6127023	9600520	6526503	3473497	0399480	3872977	48
13	6129833	9599952	6529881	3470119	0400048	3870167	47
14	6132641	9599384	6533257	3466743	0400616	3867359	46
15	6135446	9598815	6536631	3463369	0401185	3864554	45
16	6138250	9598246	6540004	3459996	0401754	3861750	44
17	6141051	9597676	6543375	3456625	0402324	3858949	43
18	6143850	9597106	6546744	3453256	0402894	3856150	42
19	6146647	9596535	6550112	3449888	0403465	3853353	41
20	6149441	9595964	6553477	3446523	0404036	3850559	40
21	6152234	9595393	6556841	3443159	0404607	3847766	39
22	6155024	9594821	6560204	3439796	0405179	3844976	38
23	6157812	9594248	6563564	3436436	0405752	3842188	37
24	6160599	9593675	6566923	3433077	0406325	3839401	36
25	6163382	9593102	6570280	3429720	0406898	3836618	35
26	6166164	9592528	6573636	3426364	0407472	3833836	34
27	6168944	9591954	6576989	3423011	0408046	3831056	33
28	6171721	9591380	6580341	3419659	0408620	3828279	32
29	6174496	9590805	6583692	3416308	0409195	3825504	31
30	6177270	9590229	6587041	3412959	0409771	3822730	30
31	6180041	9589653	6590387	3409613	0410347	3819959	29
32	6182809	9589077	6593733	3406267	0410923	3817191	28
33	6185576	9588500	6597076	3402924	0411500	3814424	27
34	6188341	9587923	6600418	3399582	0412077	3811659	26
35	6191103	9587345	6603758	3396242	0412655	3808897	25
36	6193864	9586767	6607097	3392903	0413233	3806136	24
37	6196622	9586188	6610434	3389566	0413812	3803378	23
38	6199378	9585609	6613769	3386231	0414391	3800622	22
39	6202132	9585030	6617103	3382897	0414970	3797868	21
40	6204884	9584450	6620434	3379566	0415550	3795116	20
41	6207634	9583869	6623765	3376235	0416131	3792366	19
42	6210382	9583288	6627093	3372907	0416712	3789618	18
43	6213127	9582707	6630420	3369580	0417293	3786873	17
44	6215871	9582125	6633745	3366255	0417875	3784129	16
45	6218612	9581543	6637069	3362931	0418457	3781388	15
46	6221351	9580961	6640391	3359609	0419039	3778649	14
47	6224088	9580378	6643711	3356289	0419622	3775912	13
48	6226824	9579794	6647030	3352970	0420206	3773176	12
49	6229557	9579210	6650346	3349654	0420790	3770443	11
50	6232287	9578626	6653662	3346338	0421374	3767713	10
51	6235016	9578041	6656975	3343025	0421959	3764984	9
52	6237743	9577456	6660288	3339712	0422544	3762257	8
53	6240468	9576870	6663598	3336402	0423130	3759532	7
54	6243190	9576284	6666907	3333093	0423716	3756810	6
55	6245911	9575697	6670214	3329786	0424303	3754089	5
56	6248629	9575110	6673519	3326481	0424890	3751371	4
57	6251346	9574522	6676823	3323177	0425478	3748654	3
58	6254060	9573934	6680126	3319874	0426066	3745940	2
59	6256772	9573346	6683426	3316574	0426654	3743228	1
60	6259483	9572757	6686725	3313275	0427243	3740517	0

25 Degrees. 64 Degrees.

M	Sine.		Tang.		Secant.		
0	9.6259483	9.9572757	9.6686725	10.3313275	10.0427243	10.3740517	60
1	6262191	9572168	6690023	3309977	0427832	3737809	59
2	6264897	9571578	6693319	3306681	0428422	3735103	58
3	6267601	9570988	6696613	3303387	0429012	3732399	57
4	6270303	9570397	6699906	3300094	0429603	3729697	56
5	6273003	9569806	6703197	3296803	0430194	3726997	55
6	6275701	9569215	6706486	3293514	0430785	3724299	54
7	6278397	9568623	6709774	3290226	0431377	3721603	53
8	6281090	9568030	6713060	3286940	0431970	3718910	52
9	6283782	9567437	6716345	3283655	0432563	3716218	51
10	6286472	9566844	6719628	3280372	0433156	3713528	50
11	6289160	9566250	6722910	3277090	0433750	3710840	49
12	6291845	9565656	6726190	3273810	0434344	3708155	48
13	6294529	9565061	6729468	3270532	0434939	3705471	47
14	6297211	9564466	6732745	3267255	0435534	3702789	46
15	6299890	9563870	6736020	3263980	0436130	3700110	45
16	6302568	9563274	6739294	3260706	0436726	3697432	44
17	6305243	9562678	6742566	3257434	0437322	3694757	43
18	6307917	9562081	6745836	3254164	0437919	3692083	42
19	6310589	9561483	6749105	3250895	0438517	3689411	41
20	6313258	9560886	6752372	3247628	0439114	3686742	40
21	6315926	9560287	6755638	3244362	0439713	3684074	39
22	6318591	9559689	6758903	3241097	0440311	3681409	38
23	6321255	9559089	6762165	3237835	0440911	3678745	37
24	6323916	9558490	6765426	3234574	0441510	3676084	36
25	6326576	9557890	6768686	3231314	0442110	3673424	35
26	6329233	9557289	6771944	3228056	0442711	3670767	34
27	6331889	9556688	6775201	3224799	0443312	3668111	33
28	6334542	9556087	6778456	3221544	0443913	3665458	32
29	6337194	9555485	6781709	3218291	0444515	3662806	31
30	6339844	9554882	6784961	3215039	0445118	3660156	30
31	6342491	9554280	6788211	3211789	0445720	3657509	29
32	6345137	9553676	6791460	3208540	0446324	3654863	28
33	6347780	9553073	6794708	3205292	0446927	3652220	27
34	6350422	9552469	6797953	3202047	0447531	3649578	26
35	6353062	9551864	6801198	3198802	0448136	3646938	25
36	6355699	9551259	6804440	3195560	0448741	3644301	24
37	6358335	9550653	6807682	3192318	0449347	3641665	23
38	6360969	9550047	6810921	3189079	0449953	3639031	22
39	6363601	9549441	6814160	3185840	0450559	3636399	21
40	6366231	9548834	6817396	3182604	0451166	3633769	20
41	6368859	9548227	6820632	3179368	0451773	3631141	19
42	6371484	9547619	6823865	3176135	0452381	3628516	18
43	6374108	9547011	6827098	3172902	0452989	3625892	17
44	6376731	9546402	6830328	3169672	0453598	3623269	16
45	6379351	9545793	6833557	3166443	0454207	3620649	15
46	6381969	9545184	6836785	3163215	0454816	3618031	14
47	6384585	9544574	6840011	3159989	0455426	3615415	13
48	6387199	9543963	6843236	3156764	0456037	3612801	12
49	6389812	9543352	6846459	3153541	0456648	3610188	11
50	6392422	9542741	6849681	3150319	0457259	3607578	10
51	6395030	9542129	6852901	3147099	0457871	3604970	9
52	6397637	9541517	6856120	3143880	0458483	3602363	8
53	6400241	9540904	6859338	3140662	0459096	3599759	7
54	6402844	9540291	6862553	3137447	0459709	3597156	6
55	6405445	9539677	6865768	3134232	0460323	3594555	5
56	6408044	9539063	6868981	3131019	0460937	3591956	4
57	6410640	9538448	6872192	3127808	0461552	3589360	3
58	6413235	9537833	6875402	3124598	0462167	3586765	2
59	6415828	9537218	6878611	3121389	0462782	3584172	1
60	6418420	9536602	6881818	3118182	0463398	3581580	0

26 Degrees. 63 Degrees.

M	Sine.		Tang.		Secant.		
0	9.6418420	9.9536602	9.6881818	10.3118182	10.0463398	10.3581580	60
1	6421009	9535985	6885023	3114977	0464015	3578991	59
2	6423596	9535369	6888227	3111773	0464631	3576404	58
3	6426182	9534751	6891430	3108570	0465249	3573818	57
4	6428765	9534134	6894631	3105369	0465866	3571235	56
5	6431347	9533515	6897631	3102169	0466485	3568653	55
6	6433926	9532897	6901030	3098970	0467103	3566074	54
7	6436504	9532278	6904226	3095774	0467722	3563496	53
8	6439080	9531658	6907422	3092578	0468342	3560920	52
9	6441654	9531038	6910616	3089384	0468962	3558346	51
10	6444226	9530418	6913809	3086191	0469582	3555774	50
11	6446796	9529797	6917000	3083000	0470203	3553204	49
12	6449365	9529175	6920189	3079811	0470825	3550635	48
13	6451931	9528553	6923378	3070622	0471447	3548069	47
14	6454496	9527931	6926565	3073435	0472069	3545504	46
15	6457058	9527308	6929750	3070250	0472092	3542942	45
16	6459619	9526685	6932934	3067066	0473315	3540381	44
17	6462178	9526061	6936117	3063883	0473939	3537822	43
18	6464735	9525437	6939298	3060702	0474563	3535265	42
19	6467290	9524813	6942478	3057522	0475187	3532710	41
20	6469844	9524188	6945656	3054344	0475812	3530156	40
21	6472395	9523562	6948833	3051107	0476438	3527605	39
22	6474945	9522936	6952009	3047991	0477064	3525055	38
23	6477492	9522310	6955183	3044817	0477690	3522508	37
24	6480038	9521683	6958355	3041645	0478317	3519962	36
25	6482582	9521055	6961527	3038473	0478945	3517418	35
26	6485124	9520428	6964697	3035303	0479572	3514876	34
27	6487665	9519799	6967865	3032135	0480201	3512335	33
28	6490203	9519171	6971032	3028968	0480829	3509797	32
29	6492740	9518541	6974198	3025802	0481459	3507260	31
30	6495274	9517912	6977363	3022637	0482088	3504726	30
31	6497807	9517282	6980526	3019474	0482718	3502193	29
32	6500338	951 651	6983687	3016313	0483349	3499662	28
33	6502868	9516020	6986847	3013153	0483980	3497132	27
34	6505395	9515389	6990006	3009994	0484611	3494605	26
35	6507920	9514757	6993164	3006836	0485243	3492080	25
36	6510444	9514124	6996320	3003680	0485876	3489556	24
37	6512966	9513492	6999474	3000526	0486508	3487034	23
38	6515486	9512858	7002628	2997372	0487142	3484514	22
39	6518004	9512224	7005780	2994220	0487776	3481996	21
40	6520521	9511590	7008930	2991070	0488410	3479479	20
41	6523035	9510956	7012080	2987920	0489044	3476965	19
42	6525548	9510320	7015227	2984773	0489680	3474452	18
43	6528059	9509685	7018374	2981626	0490315	3471941	17
44	6530568	9509049	7021519	2978481	0490951	3469432	16
45	6533075	9508412	7024663	2975337	0491588	3466925	15
46	6535581	9507775	7027805	2972195	0492225	3464419	14
47	6538084	9507138	7030946	2969054	0492862	3461916	13
48	6540586	9506500	7034086	2965914	0493500	3459414	12
49	6543086	9505861	7037225	2962775	0494139	3456914	11
50	6545584	9505223	7040362	2959638	0494777	3454416	10
51	6548081	9504583	7043497	2956503	0495417	3451919	9
52	6550575	9503944	7046632	2953368	0496056	3449425	8
53	6553068	9503303	7049765	2950235	0496697	3446932	7
54	6555559	9502663	7052897	2947103	0497337	3444441	6
55	6558048	9502022	7056027	2943973	0497978	3441952	5
56	6560536	9501380	7059156	2940844	0498620	3439464	4
57	6563021	9500738	7062284	2937716	0499262	3436979	3
58	6565505	9500095	7065410	2934590	0499905	3434495	2
59	6567987	9499452	7068535	2931465	0500548	3432013	1
60	6570468	9498809	7071659	2928341	0150119	3429532	0

27 Degrees. 62 Degrees.

M	Sine.		Tang.		Secant.		
0	9.6570468	9.9498809	9.7071659	10.2928341	10.0501191	10.3429532	60
1	6572946	9498165	7074781	2925219	0501835	3427054	59
2	6575423	9497521	7077902	2922098	0502479	3424577	58
3	6577898	9496876	7081022	2918978	0503124	3422102	57
4	6580371	9496230	7084141	2915859	0503770	3419629	56
5	6582842	9495585	7087258	2912742	0504415	3417158	55
6	6585312	9494938	7090374	2909626	0505062	3414688	54
7	6587780	9494292	7093488	2906512	0505708	3412220	53
8	6590246	9493645	7096601	2903399	0506355	3409754	52
9	6592710	9492997	7099713	2900287	0507003	3407290	51
10	6595173	9492349	7102824	2897176	0507651	3404827	50
11	6597633	9491700	7105933	2894067	0508300	3402367	49
12	6600093	9491051	7109041	2890959	0508949	3399907	48
13	6602550	9490402	7112148	2887852	0509598	3397450	47
14	6605005	9489752	7115254	2884746	0510248	3394995	46
15	6607459	9489101	7118358	2881642	0510899	3392541	45
16	6609911	9488450	7121461	2878539	0511550	3390089	44
17	6612361	9487799	7124562	2875438	0512201	3387639	43
18	6614810	9487147	7127662	2872338	0512853	3385190	42
19	6617257	9486495	7130761	2869239	0513505	3382743	41
20	6619702	9485842	7133859	2866141	0514158	3380298	40
21	6622145	9485189	7136956	2863044	0514811	3377855	39
22	6624586	9484535	7140051	2859949	0515465	3375414	38
23	6627026	9483881	7143145	2856855	0516119	3372974	37
24	6629464	9483227	7146237	2853763	0516773	3370536	36
25	6631900	9482572	7149329	2850671	0517428	3368100	35
26	6634335	9481916	7152419	2847581	0518084	3365665	34
27	6636768	9481260	7155508	2844492	0518740	3363232	33
28	6639199	9480604	7158595	2841405	0519396	3360801	32
29	6641628	94799-7	7161682	2838318	0520053	3358372	31
30	6644056	9479289	7164767	2835233	0520711	3355944	30
31	6646482	9478631	7167851	2832149	0521369	3353518	29
32	6648906	9477973	7170933	2829067	0522027	3351094	28
33	6651329	9477314	7174014	2825986	0522686	3348671	27
34	6653749	9476655	7177094	2822906	0523345	3346251	26
35	6656168	9475995	7180173	2819827	0524005	3343832	25
36	6658586	9475335	7183251	2816749	0524665	3341414	24
37	6661001	9474674	7186327	2813673	0525326	3338999	23
38	6663415	9474013	7189402	2810598	0525987	3336585	22
39	6665828	9473352	7192476	2807524	0526648	3334172	21
40	6668238	9472689	7195549	2804451	0527311	3331762	20
41	6670647	9472027	7198620	2801380	0527973	3329353	19
42	6673054	9471364	7201690	2798310	0528636	3326946	18
43	6675459	9470700	7204759	2795241	0529300	3324541	17
44	6677863	9470036	7207827	2792173	0529964	3322137	16
45	6680265	9469372	7210892	2789107	0530628	3319735	15
46	6682665	9468707	7213958	2786042	0531293	3317335	14
47	6685064	9468042	7217022	2782978	0531958	3314936	13
48	6687461	9467376	7220085	2779915	0532624	3312539	12
49	6689856	9466710	7223147	2776853	0533290	3310144	11
50	6692250	9466043	7226207	2773793	0533957	3307750	10
51	6694642	9465376	7229266	2770734	0534624	3305358	9
52	6697032	9464708	7232324	2767676	0535292	3302968	8
53	6699420	9464040	7235381	2764619	0535960	3300580	7
54	6701807	9463371	7238436	2761564	0536629	3298193	6
55	6704192	9462702	7241490	2758510	0537298	3295808	5
56	6706576	9462032	7244543	2755457	0537968	3293424	4
57	6708958	9461362	7247595	2752405	0538638	3291042	3
58	6711338	9460692	7250646	2749354	0539308	3288662	2
59	6713716	9460021	7253695	2746305	0539979	3286284	1
60	6716093	9459349	7256744	2743256	0540651	3283907	0

28 Degrees. 61 Degrees.

M	Sines.		Tang.		Secants.		
0	9.6716093	9.9459349	9.7256744	10.2743256	10.0540651	01.3283907	60
1	6718468	9458677	7259791	2740209	0541323	3281532	59
2	6720841	9458005	7262837	2737163	0541995	3279159	58
3	6723213	9457332	7265881	2734119	0542668	3276787	57
4	6725583	9456659	7268925	2731075	0543341	3274417	56
5	6727952	9455985	7271967	2728033	0544015	3272048	55
6	6730319	9455310	7275008	2724992	0544690	3269681	54
7	6732684	9454636	7278048	2721952	0545364	3267316	53
8	6735047	9453960	7281087	2718913	0546040	3264953	52
9	6737409	9453285	7284124	2715876	0546715	3262591	51
10	6739769	9452609	7287161	2712839	0547391	3260231	50
11	6742128	9451932	7290196	2709804	0548068	3257872	49
12	6744485	9451255	7293230	2706770	0548745	3255515	48
13	6746840	9450577	7296263	2703737	0549423	3253160	47
14	6749194	9449899	7299295	2700705	0550101	3250806	46
15	6751546	9449220	7302325	2697675	0550780	3248454	45
16	6753896	9448541	7305354	2694646	0551459	3246104	44
17	6756245	9447862	7308383	2691617	0552138	3243755	43
18	6758592	9447182	7311410	2688590	0552818	3241408	42
19	6760937	9446501	7314436	2685564	0553499	3239063	41
20	6763281	9445821	7317460	2682540	0554179	3236719	40
21	6765623	9445139	7320484	2679516	0554861	3234377	39
22	6767963	9444457	7323506	2676494	0555543	3232037	38
23	6770302	9443775	7326527	2673473	0556225	3229698	37
24	6772640	9443092	7329547	2670453	0556908	3227360	36
25	6774975	9442409	7332566	2667434	0557591	3225025	35
26	6777309	9441725	7335584	2664416	0558275	3222691	34
27	6779642	9441041	7338601	2661399	0558959	3220358	33
28	6781972	9440356	7341616	2658384	0559644	3218028	32
29	6784301	9439671	7344631	2655369	0560329	3215699	31
30	6786629	9438985	7347644	2652356	0561015	3213371	30
31	6788955	9438299	7350656	2649344	0561701	3211045	29
32	6791279	9437612	7353667	2646333	0562388	3208721	28
33	6793602	9436925	7356677	2643323	0563075	3206398	27
34	6795923	9436238	7359685	2640315	0563762	3204077	26
35	6798243	9435549	7362693	2637307	0564451	3201757	25
36	6800560	9434861	7365699	2634301	0565139	3199440	24
37	6802877	9434172	7368705	2631295	0565828	3197123	23
38	6805191	9433482	7371709	2628291	0566518	3194809	22
39	6807504	9432792	7374712	2625288	0567208	3192496	21
40	6809816	9432102	7377714	2622286	0567898	3190184	20
41	6812126	9431411	7380715	2619285	0568589	3187874	19
42	6814434	9430720	7383714	2616286	0569280	3185566	18
43	6816741	9430028	7386713	2613287	0569972	3183259	17
44	6819046	9429335	7389710	2610290	0570665	3180954	16
45	6821349	9428643	7392707	2607293	0571357	3178651	15
46	6823651	9427949	7395702	2604298	0572051	3176349	14
47	6825952	9427255	7398696	2601304	0572745	3174048	13
48	6828250	9426561	7401689	2598311	0573439	3171750	12
49	6830548	9425866	7404681	2595319	0574134	3169452	11
50	6832843	9425171	7407672	2592328	0574829	3167157	10
51	6835137	9424476	7410662	2589338	0575524	3164863	9
52	6837430	9423779	7413650	2586350	0576221	3162570	8
53	6839720	9423083	7416638	2583362	0576917	3160280	7
54	6842010	9422386	7419624	2580376	0577614	3157990	6
55	6844297	9421688	7422609	2577391	0578312	3155703	5
56	6846583	9420990	7425594	2574406	0579010	3153417	4
57	6848868	9420291	7428577	2571423	0579709	3151132	3
58	6851151	9419592	7431559	2568441	0580408	3148849	2
59	6853432	9418893	7434540	2565460	0581107	3146568	1
60	6855712	9418193	7437520	2562480	0581807	3144288	0

29 Degrees. 60 Degrees.

M	Sine.		Tang.		Secant.		
0	9.6855712	9.9418193	9.7437520	10.2562480	10.0581807	10.3144288	60
1	6857991	9417492	7440499	2559501	0582508	3142009	59
2	6860267	9416791	7443476	2556524	0583209	3139733	58
3	6862542	9416090	7446453	2553547	0583910	3137458	57
4	6864816	9415388	7449428	2550572	0584612	3135184	56
5	6867088	9414685	7452403	2547597	0585315	3132912	55
6	6869359	9413982	7455376	2544624	0586018	3130641	54
7	6871628	9413279	7458349	2541651	0586721	3128372	53
8	6873895	9412575	7461320	2538680	0587425	3126105	52
9	6876161	9411871	7464290	2535710	0588129	3123839	51
10	6878425	9411166	7467259	2532741	0588834	3121575	50
11	6880688	9410461	7470227	2529773	0589539	3119312	49
12	6882949	9409755	7473194	2526806	0590245	3117051	48
13	6885209	9409048	7476160	2523840	0590952	3114791	47
14	6887467	9408342	7479125	2520875	0591658	3112533	46
15	6889723	9407634	7482089	2517911	0592366	3110277	45
16	6891978	9406927	7485052	2514948	0593073	3108022	44
17	6894232	9406219	7488013	2511987	0593781	3105768	43
18	6896484	9405510	7490974	2509026	0594490	3103516	42
19	6898734	9404801	7493934	2506066	0595199	3101266	41
20	6900983	9404091	7496892	2503108	0595909	3099017	40
21	6903231	9403381	7499850	2500150	0596619	3096769	39
22	6905476	9402670	7502806	2497194	0597330	3094524	38
23	6907721	9401959	7505762	2494238	0598041	3092279	37
24	6909964	9401248	7508716	2491284	0598752	3090036	36
25	6912205	9400535	7511669	2488331	0599465	3087795	35
26	6914445	9399823	7514622	2485378	0600177	3085555	34
27	6916683	9399110	7517573	2482427	0600890	3083317	33
28	6918919	9398396	7520523	2479477	0601604	3081081	32
29	6921155	9397682	7523472	2476528	0602318	3078845	31
30	6923388	9396968	7526420	2473580	0603032	3076612	30
31	6925620	9396253	7529368	2470632	0603747	3074380	29
32	6927851	9395537	7532314	2467686	0604463	3072149	28
33	6930080	9394821	7535259	2464741	0605179	3069920	27
34	6932308	9394105	7538203	2461797	0605895	3067692	26
35	6934534	9393388	7541146	2458854	0606612	3065466	25
36	6936758	9392671	7544088	2455912	0607329	3063242	24
37	6938981	9391953	7547029	2452971	0608047	3061019	23
38	6941203	9391234	7549969	2450031	0608766	3058797	22
39	6943423	9390515	7552908	2447092	0609485	3056577	21
40	6945642	9389796	7555846	2444154	0610204	3054358	20
41	6947859	9389076	7558783	2441217	0610924	3052141	19
42	6950074	9388356	7561718	2438282	0611644	3049926	18
43	6952288	9387635	7564653	2435347	0612365	3047712	17
44	6954501	9386914	7567587	2432413	0613086	3045499	16
45	6956712	9386192	7570520	2429480	0613808	3043288	15
46	6958922	9385470	7573452	2426548	0614520	3041078	14
47	6961130	9384747	7576383	2423617	0615253	3038870	13
48	6963336	9384024	7579313	2420687	0615976	3036664	12
49	6965541	9383300	7582242	2417758	0616700	3034459	11
50	6967745	9382576	7585170	2414830	0617424	3032255	10
51	6969947	9381851	7588096	2411904	0618149	3030053	9
52	6972148	9381126	7591022	2408978	0618874	3027852	8
53	6974347	9380400	7593947	2406053	0619600	3025653	7
54	6976545	9379674	7596871	2403129	0620326	3023455	6
55	6978741	9378947	7599794	2400206	0621053	3021259	5
56	6980936	9378220	7602716	2397284	0621780	3019064	4
57	6983129	9377492	7605637	2394363	0622508	3016871	3
58	6985321	9376764	7608557	2391443	0623236	3014679	2
59	6987511	9376035	7611476	2388524	0623965	3012489	1
60	6989700	9375306	7614394	2385606	0624694	3010300	0

30 Degrees.	Sine.		Tang.		Secant.		59 Degrees.
M							
0	9.6989700	9.9375306	9.7614394	10 2385606	10.0624694	10.3010300	60
1	6991887	9374577	7617311	2382689	0625423	3008113	59
2	6994073	9373847	7620227	2379773	0626153	3005927	58
3	6996258	9373116	7623142	2376858	0626884	3003742	57
4	6998441	9372385	7626056	2373944	0627615	3001559	56
5	7000622	9371653	7628969	2371031	0628347	2999378	55
6	7002802	9370921	7631881	2368119	0629079	2997198	54
7	7004981	9370189	7634792	2365208	0629811	2995019	53
8	7007158	9369456	7637702	2362298	0630544	2992842	52
9	7009334	9368722	7640612	2359388	0631278	2990666	51
10	7011508	9367988	7643520	2356480	0632012	2988492	50
11	7013681	9367254	7646427	2353573	0632746	2986319	49
12	7015852	9366519	7649334	2350666	0633481	2984148	48
13	7018022	9365783	7652239	2347761	0634217	2981978	47
14	7020190	9365047	7655143	2344857	0634953	2979810	46
15	7022357	9364311	7658047	2341953	0635689	2977643	45
16	7024523	9363574	7660949	2339051	0636426	2975477	44
17	7026687	9362836	7663851	2336149	0637164	2973313	43
18	7028849	9362098	7666751	2333249	0637902	2971151	42
19	7031011	9361360	7669651	2330349	0638640	2968989	41
20	7033170	9360621	7672550	2327450	0639379	2966830	40
21	7035329	9359881	7675448	2324552	0640119	2964671	39
22	7037486	9359141	7678344	2321656	0640859	2962514	38
23	7039641	9358401	7681240	2318760	0641599	2960359	37
24	7041795	9357660	7684135	2315865	0642340	2958205	36
25	7043947	9356918	7687029	2312971	0643082	2956053	35
26	7046099	9356177	7689922	2310078	0643823	2953901	34
27	7048248	9355434	7692814	2307186	0644566	2951752	33
28	7050397	9354691	7695705	2304295	0645309	2949603	32
29	7052543	9353948	7698596	2301404	0646052	2947457	31
30	7054689	9353204	7701485	2298515	0646796	2945311	30
31	7056833	9352459	7704373	2295627	0647541	2943167	29
32	7058975	9351715	7707261	2292739	0648285	2941025	28
33	7061116	9350969	7710147	2289853	0649031	2938884	27
34	7063256	9350223	7713033	2286967	0649777	2936744	26
35	7065394	9349477	7715917	2284083	0650523	2934606	25
36	7067531	9348730	7718801	2281199	0651270	2932469	24
37	7069667	9347983	7721684	2278316	0652017	2930333	23
38	7071801	9347235	7724566	2275434	0652765	2928199	22
39	7073933	9346486	7727447	2272553	0653514	2926067	21
40	7076064	9345738	7730327	2269673	0654262	2923936	20
41	7078194	9344988	7733206	2266794	0655012	2921806	19
42	7080323	9344238	7736084	2263916	0655762	2919677	18
43	7082450	9343488	7738961	2261039	0656512	2917550	17
44	7084575	9342737	7741838	2258162	0657263	2915425	16
45	7086699	9341986	7744713	2255287	0658014	2913301	15
46	7088822	9341234	7747588	2252412	0658766	2911178	14
47	7090943	9340482	7750462	2249538	0659518	2909057	13
48	7093063	9339729	7753334	2246666	0660271	2906937	12
49	7095182	9338976	7756206	2243794	0661024	2904818	11
50	7097299	9338222	7759077	2240923	0661778	2902701	10
51	7099415	9337467	7761947	2238053	0662533	2900585	9
52	7101529	9336713	7764816	2235184	0663287	2898471	8
53	7103642	9335957	7767685	2232315	0664043	2896358	7
54	7105753	9335201	7770552	2229448	0664799	2894247	6
55	7107863	9334445	7773418	2226582	0665555	2892137	5
56	7109972	9333688	7776284	2223716	0666312	2890028	4
57	7112080	9332931	7779149	2220851	0667069	2887920	3
58	7114186	9332173	7782012	2217988	0667827	2885814	2
59	7116290	9331415	7784875	2215125	0668585	2883710	1
60	7118393	9330656	7787737	2212263	0669344	2881607	0

31 Degrees. 58 Degrees.

M	Sine.		Tang.		Secant.		M
0	9.7118393	99330656	9 7787737	10.2212263	10.0669344	10.2881607	60
1	7120495	9329897	7790599	2209401	0670103	2879505	59
2	7122596	9329137	7793459	2206541	0670863	2877404	58
3	7124695	9328376	7796318	2203682	0671624	2875305	57
4	7126792	9327616	7799177	2200823	0672384	2873208	56
5	7128889	9326854	7802034	2197966	0673146	2871111	55
6	7130983	9326092	7804891	2195109	0673908	2869017	54
7	7133077	9325330	7807747	2192253	0674670	2866923	53
8	7135169	9324567	7810602	2189398	0675433	2864831	52
9	7137260	9323804	7813456	2186544	0676196	2862740	51
10	7139349	9323040	7816309	2183691	0676960	2860651	50
11	7141437	9322276	7819162	2180838	0677724	2858563	49
12	7143524	9321511	7822013	2177987	0678489	2856476	48
13	7145609	9320746	7824864	2175136	0679254	2854391	47
14	7147693	9319980	7827713	2172287	0680020	2852307	46
15	7149776	9319213	7830562	2169438	0680787	2850224	45
16	7151857	9318447	7833410	2166590	0681553	2848143	44
17	7153937	9317679	7836258	2163742	0682321	2846063	43
18	7156015	9316911	7839104	2160896	0683089	2843985	42
19	7158092	9316143	7841949	2158051	0683857	2841908	41
20	7160168	9315374	7844794	2155206	0684626	2839832	40
21	7162243	9314605	7847638	2152362	0685395	2837757	39
22	7164316	9313835	7850481	2149519	0686165	2835684	38
23	7166387	9313065	7853323	2146677	0686935	2833613	37
24	7168458	9312294	7856164	2143836	0687706	2831542	36
25	7170526	9311522	7859004	2140996	0688478	2829474	35
26	7172594	9310750	7861844	2138156	0689250	2827406	34
27	7174660	9309978	7864682	2135318	0690022	2825340	33
28	7176725	9309205	7867520	2132480	0690795	2823275	32
29	7178789	9308432	7870357	2129643	0691568	2821211	31
30	7180851	9307658	7873193	2126807	0692342	2819149	30
31	7182912	9306883	7876028	2123972	0693117	2817088	29
32	7184971	9306109	7878863	2121137	0693891	2815029	28
33	7187030	9305333	7881696	2118304	0694667	2812970	27
34	7189086	9304557	7884529	2115471	0695443	2810914	26
35	7191142	9303781	7887361	2112639	0696219	2808858	25
36	7193196	9303004	7890192	2109808	0696996	2806804	24
37	7195249	9302226	7893023	2106977	0697774	2804751	23
38	7197300	9301448	7895852	2104148	0698552	2802700	22
39	7199350	9300670	7898681	2101319	0699330	2800650	21
40	7201399	9299891	7901508	2098492	0700109	2798601	20
41	7203447	9299112	7904335	2095665	0700888	2796553	19
42	7205493	9298332	7907161	2092839	0701668	2794507	18
43	7207538	9297551	7909987	2090013	0702449	2792462	17
44	7209581	9296770	7912811	2087189	0703230	2790419	16
45	7211623	9295989	7915635	2084365	0704011	2788377	15
46	7213664	9295207	7918458	2081542	0704793	2786336	14
47	7215704	9294424	7921280	2078720	0705576	2784296	13
48	7217742	9293641	7924101	2075899	0706359	2782258	12
49	7219779	9292857	7926921	2073079	0707143	2780221	11
50	7221814	9292073	7929741	2070259	0707927	2778186	10
51	7223848	9291289	7932560	2067440	0708711	2776152	9
52	7225881	9290504	7935378	2064622	0709496	2774119	8
53	7227913	9289718	7938195	2061805	0710282	2772087	7
54	7229943	9288932	7941011	2058989	0711068	2770057	6
55	7231972	9288145	7943827	2056173	0711855	2768028	5
56	7234000	9287358	7946641	2053359	0712642	2766000	4
57	7236026	9286571	7949455	2050545	0713429	2763974	3
58	7238051	9285783	7952268	2047732	0714217	2761949	2
59	7240075	9284994	7955081	2044919	0715006	2759925	1
60	7242097	9284205	7957892	2042108	0715795	2757903	0

32 Degrees. 57 Degrees.

M	Sines.		Tang.		Secants.		M
0	9.7242097	9.9284205	9.7957892	10 2042108	10.0715795	10.2757903	60
1	7244118	9283415	7960703	2039297	0716585	2755880	59
2	7246138	9282625	7963513	2036487	0717375	2753862	58
3	7248156	9281834	7966322	2033678	0718166	2751844	57
4	7250174	9281043	7969130	2030870	0718957	2749826	56
5	7252189	9280251	7971938	2028062	0719749	2747811	55
6	7254204	9279459	7974745	2025255	0720541	2745796	54
7	7256217	9278666	7977551	2022449	0721334	2743783	53
8	7258229	9277873	7980356	2019644	0722127	2741771	52
9	7260240	9277079	7983160	2016840	0722921	2739760	51
10	7262249	9276285	7985964	2014036	0723715	2737751	50
11	7264257	9275490	7988767	2011233	0724510	2735743	49
12	7266264	9274695	7991569	2008431	0725305	2733736	48
13	7268269	9273899	7994370	2005630	0726101	2731731	47
14	7270273	9273103	7997170	2002830	0726897	2729727	46
15	7272276	9272306	7999970	2000030	0727694	2727724	45
16	7274278	9271509	8002769	1997231	0728491	2725722	44
17	7276278	9270711	8005567	1994433	0729289	2723722	43
18	7278277	9269913	8008365	1991635	0730087	2721723	42
19	7280275	9269114	8011161	1988839	0730886	2719725	41
20	7282271	9268314	8013957	1986043	0731686	2717729	40
21	7284267	9267514	8016752	1983248	0732486	2715733	39
22	7286260	9266714	8019546	1980454	0733286	2713740	38
23	7288253	9265913	8022340	1977660	0734087	2711747	37
24	7290244	9265112	8025133	1974867	0734888	2709756	36
25	7292234	9264310	8027925	1972075	0735690	2707766	35
26	7294223	9263507	8030716	1969284	0736493	2705777	34
27	7296211	9262704	8033506	1966494	0737296	2703789	33
28	7298197	9261901	8036296	1963704	0738099	2701803	32
29	7300182	9261096	8039085	1960915	0738904	2699818	31
30	7302165	9260292	8041873	1958127	0739708	2697835	30
31	7304148	9259487	8044661	1955339	0740513	2695852	29
32	7306129	9258681	8047447	1952553	0741319	2693871	28
33	7308109	9257875	8050233	1949767	0742125	2691891	27
34	7310087	9257069	8053019	1946981	0742931	2689913	26
35	7312064	9256261	8055803	1944197	0743739	2687936	25
36	7314040	9255454	8058587	1941413	0744546	2685960	24
37	7316015	9254646	8061370	1938630	0745354	2683985	23
38	7317989	9253837	8064152	1935848	0746163	2682011	22
39	7319961	9253028	8066933	1933067	0746972	2680039	21
40	7321932	9252218	8069714	1930286	0747782	2678068	20
41	7323902	9251408	8072494	1927506	0748592	2676098	19
42	7325870	9250597	8075273	1924727	0749403	2674130	18
43	7327837	9249786	8078052	1921948	0750214	2672163	17
44	7329803	9248974	8080829	1919171	0751026	2670197	16
45	7331768	9248161	8083606	1916394	0751839	2668232	15
46	7333731	9247349	8086383	1913617	0752651	2666269	14
47	7335693	9246535	8089158	1910842	0753465	2664307	13
48	7337654	9245721	8091933	1908067	0754279	2662346	12
49	7339614	9244907	8094707	1905293	0755093	2660386	11
50	7341572	9244092	8097480	1902520	0755908	2658428	10
51	7343529	9243277	8100253	1899747	0756723	2656471	9
52	7345485	9242461	8103025	1896975	0757539	2654515	8
53	7347440	9241644	8105796	1894204	0758356	2652560	7
54	7349393	9240827	8108566	1891434	0759173	2650607	6
55	7351345	9240010	8111336	1888664	0759990	2648655	5
56	7353296	9239191	8114105	1885895	0760809	2646704	4
57	7355246	9238373	8116873	1883127	0761627	2644754	3
58	7357195	9237554	8119641	1880359	0762446	2642805	2
59	7359142	9236734	8122408	1877592	0763266	2640858	1
60	7361088	9235914	8125174	1874826	0764086	2638912	0

33 Degrees. 56 Degrees.

M	Sine.		Tang.		Secant.		M
0	9.7361088	9.9235914	9.8125174	10.1874826	10.0764086	10.2638912	60
1	7363032	9235093	8127939	1872061	0764907	2636968	59
2	7364976	9234272	8130704	1869296	0765728	2635024	58
3	7366918	9233450	8133468	1866532	0766550	2633082	57
4	7368859	9232628	8136231	1863769	0767372	2631141	56
5	7370799	9231805	8138993	1861007	0768195	2629201	55
6	7372737	9230982	8141755	1858245	0769018	2627263	54
7	7374675	9230158	8144516	1855484	0769842	2625325	53
8	7376611	9229334	8147277	1852723	0770666	2623389	52
9	7378546	9228509	8150036	1849964	0771491	2621454	51
10	7380479	9227684	8152795	1847205	0772316	2619521	50
11	7382412	9226858	8155554	1844446	0773142	2617588	49
12	7384343	9226032	8158311	1841689	0773968	2615657	48
13	7386273	9225205	8161068	1838932	0774795	2613727	47
14	7388201	9224377	8163824	1836176	0775623	2611799	46
15	7390129	9223549	8166580	1833420	0776451	2609871	45
16	7392055	9222721	8169335	1830665	0777279	2607945	44
17	7393980	9221891	8172089	1827911	0778109	2606020	43
18	7395904	9221062	8174842	1825158	0778938	2604096	42
19	7397827	9220232	8177595	1822405	0779768	2602173	41
20	7399748	9219401	8180347	1819653	0780599	2600252	40
21	7401668	9218570	8183098	1816902	0781430	2598332	39
22	7403587	9217738	8185849	1814151	0782262	2596413	38
23	7405505	9216906	8188599	1811401	0783094	2594495	37
24	7407421	9216073	8191348	1808652	0783927	2592579	36
25	7409337	9215240	8194096	1805904	0784760	2590663	35
26	7411251	9214406	8196844	1803156	0785594	2588749	34
27	7413164	9213572	8199592	1800408	0786428	2586836	33
28	7415075	9212737	8202338	1797662	0787263	2584925	32
29	7416986	9211902	8205084	1794916	0788098	2583014	31
30	7418895	9211066	8207829	1792171	0788934	2581105	30
31	7420803	9210229	8210574	1789426	0789771	2579197	29
32	7422710	9209393	8213317	1786683	0790607	2577290	28
33	7424616	9208555	8216060	1783940	0791445	2575384	27
34	7426520	9207717	8218803	1781197	0792283	2573480	26
35	7428423	9206878	8221545	1778455	0793122	2571577	25
36	7430325	9206039	8224286	1775714	0793961	2569675	24
37	7432226	9205200	8227026	1772974	0794800	2567774	23
38	7434126	9204360	8229766	1770234	0795640	2565874	22
39	7436024	9203519	8232505	1767495	0796481	2563976	21
40	7437921	9202678	8235244	1764756	0797322	2562079	20
41	7439817	9201836	8237981	1762019	0798164	2560183	19
42	7441712	9200994	8240719	1759281	0799006	2558288	18
43	7443606	9200151	8243455	1756545	0799849	2556394	17
44	7445498	9199308	8246191	1753809	0800692	2554502	16
45	7447390	9198464	8248926	1751074	0801536	2552610	15
46	7449280	9197619	8251660	1748340	0802381	2550720	14
47	7451169	9196775	8254394	1745606	0803225	2548831	13
48	7453056	9195929	8257127	1742873	0804071	2546944	12
49	7454943	9195083	8259860	1740140	0804917	2545057	11
50	7456828	9194237	8262592	1737408	0805763	2543172	10
51	7458712	9193390	8265323	1734677	0806610	2541288	9
52	7460595	9192542	8268053	1731947	0807458	2539405	8
53	7462477	9191694	8270783	1729217	0808306	2537523	7
54	7464358	9190845	8273513	1726487	0809155	2535642	6
55	7466237	9189996	8276241	1723759	0810004	2533763	5
56	7468115	9189146	8278969	1721031	0810854	2531885	4
57	7469992	9188296	8281696	1718304	0811704	2530008	3
58	7471868	9187445	8284423	1715577	0812555	2528132	2
59	7473743	9186594	8287149	1712851	0813406	2526257	1
60	7475617	9185742	8289874	1710126	0814258	2524383	0

34 Degrees. 55 Degrees.

M	Sine.		Tang.		Secant.		M
0	9.7475617	9.9185742	9.8289874	10.1710126	10.0814258	10.2524383	60
1	7477489	9184890	8292599	1707401	0815110	2522511	59
2	7479360	9184037	8295323	1704677	0815963	2520640	58
3	7481230	9183183	8298047	1701953	0816817	2518770	57
4	7483099	9182329	8300769	1699231	0817671	2516901	56
5	7484967	9181475	8303492	1696508	0818525	2515033	55
6	7486833	9180620	8306213	1693787	0819380	2513167	54
7	7488698	9179764	8308934	1691066	0820236	2511302	53
8	7490561	9178908	8311654	1688346	0821092	2509438	52
9	7492425	9178051	8314374	1685626	0821949	2507575	51
10	7494287	9177194	8317093	1682907	0822806	2505713	50
11	7496148	9176336	8319811	1680189	0823664	2503852	49
12	7498007	9175478	8322529	1677471	0824522	2501993	48
13	7499866	9174619	8325246	1674754	0825381	2500134	47
14	7501723	9173760	8327963	1672037	0826240	2498277	46
15	7503579	9172900	8330679	1669321	0827100	2496421	45
16	7505434	9172040	8333394	1666606	0827960	2494566	44
17	7507287	9171179	8336109	1663891	0828821	2492713	43
18	7509140	9170317	8338823	1661177	0829683	2490860	42
19	7510991	9169455	8341536	1658464	0830545	2489009	41
20	7512842	9168593	8344249	1655751	0831407	2487158	40
21	7514691	9167730	8346961	1653039	0832270	2485309	39
22	7516538	9166866	8349673	1650327	0833134	2483462	38
23	7518385	9166002	8352384	1647616	0833998	2481615	37
24	7520231	9165137	8355094	1644906	0834863	2479769	36
25	7522075	9164272	8357804	1642196	0835728	2477925	35
26	7523919	9163406	8360513	1639487	0836594	2476081	34
27	7525761	9162539	8363221	1636779	0837461	2474239	33
28	7527602	9161673	8365929	1634071	0838327	2472398	32
29	7529442	9160805	8368636	1631364	0839195	2470558	31
30	7531280	91599[illegible]7	8371343	1628657	0840063	2468720	30
31	7533118	9159069	8374049	1625951	0840931	2466882	29
32	7534954	9158200	8376755	1623245	0841800	2465046	28
33	7536790	9157330	8379460	1620540	0842670	2463210	27
34	7538624	9156460	8382164	1617836	0843540	2461376	26
35	7540457	9155589	8384867	1615133	0844411	2459543	25
36	7542288	9154718	8387571	1612429	0845282	2457712	24
37	7544119	9153846	8390273	1609727	0846154	2455881	23
38	7545949	9152974	8392975	1607025	0847026	2454051	22
39	7547777	9152101	8395676	1604324	0847899	2452223	21
40	7549604	9151228	8398377	1601623	0848772	2450396	20
41	7551431	9150354	8401077	1598923	0849646	2448569	19
42	7553256	9149479	8403776	1596224	0850521	2446744	18
43	7555080	9148604	8406475	1593525	0851396	2444920	17
44	7556902	9147729	8409174	1590826	0852271	2443098	16
45	7558724	9146852	8411871	1588129	0853148	2441276	15
46	7560544	9145976	8414569	1585431	0854024	2439456	14
47	7562364	9145099	8417265	1582735	0854901	2437636	13
48	7564182	9144221	8419961	1580039	0855779	2435818	12
49	7565999	9143342	8422657	1577343	0856658	2434001	11
50	7567815	9142464	8425351	1574649	0857536	2432185	10
51	7569630	9141584	8428046	1571954	0858416	2430370	9
52	7571444	9140704	8430739	1569261	0859296	2428556	8
53	7573256	9139824	8433432	1566568	0860176	2426744	7
54	7575068	9138943	8436125	1563875	0861057	2424932	6
55	7576878	9138061	8438817	1561183	0861939	2423122	5
56	7578687	9137179	8441508	1558492	0862821	2421313	4
57	7580495	9136296	8444199	1555801	0863704	2419505	3
58	7582302	9135413	8446889	1553111	0864587	2417698	2
59	7584108	9134530	8449579	1550421	0865470	2415892	1
60	7585913	9133645	8452268	1547732	0866355	2414087	0

35 Degrees. 54 Degrees.

M	Sines.		Tang.		Secant.		M
0	9.7585913	9.9133645	9.8452268	10.1547732	10.0866355	10.2414087	60
1	7587717	9132760	8454956	1545044	0867240	2412283	59
2	7589519	9131875	8457644	1542356	0868125	2410481	58
3	7591321	9130989	8460332	1539668	0869011	2408679	57
4	7593121	9130102	8463018	1536982	0869898	2406879	56
5	7594920	9129215	8465705	1534295	0870785	2405080	55
6	7596718	9128328	8468390	1531610	0871672	2403282	54
7	7598515	9127440	8471075	1528925	0872560	2401485	53
8	7600311	9126551	7473760	1526240	0873449	2399689	52
9	7602106	9125662	8476444	1523556	0874338	2397894	51
10	7603899	9124772	8479127	1520873	0875228	2396101	50
11	7605692	9123882	8481810	1518190	0876118	3394308	49
12	7607483	9122991	8484492	1515508	0877009	2392517	48
13	7609274	9122099	8487174	1512826	0877901	2390726	47
14	7611063	9121207	8489855	1510145	0878793	2388937	46
15	7612851	9120315	8492536	1507464	0879685	2387149	45
16	7614638	9119422	8495216	1504784	0880578	2385362	44
17	7616424	9118528	8497896	1502104	0881472	2383576	43
18	7618208	9117634	8500575	1499425	0882366	2381792	42
19	7619992	9116739	8503253	1496747	0883261	2380008	41
20	7621775	9115844	8505931	1494069	0884156	2378225	40
21	7623556	9114948	8508608	1491392	0885052	2376444	39
22	7625337	9114051	8511285	1488715	0885949	2374663	38
23	7627116	9113155	8513961	1486039	0886845	2372884	37
24	7628894	9112257	8516637	1483363	0887743	2371106	36
25	7630671	9111359	8519312	1480688	0888641	2369329	35
26	7632447	9110460	8521987	1478013	0889540	2367553	34
27	7634222	9109561	8524661	1475339	0890439	2365778	33
28	7635996	9108661	8527335	1472665	0891339	2364004	32
29	7637769	9107761	8530008	1469992	0892239	2362231	31
30	7639540	9106860	8532680	1467320	0893140	2360460	30
31	7641311	9105959	8535352	1464648	0894041	2358689	29
32	7643080	9105057	8538023	1461977	0894943	2356920	28
33	7644849	9104155	8540694	1459306	0895845	2355151	27
34	7646616	9103251	8543365	1456635	0896749	2353384	26
35	7648382	9102348	8546034	1453966	0897652	2351618	25
36	7650147	9101444	8548704	1451296	0898556	2349853	24
37	7651911	9120539	8551372	1448628	0899461	2348089	23
38	7653674	9099634	8554041	1445959	0900366	2346326	22
39	7655436	9098728	8556708	1443292	0901272	2344564	21
40	7657197	9097821	8559376	1440624	0902179	2342803	20
41	7658957	9096915	8562042	1437958	0903085	2341043	19
42	7660715	9096007	8564708	1435292	0903993	2339285	18
43	7662473	9095099	8567374	1432626	0904901	2337527	17
44	7664229	9094190	8570039	1429961	0905810	2335771	16
45	7665985	9093281	8572704	1427296	0906719	2334015	15
46	7667739	9092371	8575368	1424632	0907629	2332261	14
47	7669492	9091461	8578031	1421969	0908539	2330508	13
48	7671244	9090550	8580694	1419306	0909450	2328756	12
49	7672996	9089639	8583357	1416643	0910361	2317004	11
50	7674746	9088727	8586019	1413981	0911273	2325254	10
51	7676494	9087814	8588680	1411320	0912186	2323506	9
52	7678242	9086901	8591341	1408659	0913099	2321758	8
53	7679989	9085988	8594002	1405998	0914012	2320011	7
54	7681735	9085073	8596661	1403339	0914927	2318265	6
55	7683480	9084159	8599321	1400679	0915841	2316520	5
56	7685223	9083243	8601980	1398020	0916757	2314777	4
57	7686966	9082327	8604638	1395362	0917673	2351034	3
58	7688707	9081411	8607296	1392704	0918589	2311293	2
59	7690448	9080494	8609954	1390046	0919506	2309552	1
60	7692187	9079576	8612610	1387390	0920424	2307813	0

36 Degrees. 53 Degrees.

M	Sine.		Tang.		Secant.		M
0	9.7692187	9.9079576	9.8612610	10.1387390	10.0920424	10.2307813	60
1	7693925	9078658	8615267	1384733	0921342	2306075	59
2	7695662	9077740	8617923	1382077	0922260	2304338	58
3	7697398	9076820	8620578	1379422	0923180	2302602	57
4	7699134	9075901	8623233	1376767	0924099	2300866	56
5	7700868	9074980	8625887	1374113	0925020	2299132	55
6	7702601	9074059	8628541	1371459	0925941	2297399	54
7	7704332	9073138	8631195	1368805	0926862	2295668	53
8	7706063	9072216	8633848	1366152	0927784	2293937	52
9	7707793	9071293	8636500	1363500	0928707	2292207	51
10	7709522	9070370	8639152	1360848	0929630	2290478	50
11	7711249	9069446	8641803	1358197	0930554	2288751	49
12	7712976	9068522	8644454	1355546	0931478	2287024	48
13	7714702	9067597	8647105	1352895	0932403	2285298	47
14	7716426	9066671	8649755	1350245	0933329	2283574	46
15	7718150	9065745	8652404	1347596	0934255	2281850	45
16	7719872	9064819	8655053	1344947	0935181	2280128	44
17	7721593	9063892	8657702	1342298	0936108	2278407	43
18	7723314	9062964	8660350	1339650	0937036	2276686	42
19	7725033	9062036	8662997	1337003	0937964	2274967	41
20	7726751	9061107	8665644	1334356	0938893	2273249	40
21	7728468	9060177	8668291	1331709	0939823	2271532	39
22	7730185	9059247	8670937	1329063	0940753	2269815	38
23	7731900	9058317	8673583	1326417	0941683	2268100	37
24	7733614	9057386	8676228	1323772	0942614	2266386	36
25	7735327	9056454	8678873	1321127	0943546	2264673	35
26	7737039	9055522	8681517	1318483	0944478	2262961	34
27	7738749	9054589	8684160	1315840	0945411	2261251	33
28	7740459	9053656	8686804	1313196	0946344	2259541	32
29	7742168	9052722	8689446	1310554	0947278	2257832	31
30	7743876	9051787	8692089	1307911	0948213	2256124	30
31	7745583	9050852	8694731	1305269	0949148	2254417	29
32	7747288	9049916	8697372	1302628	0950984	2252712	28
33	7748993	9048980	8700013	1299987	0951020	2251007	27
34	7750697	9048043	8702653	1297347	0951957	2249303	26
35	7752399	9047106	8705293	1294707	0952894	2247601	25
36	7754101	9046168	8707933	1292067	0953832	2245899	24
37	7755801	9045230	8710572	1289428	0954770	2244199	23
38	7757501	9044291	8713210	1286790	0955709	2242499	22
39	7759199	9043351	8715848	1284152	0956649	2240801	21
40	7760897	9042411	8718486	1281514	0957589	2239103	20
41	7762593	9041470	8721123	1278877	0958530	2237407	19
42	7764289	9040529	8723760	1276240	0959471	2235711	18
43	7765983	9039587	8726396	1273604	0960413	2234017	17
44	7767676	9038644	8729032	1270968	0961356	2232324	16
45	7769369	9037701	8731668	1268332	0962299	2230631	15
46	7771060	9036757	8734302	1265698	0963243	2228940	14
47	7772750	9035813	8736937	1263063	0964187	2227250	13
48	7774439	9034868	8739571	1260429	0965132	2225561	12
49	7776128	9033923	8742204	1257796	0966077	2223872	11
50	7777815	9032977	8744838	1255162	0967023	2222185	10
51	7779501	9032031	8747470	1252530	0967969	2220499	9
52	7781186	9031084	8750102	1249898	0968916	2218814	8
53	7782870	9030136	8752734	1247266	0969864	2217130	7
54	7784553	9029188	8755365	1244635	0970812	2215447	6
55	7786235	9028239	8757996	1242004	0971761	2213765	5
56	7787916	9027289	8760627	1239373	0972711	2212084	4
57	7789596	9026339	8763257	1236743	0973661	2210404	3
58	7791275	9025389	8765886	1234114	0974611	2208725	2
59	7792953	9024438	8768515	1231485	0975562	2207047	1
60	7794630	9023486	8771144	1228856	0976514	2205370	0

37 Degrees. 52 Degrees.

M	Sine.		Tang.		Secant.		M
0	9.7794630	9.9023486	9.8771144	10.1228856	10.0976514	10.2205370	60
1	7796306	9022534	8773772	1226228	0977466	2203694	59
2	7797981	9021581	8776400	1223600	0978419	2202019	58
3	7799655	9020628	8779027	1220973	0979372	2200345	57
4	7801328	9019674	8781654	1218346	0980326	2198672	56
5	7803000	9018719	8784281	1215719	0981281	2197000	55
6	7804671	9017764	8786907	1213093	0982236	2195329	54
7	7806341	9016808	8789533	1210467	0983192	2193659	53
8	7808010	9015852	8792158	1207842	0984148	2191990	52
9	7809677	9014895	8794782	1205218	0985105	2190323	51
10	7811344	9013938	8797407	1202593	0986062	2188656	50
11	7813010	9012980	8800031	1199969	0987020	2186990	49
12	7814675	9012021	8802654	1197346	0987979	2185325	48
13	7816339	9011062	8805277	1194723	0988938	2183661	47
14	7818002	9010102	8807900	1192100	0989898	2181998	46
15	7819664	9009142	8810522	1189478	0990858	2180336	45
16	7821324	9008181	8813144	1186856	0991819	2178676	44
17	7822984	9007219	8815765	1184235	0992781	2177016	43
18	7824643	9006257	8818386	1181614	0993743	2175357	42
19	7826301	9005294	8821007	1178993	0994706	2173699	41
20	7827958	9004331	8823627	1176373	0995669	2172042	40
21	7829614	9003367	8826246	1173754	0996633	2170386	39
22	7831268	9002403	8828866	1171134	0997597	2168732	38
23	7832922	9001438	8831484	1168516	0998562	2167078	37
24	7834575	9000472	8834103	1165897	0999528	2165425	36
25	7836227	8999506	8836721	1163279	1000494	2163773	35
26	7837878	8998539	8839338	1160662	1001461	2162122	34
27	7839528	8997572	8841956	1158044	1002428	2160472	33
28	7841177	8996604	8844572	1155428	1003396	2158823	32
29	7842824	8995636	8847189	1152811	1004364	2157176	31
30	7844471	8994667	8849805	1150195	1005333	2155529	30
31	7846117	8993697	8852420	1147580	1006303	2153883	29
32	7847762	8992727	8855035	1144965	1007271	2152238	28
33	7849406	8991755	8857650	1142350	1008244	2150594	27
34	7851049	8990784	8860264	1139736	1009216	2148951	26
35	7852691	8989812	8862878	1137122	1010188	2147309	25
36	7854332	8988840	8865492	1134508	1011160	2145668	24
37	7855972	8987867	8868105	1131895	1012133	2144028	23
38	7857611	8986893	8870718	1129282	1013107	2142389	22
39	7859249	8985919	8873330	1126670	1014081	2140751	21
40	7860886	8984944	8875942	1124058	1015056	2139114	20
41	7862522	8983968	8878554	1121446	1016032	2137478	19
42	7864157	8982992	8881165	1118835	1017008	2135843	18
43	7865791	8982015	8883775	1116225	1017985	2134209	17
44	7867424	8981038	8886386	1113614	1018962	2132576	16
45	7869056	8980060	8888996	1111004	1019940	2130944	15
46	7870687	8979082	8891605	1108395	1020918	2129313	14
47	7872317	8978103	8894214	1105786	1021897	2127683	13
48	7873946	8977123	8896823	1103177	1022877	2126054	12
49	7875574	8976143	8899432	1100568	1023857	2124426	11
50	7877202	8975162	8902420	1097960	1024838	2122798	10
51	7878828	8974181	8904647	1095353	1025819	2121172	9
52	7880453	8973199	8907254	1092746	1026801	2119547	8
53	7882077	8972216	8909861	1090139	1027784	2117923	7
54	7883701	8971233	8912468	1087532	1028767	2116299	6
55	7885323	8970249	8915074	1084926	1029751	2114677	5
56	7886944	8969265	8917679	1082321	1030735	2113056	4
57	7888565	8968280	8920285	1079715	1031720	2111435	3
58	7890184	8967294	8922890	1077110	1032706	2109816	2
59	7891802	8966308	8925494	1074506	1033692	2108198	1
60	7893420	8965321	8928098	1071902	1034679	2106580	0

38 Degrees. 51 Degrees.

M	Sine.		Tang.		Secant.		M
0	9.7893420	9.8965321	9.8928098	10.1071902	10.1034679	10.2106580	60
1	7895036	8964334	8930702	1069298	1035666	2104964	59
2	7896652	8963346	8933306	1066694	1036654	2103348	58
3	7898266	8962358	8935909	1064091	1037642	2101734	57
4	7899880	8961369	8938511	1061489	1038631	2100120	56
5	7901493	8960379	8941114	1058886	1039621	2098507	55
6	7903104	8959389	8943715	1056285	1040611	2096896	54
7	7904715	8958398	8946317	1053683	1041602	2095285	53
8	7906325	8957406	8948918	1051082	1042594	2093675	52
9	7907933	8956414	8951519	1048481	1043586	2092067	51
10	7909541	8955422	8954119	1045881	1044578	2090459	50
11	7911148	8954429	8956719	1043281	1045571	2088852	49
12	7912754	8953435	8959319	1040681	1046565	2087246	48
13	7914359	8952440	8961918	1038082	1047560	2085641	47
14	7915963	8951445	8964517	1035483	1048555	2084037	46
15	7917566	8950450	8967116	1032884	1049550	2082434	45
16	7919168	8949453	8969714	1030286	1050547	2080832	44
17	7920769	8948457	8972312	1027688	1051543	2079231	43
18	7922369	8947459	8974910	1025090	1052541	2077631	42
19	7923968	8946461	8977507	1022493	1053539	2076032	41
20	7925566	8945463	8980104	1019896	1054537	2074434	40
21	7927163	8944463	8982700	1017300	1055537	2072837	39
22	7928760	8943464	8985296	1014704	1056536	2071240	38
23	7930355	8942463	8987892	1012108	1057537	2069645	37
24	7931949	8941462	8990487	1009513	1058538	2068051	36
25	7933543	8940461	8993082	1006918	1059539	2066457	35
26	7935135	8939458	8995677	1004323	1060542	2064865	34
27	7936727	8938456	8998271	1001729	1061544	2063273	33
28	7938317	8937452	9000865	0999135	1062548	2061683	32
29	7939907	8936448	9003459	0996541	1063552	2060093	31
30	7941496	8935444	9006052	0993948	1064556	2058504	30
31	7943083	8934439	9008654	0991355	1065561	2056917	29
32	7944670	8933433	9011237	0988763	1066567	2055330	28
33	7946256	8932426	9013830	0986170	1067574	2053744	27
34	7947841	8931419	9016422	0983578	1068581	2052159	26
35	7949425	8930412	9019013	0980987	1069588	2050575	25
36	7951008	8929404	9021604	0978396	1070596	2048992	24
37	7952590	8928395	9024195	0975805	1071605	2047410	23
38	7954171	8927385	9026786	0973214	1072615	2045829	22
39	7955751	8926375	9029376	0970624	1073625	2044249	21
40	7957330	8925365	9031966	0968034	1074635	2042670	20
41	7958909	8924354	9034555	0965445	1075646	2041091	19
42	7960486	8923342	9037144	0962856	1076658	2039514	18
43	7962062	8922329	9039733	0960267	1077671	2037938	17
44	7963638	8921316	9042321	0957679	1078684	2036362	16
45	7965212	8920303	9044910	0955090	1079697	2034788	15
46	7966786	8919289	9047497	0952503	1080711	2033214	14
47	7968359	8918274	9050085	0949915	1081726	2031641	13
48	7969930	8917258	9052672	0947328	1082742	2030070	12
49	7971501	8916242	9055259	0944741	1083758	2028499	11
50	7973071	8915226	9057845	0942155	1084774	2026929	10
51	7974640	8914208	9060431	0939569	1085792	2025360	9
52	7976208	8913191	9063017	0936983	1086809	2023792	8
53	7977775	8912172	9065603	0934397	1087828	2022225	7
54	7979341	8911153	9068188	0931812	1088847	2020659	6
55	7980906	8910133	9070773	0929227	1089867	2019094	5
56	7982470	8909113	9073357	0926643	1090887	2017530	4
57	7984034	8908092	9075941	0924059	1091908	2015966	3
58	7985596	8907071	9078525	0921475	1092929	2014404	2
59	7987158	8906049	9081109	0918891	1093951	2012842	1
60	7988718	8905026	9083692	0916308	1094974	2011282	0

M	39 Degrees. Sine.		Tang.		Secant.	50 Degrees.	M
0	9.7988718	9.8905026	9.9083692	10.0916308	10.1094974	10.2011282	60
1	7990278	8904003	9086275	0913725	1095997	2009722	59
2	7991836	8902979	9088858	091114	1097021	2008164	58
3	7993394	8901954	9091440	0908560	1098046	2006606	57
4	7994951	8900929	9094022	0905978	1099071	2005049	56
5	7996507	8899903	9096603	0903397	1100097	2003493	55
6	7998062	8898877	9099185	0900815	1101123	2001938	54
7	7999616	8897850	9101766	0898234	1102150	2000384	53
8	8001169	8896822	9104347	0895653	1103178	1998831	52
9	8002721	8895794	9106927	0893073	1104206	1997279	51
10	8004272	8894765	9109507	0890493	1105235	1995728	50
11	8005823	8893736	9112087	0887913	1106264	1994177	49
12	8007372	8892706	9114666	0885334	1107294	1992628	48
13	8008921	8891675	9117245	0882755	1108325	1991079	47
14	8010468	8890644	9119824	0880176	1109356	1989532	46
15	8012015	8889612	9122403	0877597	1110388	1987985	45
16	8013561	8888580	9124981	0875019	1111420	1986439	44
17	8015106	8887547	9127559	0872441	1112453	1984894	43
18	8016649	8886513	9130137	0869863	1113487	1983351	42
19	8018192	8885479	9132714	0867286	1114521	1981808	41
20	8019735	8884444	9135291	0864709	1115556	1980265	40
21	8021276	8883408	9137868	0862132	1116592	1978724	39
22	8022816	8882372	9140444	0859556	1117628	1977184	38
23	8024355	8881335	9143020	0856980	1118665	1975645	37
24	8025894	8880298	9145596	0854404	1119702	1974106	36
25	8027431	8879260	9148171	0851829	1120740	1972569	35
26	8028968	8878221	9150747	0849253	1121779	1971032	34
27	8030504	8877182	9153322	0846678	1122818	1969496	33
28	8032038	8876142	9155896	0844104	1123858	1967962	32
29	8033572	8875102	9158471	0841529	1124898	1966428	31
30	8035105	8874061	9161045	0838955	1125939	1964895	30
31	8036637	8873019	9163618	0836382	1126981	1963363	29
32	8038168	8871977	9166192	0833808	1128023	1961832	28
33	8039699	8870934	9168765	0831235	1129066	1960301	27
34	8041228	8869890	9171338	0828662	1130110	1958772	26
35	8042757	8868846	9173911	0826089	1131154	1957243	25
36	8044284	8867801	9176483	0823517	1132199	1955716	24
37	8045811	8866756	9179055	0820945	1133244	1954189	23
38	8047336	8865710	9181627	0818373	1134290	1952664	22
39	8048861	8864663	9184198	0815802	1135337	1951139	21
40	8050385	8863616	9186769	0813231	1136384	1949615	20
41	8051908	8862568	9189340	0810660	1137432	1948092	19
42	8053430	8861519	9191911	0808089	1138481	1946570	18
43	8054951	8860470	9194481	0805519	1139530	1945049	17
44	8056472	8859420	9197051	0802949	1140580	1943528	16
45	8057991	8858370	9199621	0800379	1141630	1942009	15
46	8059510	8857319	9202191	0797809	1142681	1940490	14
47	8061027	8856267	9204760	0795240	1143733	1938973	13
48	8062544	8855215	9207329	0792671	1144785	1937456	12
49	8064060	8854162	9209898	0790102	1145838	1935940	11
50	8065575	8853109	9212466	0787534	1146891	1934425	10
51	8067089	8852055	9215034	0784966	1147945	1932911	9
52	8068602	8851000	9217602	0782398	1149000	1931398	8
53	8070114	8849945	9220170	0779830	1150055	1929886	7
54	8071626	8848889	9222737	0777263	1151111	1928374	6
55	8073136	8847832	9225304	0774696	1152168	1926864	5
56	8074646	8846775	9227871	0772129	1153225	1925354	4
57	8076154	8845717	9230437	0769563	1154283	1923846	3
58	8077662	8844659	9233004	0766996	1155341	1922338	2
59	8079169	8843599	9235570	0764430	1156401	1920831	1
60	8080675	8842540	9238135	0761865	1157460	1919325	0

40 Degrees. 49 Degrees.

M	Sines.		Tang.		Secant		M
0	9.8080675	9.8842540	9.9238135	10.0761865	10.1157460	10.1919325	60
1	8082180	8841479	9240701	0759299	1158521	1917820	59
2	8083684	8840418	9243266	0756734	1159582	1916316	58
3	8085188	8839357	9245831	0754169	1160643	1914812	57
4	8086690	8838294	9248396	0751604	1161706	1913310	56
5	8088192	8837232	9250960	0749040	1162768	1911808	55
6	8089692	8836168	9253524	0746476	1163832	1910308	54
7	8091192	8835104	9256088	0743912	1164896	1908808	53
8	8092691	8834039	9258652	0741348	1165961	1907309	52
9	8094189	8832974	9261215	0738785	1167026	1905811	51
10	8095686	8831908	9263778	0736222	1168092	1904314	50
11	8097182	8830841	9266341	0733659	1169159	1902818	49
12	8098678	8829774	9268904	0731096	1170226	1901322	48
13	8100172	8828706	9271466	0728534	1171294	1899828	47
14	8101666	8827638	9274028	0725972	1172362	1898334	46
15	8103159	8826568	9276590	0723410	1173432	1896841	45
16	8104650	8825499	9279152	0720848	1174501	1895350	44
17	8106141	8824428	9281713	0718287	1175572	1893859	43
18	8107631	8823357	9284274	0715726	1176643	1892369	42
19	8109121	8822285	9286835	0713165	1177715	1890879	41
20	8110609	8821213	9289396	0710604	1178787	1889391	40
21	8112096	8820140	9291956	0708044	1179860	1887904	39
22	8113583	8819067	9294516	0705484	1180933	1886417	38
23	8115069	8817992	9297076	0702924	1182008	1884931	37
24	8116554	8816918	9299636	0700364	1183082	1883446	36
25	8118038	8815842	9302195	0697805	1184158	1881962	35
26	8119521	8814766	9304755	0695245	1185234	1880479	34
27	8121003	8813689	9307314	0692686	1186311	1878997	33
28	8122484	8812612	9309872	0690128	1187388	1877516	32
29	8123965	8811534	9312431	0687569	1188466	1876035	31
30	8125444	8810455	9314989	0685011	1189545	1874550	30
31	8126923	8809376	9317547	0682453	1190624	1873077	29
32	8128401	8808296	9320105	0679895	1191704	1871599	28
33	8129878	8807215	9322662	0677338	1192785	1870122	27
34	8131354	8806134	9325220	0674780	1193866	1868646	26
35	8132829	8805052	9327777	0672223	1194948	1867171	25
36	8134303	8803970	9330334	0669666	1196030	1865697	24
37	8135777	8802887	9332890	0667110	1197113	1864223	23
38	8137250	8801803	9335446	0664554	1198197	1862750	22
39	8138721	8800719	9338003	0661997	1199281	1861279	21
40	8140192	8799634	9340559	0659941	1200366	1859808	20
41	8141662	8798548	9343114	0656886	1201452	1858338	19
42	8143131	8797462	9345670	0654330	1202538	1856869	18
43	8144600	8796375	9348225	0651775	1203625	1855400	17
44	8146067	8795287	9350780	0649220	1204713	1853933	16
45	8147534	8794199	9353335	0646665	1205801	1852466	15
46	8148999	8793110	9355889	0644111	1206890	1851001	14
47	8150464	8792021	9358444	0641556	1207979	1849536	13
48	8151928	8790930	9360998	0639002	1209070	1848072	12
49	8153391	8789840	9363552	0636448	1210160	1846609	11
50	8154854	8788748	9366105	0633895	1211252	1845146	10
51	8156315	8787656	9368659	0631341	1212344	1843685	9
52	8157776	8786563	9371212	0628788	1213437	1842224	8
53	8159235	8785470	9373765	0626235	1214530	1840765	7
54	8160694	8784376	9376318	0623682	1215624	1839306	6
55	8162152	8783281	9378871	0621129	1216719	1837848	5
56	8163609	8782186	9381423	0618577	1217814	1836391	4
57	8165066	8781090	9383975	0616025	1218910	1834934	3
58	8166521	8779994	9386527	0613473	1220006	1833479	2
59	8167975	8778896	9389079	0610921	1221104	1 32025	1
60	8169429	8777799	9391631	0608369	1222201	1830571	0

41 Degrees. 48 Degrees.

M	Sine.		Tang.		Secant.		M
0	9.8169429	9.8777799	9.9391631	10.0608369	10.1222201	10.1830571	60
1	8170882	8776700	9394182	0605818	1223300	1829118	59
2	8172334	8775601	9396733	0603267	1224399	1827666	58
3	8173785	8774501	9399284	0600716	1225499	1826215	57
4	8175235	8773401	9401835	0598165	1226599	1824765	56
5	8176685	8772300	9404385	0595615	1227700	1823315	55
6	8178135	8771198	9406936	0593064	1228802	1821867	54
7	8179581	8770096	9409486	0590514	1229904	1820419	53
8	8181028	8768993	9412036	0587964	1231007	1818172	52
9	8182474	8767889	9414585	0585415	1232111	1817526	51
10	8183919	8766785	9417135	0582865	1233215	1816081	50
11	8185364	8765680	9419684	0580316	1234320	1814636	49
12	8186807	8764574	9422233	0577767	1235426	1813193	48
13	8188250	8763468	9424782	0575218	1236532	1811750	47
14	8189692	8762361	9427331	0572669	1237639	1810308	46
15	8191133	8761253	9429879	0570121	1238747	1808867	45
16	8192573	8760145	9432428	0567572	1239855	1807427	44
17	8194012	8759036	9434976	0565024	1240964	1805988	43
18	8195450	8757927	9437524	0562476	1242073	1804550	42
19	8196888	8756816	9440072	0559928	1243184	1803112	41
20	8198325	8755706	9442619	0557381	1244294	1801675	40
21	8199761	8754594	9445166	0554834	1245406	1800239	39
22	8201196	8753482	9447714	0552286	1246518	1798804	38
23	8202630	8752369	9450261	0549739	1247631	1797370	37
24	8204063	8751256	9452807	0547193	1248744	1795937	36
25	8205496	8750142	9455354	0544646	1249858	1794504	35
26	8206927	8749027	9457900	0542100	1250973	1793073	34
27	8208358	8747912	9460447	0539553	1252088	1791642	33
28	8209788	8746795	9462993	0537007	1253205	1790212	32
29	8211217	8745679	9465539	0534461	1254321	1788783	31
30	8212646	8744561	9468084	0531916	1255439	1787354	30
31	8214073	8743443	9470630	0529370	1256557	1785927	29
32	8215500	8742325	9473175	0526825	1257675	1784500	28
33	8216926	8741205	9475720	0524280	1258795	1783074	27
34	8218351	8740085	9478265	0521735	1259915	1781649	26
35	8219775	8738965	9480810	0519190	1261035	1780225	25
36	8221198	8737844	9483355	0516645	1262156	1778802	24
37	8222621	8736722	9485899	0514101	1263278	1777379	23
38	8224042	8735599	9488443	0511557	1264401	1779758	22
39	8225463	8734476	9490987	0509013	1265524	1774537	21
40	8226883	8733352	9493531	0506469	1266648	1773117	20
41	8228302	8732227	9496075	0503925	1267773	1771698	19
42	8229721	8731102	9498619	0501381	1268898	1770279	18
43	8231138	8729976	9501162	0498838	1270024	1768862	17
44	8232555	8728849	9503705	0496295	1271151	1767445	16
45	8233971	8727722	9506248	0493752	1272278	1766029	15
46	8235386	8726594	9508791	0491209	1273406	1764614	14
47	8236800	8725466	9511334	0488666	1274534	1763200	13
48	8238213	8724337	9513876	0486124	1275663	1761787	12
49	8239626	8723207	9516419	0483581	1276793	1760374	11
50	8241037	8722076	9518961	0481039	1277924	1758963	10
51	8242448	8720945	9521503	0478497	1279055	1757552	9
52	8243858	8719813	9524045	0475955	1280187	1756142	8
53	8245267	8718681	9526587	0473413	1281319	1754733	6
54	8246676	8717548	9529128	0470872	1282452	1753324	7
55	8248083	8716414	9531670	0468330	1283586	1751917	5
56	8249490	8715279	9534211	0465789	1284721	1750510	4
57	8250896	8714144	9536752	0463248	1285856	1749104	3
58	8252301	8713008	9539293	0460707	1286992	1747699	2
59	8253705	8711872	9541834	0458166	1288128	1746295	1
60	8255109	8710735	9544374	0455626	1289265	1744891	0

42 Degrees. **47 Degrees.**

M	Sine.		Tang.		Secant.		M
0	9.8255109	9.8710735	9.9544374	10.0455626	10.1289265	10.1744891	60
1	8256512	8709597	9546915	0453085	1290403	1743488	59
2	8257913	8708458	9549455	0450545	1291542	1742087	58
3	8259314	87073·9	9551995	0448005	1292681	1740686	57
4	8260715	8706179	9554535	0445465	1293821	1739285	56
5	8262114	8705039	9557075	0442925	1294961	1737886	55
6	8263512	8703898	9559615	0440385	1296102	1736488	54
7	8264910	8702756	9562154	0437846	1297244	1735090	53
8	8266307	8701613	9564694	0435306	1298387	1733693	52
9	8267703	8700470	9567233	0432767	1299530	1732297	51
10	8269098	8699326	9569772	0430228	1300674	1730902	50
11	8270493	8698182	9572311	0427689	1301818	1729507	49
12	8271887	8697037	9574850	0425150	1302963	1728113	48
13	8273279	8695891	9577389	0422611	1304109	1726721	47
14	8274671	8694744	9579927	0420073	1305256	1725329	46
15	8276063	8693597	9582465	0417535	1306403	1723937	45
16	8277453	8692449	9585004	0414996	1307551	1722547	44
17	8278843	8691301	9587542	0412458	1308699	1721157	43
18	8280231	8690152	9590080	0409920	1309848	1719769	42
19	8281619	8689002	9592618	0407382	1310998	1718381	41
20	8283006	8687851	9595155	0404845	1312149	1716994	40
21	8284393	8686700	9597693	0402307	1313300	1715607	39
22	8285778	8685548	9600230	0399770	1314452	1714222	38
23	8287163	8684396	9602767	0397233	1315604	1712837	37
24	8288547	8683242	9605305	0393965	1316758	1711453	36
25	8289930	8682088	9607842	0392158	1317912	1710070	35
26	8291312	8680934	9610378	0389622	1319066	1708688	34
27	8292694	8679779	9612915	0387085	1320221	1707306	33
28	8294075	8678623	9615452	0384548	1321377	1705925	32
29	8295454	8677466	9617988	0382012	1322534	1704546	31
30	8296833	8676309	9620525	0379475	1323691	1703167	30
31	8298212	8675151	9623061	0376939	1324849	1701788	29
32	8299589	8673992	9625597	0374403	1326008	1700411	28
33	8300966	8672833	9628133	0371867	1327167	1699034	27
34	8302342	8671673	9630669	0369331	1328327	1697658	26
35	8303717	8670512	9633204	0366796	1329488	1696283	25
36	8305091	8669351	9635740	0364260	1330649	1694909	24
37	8306464	8668189	9638275	0361725	1331811	1693536	23
38	8307837	8667026	9640811	0359189	1332974	1692163	22
39	8309209	8665863	9643346	0356654	1334137	1690791	21
40	8310580	8664699	9645881	0354119	1335301	1689420	20
41	8311950	8663534	9648416	0351584	1336466	1688050	19
42	8313320	8662369	9650951	0349049	1337631	1686680	18
43	8314688	8661203	9653486	0346514	1338797	1685312	17
44	8316056	8660036	9656020	0343980	1339964	1683944	16
45	8317423	8658868	9658555	0341445	1341132	1682577	15
46	8318789	8657700	9661089	0338911	13,2300	1681211	14
47	8320155	8656531	9663623	0336377	1343469	1679845	13
48	8321519	8655362	9666157	0333843	1344638	1678481	12
49	8322883	8654192	9668692	0331308	1345808	1677117	11
50	8324246	8653021	9671225	0328775	1346979	1675754	10
51	8325609	8651849	9673759	0326241	1348151	1674391	9
52	8326970	8650677	9676293	0323707	1349323	1673030	8
53	8328331	8649504	9678827	0321173	1350496	1671669	7
54	8329691	8648331	9681360	0318640	1351669	1670309	6
55	8331050	8647156	9683893	0316107	1352844	1668950	5
56	8332408	8645981	9686427	0313573	13540·9	1667592	4
57	8333766	8644806	9688960	0311040	1355194	1666234	3
58	8335122	8643629	9691493	0308507	1356371	1664878	2
59	8336478	8642452	9694026	0305974	1357548	1663522	1
60	8337833	8641275	9696559	0303441	1358725	1662167	0

43 Degrees. 46 Degrees.

M	Sine.		Tang.		Secant.		M
0	9.8337833	9.8641275	9.9696559	10.0303441	10.1358725	10.1662167	60
1	8339188	8640096	9699091	0300909	1359904	1660812	59
2	8340541	8638917	9701624	0298376	1361083	1659459	58
3	8341894	8637737	9704157	0295843	1362263	1658106	57
4	8343246	8636557	9706689	0293311	1363443	1656754	56
5	8344597	8635376	9709221	0290779	1364624	1655403	55
6	8345948	8634194	9711754	0288246	1365806	1654052	54
7	8347297	8633011	9714286	0285714	1366989	1652703	53
8	8348646	8631828	9716818	0283182	1368172	1651354	52
9	8349994	8630644	9719350	0280650	1369356	1650006	51
10	8351341	8629460	9721882	0278118	1370540	1648659	50
11	8352688	8628274	9724413	0275587	1371726	1647312	49
12	8354033	8627088	9726945	0273055	1372912	1645967	48
13	8355378	8625902	9729477	0270523	1374098	1644622	47
14	8356722	8624714	9732008	0267992	1375286	1643278	46
15	8358066	8623526	9734539	0265461	1376474	1641934	45
16	8359408	8622338	9737071	0262929	1377662	1640592	44
17	8360750	8621148	9739602	0260398	1378852	1639250	43
18	8362091	8619958	9742133	0257867	1380042	1637909	42
19	8363431	8618767	9744664	0255336	1381233	1636569	41
20	8364771	8617576	9747195	0252805	1382424	1635229	40
21	8366109	8616383	9749726	0250274	1383617	1633891	39
22	8367447	8615190	9752257	0247743	1384810	1632553	38
23	8368784	8613997	9754787	0245213	1386003	1631216	37
24	8370121	8612803	9757318	0242682	1387197	1629879	36
25	8371456	8611608	9759849	0240151	1388392	1628544	35
26	8372791	8610412	9762379	0237621	1389588	1627209	34
27	8374125	8609215	9764909	0235091	1390785	1625875	33
28	8375458	8608018	9767440	0232560	1391982	1624542	32
29	8376790	8606821	9769970	0230030	1393179	1623210	31
30	8378122	8605622	9772500	0227500	1394378	1621878	30
31	8379453	8604423	9775030	0224970	1395577	1620547	29
32	8380783	8603223	9777560	0222440	1396777	1619217	28
33	8382112	8602022	9780090	0219910	1397978	1617888	27
34	8383441	8600821	9782620	0217380	1399179	1616559	26
35	8384769	8599619	9785149	0214851	1400381	1615231	25
36	8386096	8598416	9787679	0212321	1401584	1613904	24
37	8387422	8597213	9790209	0209791	1402787	1612578	23
38	8388747	8596009	9792738	0207262	1403991	1611253	22
39	8390072	8594804	9795268	0204732	1405196	1609928	21
40	8391396	8593599	9797797	0202203	1406401	1608604	20
41	8392719	8592393	9800326	0199674	1407607	1607281	19
42	8394041	8591186	9802856	0197144	1408814	1605959	18
43	8395363	8589978	9805385	0194615	1410022	1604637	17
44	8396684	8588770	9807914	0192086	1411230	1603316	16
45	8398004	8587561	9810443	0189557	1412439	1601996	15
46	8399323	8586351	9812972	0187028	1413649	1600677	14
47	8400642	8585141	9815501	0184499	1414859	1599358	13
48	8401959	8583929	9818030	0181970	1416071	1598041	12
49	8403276	8582718	9820559	0179441	1417282	1596724	11
50	8404593	8581505	9823087	0176913	1418495	1595407	10
51	8405908	8580292	9825616	0174384	1419708	1594092	9
52	8407223	8579078	9828145	0171855	1420922	1592777	8
53	8408537	8577863	9830673	0169327	1422137	1591463	7
54	8409850	8576648	9833202	0166798	1423352	1590150	6
55	8411162	8575432	9835730	0164270	1424568	1588838	5
56	8412474	8574215	9838259	0161741	1425785	1587526	4
57	8413785	8572998	9840787	0159213	1427002	1586215	3
58	8415095	8571779	9843315	0156685	1428221	1584905	2
59	8416404	8570561	9845844	0154156	1429439	1583596	1
60	8417713	8569341	9848372	0151628	1430659	1582287	0

44 Degrees. 45 Degrees.

M	Sin:.		Tang.		Secant.		M
0	9.8417713	9.8569341	9.9848372	10.0151628	10 1430659	10.1582287	60
1	8419021	8568121	9850900	0149100	1431879	1580979	59
2	8420328	8566900	9853428	0146572	1433100	1579672	58
3	8421634	8565678	9855956	0144044	1434322	1578366	57
4	8422939	8564455	9858484	0141516	1435545	1577061	56
5	8424244	8563232	9861012	0138988	1436768	1575756	55
6	8425548	8562008	9863540	0136460	1437992	1574452	54
7	8426851	8560784	9866068	0133932	1439216	1573149	53
8	8428154	8559558	9868596	0131404	1440442	1571846	52
9	8429456	8558332	9871123	0128877	1441668	1570544	51
10	8430757	8557106	9873651	0126349	1442894	1569243	50
11	8432057	8555878	9876179	0123821	1444122	1567943	49
12	8433356	8554650	9878706	0121294	1445350	1566644	48
13	8434655	8553421	9881234	0118766	1446579	1565345	47
14	8435953	8552192	9883761	0116239	1447808	1564047	46
15	8437250	8550961	9886289	0113711	1449039	1562750	45
16	8438547	8549730	9888816	0111184	1450270	1561453	44
17	8439842	8548499	9891344	0108656	1451501	1560158	43
18	8441137	8547266	9893871	0106129	1452734	1558863	42
19	8442432	8546033	9896399	0103601	1453967	1557568	41
20	8443725	8544799	9898926	0101074	1455201	1556275	40
21	8445018	8543564	9901453	0098547	1456436	1554982	39
22	8446310	8542329	9903981	0096019	1457671	1553690	38
23	8447601	8541093	9906508	0093492	1458907	1552399	37
24	8448891	8539856	9909035	0090965	1460144	1551109	36
25	8450181	8538619	9911562	0088438	1461381	1549819	35
26	8451470	8537381	9914089	0085911	1462619	1548530	34
27	8452758	8536142	9916616	0083384	1463858	1547242	33
28	8454045	8534902	9919143	0080857	1465098	1545955	32
29	8455332	8533662	9921670	0078330	1466338	1544668	31
30	8456618	8532421	9924197	0075803	1467579	1543382	30
31	8457903	8531179	9926724	0073276	1468821	1542097	29
32	8459188	8529936	9929251	0070749	1470064	1540812	28
33	8460471	8528693	9931778	0068222	1471307	1539529	27
34	8461754	8527449	9934305	0065695	1472551	1538246	26
35	8463036	8526204	9936832	0063168	1473796	1536964	25
36	8464318	8524959	9939359	0060641	1475041	1535682	24
37	8465599	8523713	9941886	0058114	1476287	1534401	23
38	8466879	8522466	9944413	0055587	1477534	1533121	22
39	8468158	8521218	9946940	0053060	1478782	1531842	21
40	8469436	8519970	9949466	0050534	1480030	1530564	20
41	8470714	8518721	9951993	0048007	1481279	1529286	19
42	8471991	8517471	9954520	0045480	1482529	1528009	18
43	8473267	8516220	9957047	0042953	1483780	1526733	17
44	8474543	8514969	9959573	0040427	1485031	1525457	16
45	8475817	8513717	9962100	0037900	1486283	1524183	15
46	8477091	8512465	9964627	0035373	1487535	1522909	14
47	8478365	8511211	9967154	0032846	1488789	1521635	13
48	8479637	8509957	9969680	0030320	1490043	1520363	12
49	8480909	8508702	9972207	0027793	1491298	1519091	11
50	8482180	8507446	9974734	0025266	1492554	1517820	10
51	8483450	8506190	9977260	0022740	1493810	1516550	9
52	8484720	8504933	9979787	0020213	1495067	1515280	8
53	8485989	8503675	9982314	0017686	1496325	1514011	7
54	8487257	8502417	9984840	0015160	1497583	1512743	6
55	8488524	8501157	9987367	0012633	1498843	1511476	5
56	8489791	8499897	9989893	0010107	1500103	1510209	4
57	8491057	8498637	9992420	0007580	1501363	1508943	3
58	8492322	8497375	9994947	0005053	1502625	1507678	2
59	8493586	8496113	9997473	0002527	1503887	1506414	1
60	8494850	8494850	0000000	0000000	1505150	1505150	0

A
T A B L E
O F
NATURAL SINES.

M	0 Deg.	89 Deg.	1 Deg.	88 Deg.	2 Deg.	87 Deg.	3 Deg.	86 Deg.	M
0	0.000	10000.000	174.524	9998 477	348.995	9993.908	523.360	9986 295	60
1	2 909	9999 996	177 432	8 426	351 902	93 806	526 264	86 143	59
2	5 818	998	180 341	8 374	354 809	93 704	529 169	85 989	58
3	8 727	996	183 249	8 321	357 716	93 600	532 074	85 835	57
4	11 636	993	186 158	8 267	360 623	93 495	534 979	85 680	56
5	14 544	989	189 066	8 213	363 530	93 390	537 883	85 524	55
6	17 453	984	191 974	8 157	366 437	93 284	540 788	85 367	54
7	20 362	979	194 885	8 101	369 344	93 177	543 693	85 209	53
8	23 271	973	197 791	8 044	372 251	93 069	546 597	85 050	52
9	26 180	966	200 699	7 986	375 158	92 960	549 502	84 891	51
10	29 089	958	203 608	7 927	378 065	92 851	552 406	84 731	50
11	31 998	949	206 516	7 867	380 971	92 740	555 311	84 570	49
12	34 906	939	209 424	7 807	383 878	92 629	558 215	84 408	48
13	37 815	928	212 332	7 745	386 785	92 517	561 119	84 245	47
14	40 724	917	215 241	7 683	389 692	92 404	564 024	84 081	46
15	43 633	905	218 149	7 620	392 598	92 290	566 928	83 917	45
16	46 542	892	221 057	7 556	395 505	92 176	569 832	83 751	44
17	49 451	878	223 965	7 491	398 411	92 060	572 736	83 585	43
18	52 360	863	226 873	7 426	401 318	91 944	575 640	83 418	42
19	55 268	847	229 781	7 360	404 224	91 827	578 544	83 250	41
20	58 177	831	232 690	7 292	407 131	91 709	581 448	83 082	40
21	61 086	813	235 598	7 224	410 037	91 590	584 352	82 912	39
22	63 995	795	238 506	7 156	412 944	91 470	587 256	82 742	38
23	66 904	776	241 414	7 086	415 850	91 350	590 160	82 570	37
24	69 813	756	244 322	7 015	418 757	91 228	593 064	82 398	36
25	72 721	736	247 230	6 943	421 663	91 106	595 967	82 225	35
26	75 630	713	250 138	6 871	424 569	90 983	598 871	82 052	34
27	78 539	691	253 046	6 798	427 475	90 859	601 775	81 877	33
28	81 448	668	255 954	6 724	430 382	90 734	604 678	81 701	32
29	84 357	644	258 862	6 649	433 288	90 609	607 582	81 525	31
30	87 265	619	261 769	6 573	436 194	90 482	610 485	81 348	30
31	90 174	593	264 677	6 497	439 100	90 355	613 389	81 170	29
32	93 083	566	267 585	6 419	442 006	90 227	616 292	80 991	28
33	95 992	539	270 493	6 341	444 912	90 098	619 196	80 811	27
34	98 900	511	273 401	6 262	447 818	89 968	622 099	80 631	26
35	101 809	482	276 309	6 182	450 724	89 837	625 002	80 450	25
36	104 718	452	279 216	6 101	453 630	89 706	627 905	80 267	24
37	107 627	421	282 124	5 020	456 536	89 573	630 808	80 084	23
38	110 535	389	285 032	5 937	459 442	89 440	633 711	79 900	22
39	113 444	357	287 940	5 854	462 347	89 306	636 614	79 716	21
40	116 353	323	290 847	5 770	465 253	89 171	639 517	79 530	20
41	119 261	289	293 755	5 684	468 159	89 035	642 420	79 343	19
42	122 170	254	296 662	5 599	471 065	88 899	645 323	79 156	18
43	125 079	218	299 570	5 512	473 970	88 761	648 226	78 968	17
44	127 987	181	302 478	5 424	476 876	88 623	651 129	78 779	16
45	130 896	143	305 385	5 336	479 781	88 484	654 031	78 589	15
46	133 805	104	308 293	5 247	482 687	88 344	656 934	78 399	14
47	136 713	065	311 200	5 157	485 592	88 203	659 836	78 207	13
48	139 622	025	314 108	5 066	488 498	88 061	662 739	78 015	12
49	142 530	9998 984	317 015	4 974	491 403	87 919	665 641	77 821	11
50	145 439	8 942	319 922	4 881	494 308	87 775	668 544	77 627	10
51	148 348	8 899	322 830	4 788	497 214	87 631	671 446	77 433	9
52	151 256	8 855	325 737	4 693	500 119	87 486	674 349	77 237	8
53	154 165	8 811	328 644	4 598	503 024	87 340	677 251	77 040	7
54	157 073	8 766	331 552	4 502	505 929	87 194	680 153	76 843	6
55	159 982	8 720	334 459	4 405	508 835	87 046	683 055	76 645	5
56	162 890	8 673	337 366	4 308	511 740	86 898	685 957	76 445	4
57	165 799	8 625	340 273	4 209	514 645	86 748	688 859	76 245	3
58	168 707	8 577	343 181	4 110	517 550	86 598	691 761	76 045	2
59	171 616	8.527	346 088	4 009	520 455	86 447	694 663	75 843	1
60	174 524	8 477	348 995	3 908	523 360	86 295	697 565	75 641	0

M	4 Deg.	85 Deg.	5 Deg.	84 Deg.	6 Deg.	83 Deg.	7 Deg.	82 Deg.	M
0	697.565	9975 641	871.557	9961.947	1045.285	9945.219	1218.693	9925.462	60
1	700 467	75 437	874 455	61 693	1048 178	44 914	1221 581	25 107	59
2	703 368	75 233	877 353	61 438	1051 070	44 609	1224 468	24 751	58
3	706 270	75 028	880 251	61 183	1053 963	44 303	1227 355	24 394	57
4	709 171	74 822	883 148	60 926	1056 856	43 996	1230 241	24 037	56
5	712 073	74 615	886 046	60 669	1059 748	43 688	1233 128	23 679	55
6	714 974	74 408	888 943	60 411	1062 641	43 379	1236 015	23 319	54
7	717 876	74 199	891 840	60 152	1065 533	43 070	1238 901	22 959	53
8	720 777	73 990	894 738	59 892	1068 425	42 760	1241 788	22 599	52
9	723 678	73 780	897 635	59 631	1071 318	42 448	1244 674	22 237	51
10	726 580	73 569	900 532	59 370	1074 210	42 136	1247 560	21 874	50
11	729 481	73 357	903 429	59 107	1077 102	41 823	1250 446	21 511	49
12	732 382	73 145	906 326	58 844	1079 994	41 510	1253 332	21 147	48
13	735 283	72 931	909 223	58 580	1082 885	41 195	1256 218	20 782	47
14	738 184	72 717	912 119	58 315	1085 777	40 880	1259 104	20 416	46
15	741 085	72 502	915 016	58 049	1088 669	40 563	1261 990	20 049	45
16	743 986	72 286	917 913	57 783	1091 560	40 246	1264 875	19 682	44
17	746 887	72 069	920 809	57 515	1094 452	39 928	1267 761	19 314	43
18	749 787	71 851	923 706	57 247	1097 343	39 610	1270 646	18 944	42
19	752 688	71 633	926 602	56 978	1100 234	39 290	1273 531	18 574	41
20	755 589	71 413	929 499	56 708	1103 126	38 969	1276 416	18 204	40
21	758 489	71 193	932 395	56 437	1106 017	38 648	1279 302	17 832	39
22	761 390	70 972	935 291	56 165	1108 908	38 326	1282 186	17 459	38
23	764 290	70 750	938 187	55 893	1111 799	38 003	1285 071	17 086	37
24	767 190	70 528	941 083	55 620	1114 689	37 679	1287 956	16 712	36
25	770 091	70 304	943 979	55 345	1117 580	37 355	1290 841	16 337	35
26	772 991	70 080	946 875	55 070	1120 471	37 029	1293 725	15 961	34
27	775 891	69 854	949 771	54 795	1123 361	36 703	1296 609	15 584	33
28	778 791	69 628	952 666	54 518	1126 252	36 375	1299 494	15 206	32
29	781 691	69 401	955 562	54 240	1129 142	36 047	1302 378	14 828	31
30	784 591	69 173	958 458	53 962	1132 032	35 719	1305 262	14 449	30
31	787 491	68 945	961 353	53 683	1134 922	35 389	1308 146	14 069	29
32	790 391	68 715	964 248	53 403	1137 812	35 058	1311 030	13 688	28
33	793 290	68 485	967 144	53 122	1140 702	34 727	1313 913	13 306	27
34	796 190	68 254	970 039	52 840	1143 592	34 395	1316 797	12 923	26
35	799 090	68 022	972 934	52 557	1146 482	34 062	1319 681	12 540	25
36	801 989	67 789	975 829	52 274	1149 372	33 728	1322 564	12 155	24
37	804 889	67 555	978 724	51 990	1152 261	33 393	1325 447	11 770	23
38	807 788	67 321	981 619	51 705	1155 151	33 057	1328 330	11 384	22
39	810 687	67 085	984 514	51 419	1158 040	32 721	1331 213	10 997	21
40	813 587	66 849	987 408	51 132	1160 929	32 384	1334 096	10 610	20
41	816 486	66 612	990 303	50 844	1163 818	32 045	1336 979	10 221	19
42	819 385	66 374	993 197	50 556	1166 707	31 706	1339 862	09 832	18
43	822 284	66 135	996 092	50 266	1169 596	31 367	1342 744	09 442	17
44	825 183	65 895	998 986	49 976	1172 485	31 026	1345 627	09 051	16
45	828 082	65 655	1001 881	49 685	1175 374	30 685	1348 509	08 659	15
46	830 981	65 414	1004 775	49 393	1178 263	30 342	1351 392	08 266	14
47	833 880	65 172	1007 669	49 101	1181 151	29 999	1354 274	07 873	13
48	836 778	64 929	1010 563	48 807	1184 040	29 655	1357 156	07 478	12
49	839 677	64 685	1013 457	48 513	1186 928	29 310	1360 038	07 083	11
50	842 576	64 440	1016 351	48 217	1189 816	28 965	1362 919	06 687	10
51	845 474	64 195	1019 245	47 921	1192 704	28 618	1365 801	06 290	9
52	848 373	63 948	1022 138	47 625	1195 593	28 271	1368 683	05 893	8
53	851 271	63 701	1025 032	47 327	1198 481	27 922	1371 564	05 494	7
54	854 169	63 453	1027 925	47 028	1201 368	27 573	1374 445	05 095	6
55	857 067	63 204	1030 819	46 729	1204 256	27 224	1377 327	04 694	5
56	859 966	62 954	1033 712	46 428	1207 144	26 873	1380 208	04 293	4
57	862 864	62 704	1036 605	46 127	1210 031	26 521	1383 089	03 891	3
58	865 762	62 452	1039 499	45 825	1212 919	26 169	1385 970	03 489	2
59	868 660	62 200	1042 392	45 523	1215 806	25 810	1388 850	03 085	1
60	871 557	61 947	1045 285	45 219	1218 693	25 462	1391 731	02 681	0

M	8 Deg.	81 Deg.	9 Deg.	80 Deg.	10 Deg.	79 Deg.	11 Deg.	78 Deg	M
0	1391.731	9902.681	1564.345	9876.883	1736 482	9848.078	1908 090	9816.272	60
1	1394 612	9902 275	1567 218	9876 428	1739 346	9847 572	1910 945	9815 716	59
2	1397 492	9901 869	1570 091	9875 972	1742 211	9847 066	1913 801	9815 160	58
3	1400 372	9901 462	1572 963	9875 514	1745 075	9846 558	1916 656	9814 603	57
4	1403 252	9901 055	1575 836	9875 057	1747 939	9846 050	1919 510	9814 045	56
5	1406 132	9900 646	1578 708	9874 598	1750 803	9845 542	1922 365	9813 486	55
6	1409 012	9900 237	1581 581	9874 138	1753 667	9845 032	1925 220	9812 927	54
7	1411 892	9899 826	1584 453	9873 678	1756 531	9844 521	1928 074	9812 366	53
8	1414 772	9899 415	1587 325	9873 216	1759 395	9844 010	1930 928	9811 805	52
9	1417 651	9899 003	1590 197	9872 754	1762 258	9843 498	1933 782	9811 243	51
10	1420 531	9898 590	1593 069	9872 291	1765 121	9842 985	1936 636	9810 680	50
11	1423 410	9898 177	1595 940	9871 827	1767 984	9842 471	1939 490	9810 116	49
12	1426 289	9897 762	1598 812	9871 363	1770 847	9841 956	1942 344	9809 552	48
13	1429 168	9897 347	1601 683	9870 897	1773 710	9841 441	1945 197	9808 986	47
14	1432 047	9896 931	1604 555	9870 431	1776 573	9840 924	1948 050	9808 420	46
15	1434 926	9896 514	1607 426	9869 964	1779 435	9840 407	1950 903	9807 853	45
16	1437 805	9896 096	1610 297	9869 496	1782 298	9839 889	1953 756	9807 285	44
17	1440 684	9895 677	1613 167	9869 027	1785 160	9839 370	1956 609	9806 716	43
18	1443 562	9895 258	1616 038	9868 557	1788 022	9838 850	1959 461	9806 147	42
19	1446 440	9894 838	1618 909	9868 087	1790 884	9838 330	1962 314	9805 576	41
20	1449 319	9894 416	1621 779	9867 615	1793 746	9837 808	1965 166	9805 005	40
21	1452 197	9893 994	1624 650	9867 143	1796 607	9837 286	1968 018	9804 433	39
22	1455 075	9893 572	1627 520	9866 670	1799 469	9836 763	1970 870	9803 860	38
23	1457 953	9893 148	1630 390	9866 196	1802 330	9836 239	1973 722	9803 286	37
24	1460 83[illegible]	9892 723	1633 260	9865 722	1805 191	9835 715	1976 573	9802 712	36
25	1463 708	9892 298	1636 129	9865 246	1808 052	9835 189	1979 425	9802 136	35
26	1466 585	9891 872	1638 999	9864 770	1810 913	9834 663	1982 276	9801 560	34
27	1469 463	9891 445	1641 868	9864 293	1813 774	9834 136	1985 127	9800 983	33
28	1472 340	9891 017	1644 738	9863 815	1816 635	9833 608	1987 978	9800 405	32
29	1475 217	9890 588	1647 607	9863 336	1819 495	9833 079	1990 829	9799 827	31
30	1478 094	9890 159	1650 476	9862 856	1822 355	9832 549	1993 679	9799 247	30
31	1480 971	9889 728	1653 345	9862 375	1825 215	9832 019	1996 530	9798 667	29
32	1483 848	9889 297	1656 214	9861 894	1828 075	9831 487	1999 380	9798 086	28
33	1486 724	9888 865	1659 082	9861 412	1830 935	9830 955	2002 230	9797 504	27
34	1489 601	9888 432	1661 951	9860 929	1833 795	9830 422	2005 080	9796 921	26
35	1492 477	9887 998	1664 819	9860 445	1836 654	9829 888	2007 930	9796 337	25
36	1495 353	9887 564	1667 687	9859 960	1839 514	9829 353	2010 779	9795 752	24
37	1498 230	9887 128	1670 556	9859 475	1842 373	9828 818	2013 629	9795 167	23
38	1501 106	9886 692	1673 423	9858 988	1845 232	9828 282	2016 478	9794 581	22
39	1503 981	9886 255	1676 291	9858 501	1848 091	9827 744	2019 327	9793 994	21
40	1506 857	9885 817	1679 159	9858 013	1850 949	9827 206	2022 176	9793 406	20
41	1509 733	9885 378	1682 026	9857 524	1853 806	9826 668	2025 024	9792 818	19
42	1512 608	9884 939	1684 894	9857 035	1856 666	9826 128	2027 873	9792 228	18
43	1515 484	9884 498	1687 761	9856 544	1859 524	9825 587	2030 721	9791 638	17
44	1518 359	9884 057	1690 628	9856 053	1862 382	9825 046	2033 569	9791 047	16
45	1521 234	9883 615	1693 495	9855 561	1865 240	9824 504	2036 418	9790 455	15
46	1524 109	9883 172	1696 362	9855 068	1868 098	9823 961	2039 265	9789 862	14
47	1526 984	9882 728	1699 228	9854 574	1870 956	9823 417	2042 113	9789 268	13
48	1529 858	9882 284	1702 095	9854 079	1873 813	9822 873	2044 961	9788 674	12
49	1532 733	9881 838	1704 961	9853 583	1876 670	9822 327	2047 808	9788 079	11
50	1535 607	9881 392	1707 828	9853 087	1879 528	9821 781	2050 655	9787 483	10
51	1538 482	9880 945	1710 694	9852 590	1882 385	9821 234	2053 502	9786 886	9
52	1541 356	9880 497	1713 560	9852 092	1885 241	9820 686	2056 349	9786 288	8
53	1544 230	9880 048	1716 425	9851 593	1880 098	9820 137	2059 195	9785 689	6
54	1547 104	9879 599	1719 291	9851 093	1893 954	9819 587	2062 042	9785 090	7
55	1549 978	9879 148	1722 156	9850 593	1896 811	9819 037	2064 888	9784 490	5
56	1552 851	9878 697	1725 022	9850 091	1898 667	9818 485	2067 734	9783 889	4
57	1555 725	9878 245	1727 887	9849 589	1889 523	9817 933	2070 580	9783 287	3
58	1558 598	9877 792	1730 752	9849 086	1902 379	9817 380	2073 426	9782 684	2
59	1561 472	9877 338	1733 617	9848 582	1905 234	9816 826	2076 272	9782 080	1
60	1564 345	9876 883	1736 482	9848 078	1908 090	9816 272	2079 117	9781 476	0

M	12 Deg.	77 Deg.	13 Deg.	76 Deg.	14 Deg.	75 Deg.	15 Deg.	74 Deg.	M
0	2079.117	9781.476	2249.511	9743.701	2419.219	9702.957	2588.190	9659.258	60
1	2081 962	9780 871	2252 345	9743 046	2422 041	9702 253	2591 000	9658 505	59
2	2084 807	9780 265	2255 179	9742 390	2424 863	9701 548	2593 810	9657 751	58
3	2087 652	9779 658	2258 013	9741 734	2427 685	9700 842	2596 619	9656 996	57
4	2090 497	9779 050	2260 846	9741 077	2430 507	9700 136	2599 428	9656 240	56
5	2093 341	9778 442	2263 680	9740 419	2433 329	9699 428	2602 237	9655 484	55
6	2096 186	9777 832	2266 513	9739 760	2436 150	9698 720	2605 045	9654 726	54
7	2099 030	9777 222	2269 346	9739 100	2438 971	9698 011	2607 853	9653 968	53
8	2101 874	9776 611	2272 179	9738 439	2441 792	9697 301	2610 662	9653 209	52
9	2104 718	9775 999	2275 012	9737 778	2444 613	9696 591	2613 469	9652 449	51
10	2107 561	9775 387	2277 844	9737 116	2447 433	9695 879	2616 277	9651 689	50
11	2110 405	9774 773	2280 677	9736 453	2450 254	9695 167	2619 085	9650 927	49
12	2113 248	9774 159	2283 509	9735 789	2453 074	9694 453	2621 892	9650 165	48
13	2116 091	9773 544	2286 341	9735 124	2455 894	9693 740	2624 699	9649.402	47
14	2118 934	9772 928	2289 172	9734 459	2458 713	9693 025	2627 506	9648 638	46
15	2121 777	9772 311	2292 004	9733 793	2461 533	9692 309	2630 312	9647 873	45
16	2124 619	9771 693	2294 835	9733 125	2464 352	9691 593	2633 118	9647 108	44
17	2127 462	9771 075	2297 666	9732 458	2467 171	9690 875	2635 925	9646 341	43
18	2130 304	9770 456	2300 497	9731 789	2469 990	9690 157	2638 730	9645 574	42
19	2133 146	9769 836	2303 328	9731 119	2472 809	9689 438	2641 536	9644 806	41
20	2135 988	9769 215	2306 159	9730 449	2475 627	9688 719	2644 342	9644 037	40
21	2138 829	9768 593	2308 989	9729 777	2478 445	9687 998	2647 147	9643 268	39
22	2141 671	9767 970	2311 819	9729 105	2481 263	9687 277	2649 952	9642 497	38
23	2144 512	9767 347	2314 649	9728 432	2484 081	9686 555	2652 757	9641 726	37
24	2147 353	9766 723	2317 479	9727 759	2486 899	9685 832	2655 561	9640 954	36
25	2150 194	9766 098	2320 309	9727 084	2489 716	9685 108	2658 366	9640 181	35
26	2153 035	9765 472	2323 138	9726 409	2492 533	9684 383	2661 170	9639 407	34
27	2155 876	9764 845	2325 967	9725 733	2495 350	9683 658	2663 973	9638 633	33
28	2158 716	9764 218	2328 796	9725 056	2498 167	9682 931	2666 777	9637 858	32
29	2161 556	9763 589	2331 625	9724 378	2500 984	9682 204	2669 581	9637 081	31
30	2164 396	9762 960	2334 454	9723 699	2503 800	9681 476	2672 384	9636 305	30
31	2167 236	9762 330	2337 282	9723 020	2506 616	9680 748	2675 187	9635 527	29
32	2170 076	9761 699	2340 110	9722 339	2509 432	9680 018	2677 989	9634 748	28
33	2172 915	9761 068	2342 938	9721 658	2512 248	9679 288	2680 792	9633 969	27
34	2175 754	9760 435	2345 766	9720 976	2515 063	9678 557	2683 594	9633 189	26
35	2178 593	9759 802	2348 594	9720 294	2517 879	9677 825	2686 396	9632 408	25
36	2181 432	9759 168	2351 421	9719 610	2520 694	9677 092	2689 198	9631 626	24
37	2184 271	9758 533	2354 248	9718 926	2523 508	9676 358	2692 000	9630 843	23
38	2187 110	9757 897	2357 075	9718 240	2526 323	9675 624	2694 801	9630 060	22
39	2189 948	9757 260	2359 902	9717 554	2529 137	9674 888	2697 602	9629 275	21
40	2192 786	9756 623	2362 729	9716 867	2531 952	9674 152	2700 403	9628 490	20
41	2195 624	9755 985	2365 555	9716 180	2534 766	9673 415	2703 204	9627 704	19
42	2198 462	9755 345	2368 381	9715 491	2537 579	9672 678	2706 004	9626 917	18
43	2201 300	9754 706	2371 207	9714 802	2540 393	9671 939	2708 805	9626 130	17
44	2204 137	9754 065	2374 033	9714 112	2543 206	9671 200	2711 605	9625 342	16
45	2206 974	9753 423	2376 859	9713 421	2546 019	9670 459	2714 404	9624 552	15
46	2209 811	9752 781	2379 684	9712 729	2548 832	9669 718	2717 204	9623 762	14
47	2212 648	9752 138	2382 510	9712 036	2551 645	9668 977	2720 003	9622 972	13
48	2215 485	6751 494	2385 335	9711 343	2554 458	9668 234	2722 802	9622 180	12
49	2218 321	9750 849	2388 159	9710 649	2557 270	9667 490	2725 601	9621 387	11
50	2221 158	9750 203	2390 984	9709 953	2560 082	9666 746	2728 400	9620 594	10
51	2223 994	9749 556	2393 808	9709 258	2562 894	9666 001	2731 198	9619 800	9
52	2226 830	9748 909	2396 633	9708 561	2565 705	9665 255	2733 997	9619 005	8
53	2229 666	9748 261	2399 457	9707 863	2568 517	9664 508	2736 794	9618 210	7
54	2232 501	9747 612	2402 280	9707 165	2571 328	9663 761	2739 592	9617 413	6
55	2235 337	9746 962	2405 104	9706 466	2574 139	9663 012	2742 390	9616 616	5
56	2238 172	9746 311	2407 927	9705 766	2576 950	9662 263	2745 187	9615 818	4
57	2241 007	9745 660	2410 751	9705 065	2579 760	9661 513	2747 984	9615 019	3
58	2243 842	9745 008	2413 574	9704 363	2582 570	9660 762	2750 781	9614 219	2
59	2246 676	9744 355	2416 396	9703 661	2585 381	9660 011	2753 577	9613 418	1
60	2249 511	9743 701	2419 219	9702 957	2588 190	9659 258	2756 374	9612 617	0

M	16 Deg.	73 Deg.	17 Deg.	72 Deg.	18 Deg.	71 Deg.	19 Deg.	70 Deg.	M
0	2756.374	9612.617	2923.717	9563.048	3090.170	9510.565	3255.682	9455.186	60
1	2759 170	9611 815	2926 499	9562 197	3092 936	9509 666	3258 432	9454 238	59
2	2761 965	9611 012	2929 280	9561 345	3095 702	9508 766	3261 182	9453 290	58
3	2764 761	9610 208	2932 061	9560 492	3098 468	9507 865	3263 932	9452 341	57
4	2767 556	9609 403	2934 842	9559 639	3001 234	9506 963	3266 681	9451 391	56
5	2770 352	9608 598	2937 623	9558 785	3103 999	9506 061	3269 430	9450 441	55
6	2773 147	9607 792	2940 403	9557 930	3106 764	9505 157	3272 179	9449 489	54
7	2775 941	9606 984	2943 183	9557 074	3109 529	9504 253	3274 928	9448 537	53
8	2778 736	9606 177	2945 963	9556 218	3112 294	9503 348	3277 676	9447 584	52
9	2781 530	9605 368	2948 743	9555 361	3115 058	9502 443	3280 424	9446 630	51
10	2784 324	9604 558	2951 522	9554 502	3117 822	9501 536	3283 172	9445 675	50
11	2787 118	9603 748	2954 302	9553 643	3120 586	9500 629	3285 919	9444 720	49
12	2789 911	9602 937	2957 081	9552 784	3123 349	9499 721	3288 666	9443 764	48
13	2792 704	9602 125	2959 859	9551 923	3126 112	9498 812	3291 413	9442 807	47
14	2795 497	9601 312	2962 638	9551 062	3128 875	9497 902	3294 160	9441 849	46
15	2798 290	9600 499	2965 416	9550 199	3131 638	9496 991	3296 906	9440 890	45
16	2801 083	9599 684	2968 194	9549 336	3134 400	9496 080	3299 653	9439 931	44
17	2803 875	9598 869	2970 971	9548 473	3137 163	9495 168	3302 398	9438 971	43
18	2806 667	9598 053	2973 749	9547 608	3139 925	9494 255	3305 144	9438 010	42
19	2809 459	9597 236	2976 526	9546 743	3142 686	9493 341	3307 889	9437 048	41
20	2812 251	9596 418	2979 303	9545 876	3145 448	9492 426	3310 634	9436 085	40
21	2815 042	9595 600	2982 079	9545 009	3148 209	9491 511	3313 379	9435 122	39
22	2817 833	9594 781	2984 856	9544 141	3150 969	9490 595	3316 123	9434 157	38
23	2820 624	9593 961	2987 632	9543 273	3153 730	9489 678	3318 867	9433 192	37
24	2823 415	9593 140	2990 408	9542 403	3156 490	9488 760	3321 611	9432 227	36
25	2826 205	9592 318	2993 184	9541 533	3159 250	9487 842	3324 355	9431 260	35
26	2828 995	9591 496	2995 959	9540 662	3162 010	9486 922	3327 098	9430 293	34
27	2831 785	9590 672	2998 734	9539 790	3164 770	9486 002	3329 841	9429 324	33
28	2834 575	9589 848	3001 509	9538 917	3167 529	9485 081	3332 584	9428 355	32
29	2837 364	9589 023	3004 284	9538 044	3170 288	9484 159	3335 326	9427 386	31
30	2840 153	9588 197	3007 058	9537 170	3173 047	9483 237	3338 069	9426 415	30
31	2842 942	9587 371	3009 832	9536 294	3175 805	9482 313	3340 810	9425 444	29
32	2845 731	9586 543	3012 606	9535 418	3178 563	9481 389	3343 552	9424 471	28
33	2848 520	9585 715	3015 380	9534 542	3181 321	9480 464	3346 293	9423 498	27
34	2851 308	9584 886	3018 153	9533 664	3184 079	9479 538	3349 034	9422 525	26
35	2854 096	9584 056	3020 926	9532 786	3186 836	9478 612	3351 775	9421 550	25
36	2856 884	9583 226	3023 699	9531 907	3189 593	9477 684	3354 516	9420 575	24
37	2859 671	9582 394	3026 471	9531 027	3192 350	9476 756	3357 256	9419 598	23
38	2862 458	9581 562	3029 244	9530 146	3195 106	9475 827	3359 996	9418 621	22
39	2865 246	9580 729	3032 016	9529 264	3197 863	9474 897	3362 735	9417 644	21
40	2868 032	9579 895	3034 788	9528 382	3200 619	9473 966	3365 475	9416 665	20
41	2870 819	9579 060	3037 559	9527 499	3203 374	9473 035	3368 214	9415 686	19
42	2873 605	9578 225	3040 331	9526 615	3206 130	9472 103	3370 953	9414 705	18
43	2876 391	9577 389	3043 102	9525 730	3208 885	9471 170	3373 691	9413 724	17
44	2879 177	9576 552	3045 872	9524 844	3211 640	9470 236	3376 429	9412 743	16
45	2881 963	9575 714	3048 643	9523 958	3214 395	9469 301	3379 167	9411 760	15
46	2884 748	9574 875	3051 413	9523 071	3217 149	9468 366	3381 905	9410 777	14
47	2887 533	9574 035	3054 183	9522 183	3219 903	9467 430	3384 642	9409 793	13
48	2890 318	9373 195	3056 953	9521 294	3222 657	9466 493	3387 379	9408 808	12
49	2893 103	9572 354	3059 723	9520 404	3225 411	9465 555	3390 116	9407 822	11
50	2895 887	9571 512	3062 492	9519 514	3228 164	9464 616	3392 852	9406 835	10
51	2898 671	9570 669	3065 261	9518 623	3 30 917	9463 677	3395 589	9405 848	9
52	2901 455	9569 825	3068 030	9517 731	3233 670	9462 736	3398 325	9404 860	8
53	2904 239	9568 981	3070 798	9516 838	3236 422	9461 795	3401 060	9403 871	7
54	2907 022	9568 136	3073 566	9515 944	3239 174	9460 854	3403 796	9402 881	6
55	2909 805	9567 290	3076 334	9515 050	3241 926	9459 911	3406 531	9401 891	5
56	2912 588	9566 445	3079 102	9514 154	3244 678	9458 968	3409 265	9400 899	4
57	2915 371	9565 595	3081 869	9513 258	3247 429	9458 023	3412 000	9399 907	3
58	2918 153	9564 747	3084 636	9512 361	3250 180	9457 078	3414 734	9398 914	2
59	2920 935	9563 898	3087 403	9511 464	3252 931	9456 132	3417 468	9397 921	1
60	2923 717	9563 048	3090 170	9510 565	3255 682	9455 186	3420 201	9396 926	0

M	20 Deg.	69 Deg.	21 Deg.	68 Deg.	22 Deg.	67 Deg.	23 Deg.	66 Deg.	M
0	3420.201	9396.926	3583.679	9335.804	3746.066	9271.839	3907.311	9205.049	60
1	3422 935	9395 931	3586 395	9334 761	3748 763	9270 748	3909 989	9203 912	59
2	3425 668	9394 935	3589 110	9333 718	3751 459	9269 658	3912 666	9202 774	58
3	3428 400	9393 938	3591 825	9332 673	3754 156	9268 566	3915 343	9201 635	57
4	3431 133	9392 940	3594 540	9331 628	3756 852	9267 474	3918 019	9200 496	56
5	3433 865	9391 942	3597 254	9330 582	3759 547	9266 380	3920 695	9199 356	55
6	3436 597	9390 943	3599 968	9329 535	3762 243	9265 286	3923 371	9198 215	54
7	3439 329	9389 942	3602 682	9328 488	3764 938	9264 192	3926 047	9197 073	53
8	3442 060	9388 942	3605 395	9327 439	3767 632	9263 096	3928 722	9195 931	52
9	3444 791	9387 940	3608 108	9326 390	3770 327	9262 000	3931 397	9194 788	51
10	3447 521	9386 938	3610 821	9325 340	3773 021	9260 902	3934 071	9193 644	50
11	3450 252	9385 934	3613 534	9324 290	3775 714	9259 805	3936 745	9192 499	49
12	3452 982	9384 930	3616 246	9323 238	3778 408	9258 706	3939 419	9191 353	48
13	3455 712	9383 925	3618 958	9322 186	3781 101	9257 606	3942 093	9190 207	47
14	3458 441	9382 920	3621 669	9321 133	3783 794	9256 506	3944 766	9189 060	46
15	3461 171	9381 913	3624 380	9320 079	3786 486	9255 405	3947 439	9187 912	45
16	3463 909	9380 906	3627 091	9319 024	3789 178	9254 303	3950 111	9186 763	44
17	3466 628	9379 898	3629 802	9317 969	3791 870	9253 201	3952 783	9185 614	43
18	3469 357	9378 889	3632 512	9316 912	3794 562	9252 097	3955 455	9184 464	42
19	3472 085	9377 880	3635 222	9315 855	3797 253	9250 993	3958 127	9183 313	41
20	3474 812	9376 869	3637 932	9314 797	3799 944	9249 888	3960 798	9182 161	40
21	3477 540	9375 858	3640 641	9313 739	3802 634	9248 782	3963 468	9181 009	39
22	3480 267	9374 846	3643 351	9312 679	3805 324	9247 676	3966 139	9179 855	38
23	3482 994	9373 833	3646 059	9311 619	3808 014	9246 568	3968 809	9178 701	37
24	3485 720	9372 820	3648 768	9310 558	3810 704	9245 460	3971 479	9177 546	36
25	3488 447	9371 806	3651 476	9309 496	3813 393	9244 351	3974 148	9176 391	35
26	3491 173	9370 790	3654 184	9308 434	3816 082	9243 242	3976 818	9175 234	34
27	3493 898	9369 774	3656 891	9307 370	3818 770	9242 131	3979 486	9174 077	33
28	3496 624	9368 758	3659 599	9306 306	3821 459	9241 020	3982 155	9172 919	32
29	3499 349	9367 740	3662 306	9305 241	3824 147	9239 908	3984 823	9171 760	31
30	3502 074	9366 722	3665 012	9304 176	3826 834	9238 795	3987 491	9170 601	30
31	3504 798	9365 703	3667 719	9303 109	3829 522	9237 682	3990 158	9169 440	29
32	3507 523	9364 683	3670 425	9302 042	3832 209	9236 567	3992 825	9168 279	28
33	3510 246	9363 662	3673 130	9300 974	3834 895	9235 452	3995 492	9167 118	27
34	3512 970	9362 641	3675 836	9299 905	3837 582	9234 336	3998 158	9165 955	26
35	3515 693	9361 618	3678 541	9298 835	3840 268	9233 220	4000 825	9164 791	25
36	3518 416	9360 595	3681 246	9297 765	3842 953	9232 102	4003 490	9163 627	24
37	3521 139	9359 571	3683 950	9296 694	3845 639	9230 984	4006 156	9162 462	23
38	3523 862	9358 547	3686 654	9295 622	3848 324	9229 865	4008 821	9161 297	22
39	3526 584	9357 521	3689 358	9294 549	3851 008	9228 745	4011 486	9160 130	21
40	3529 306	9356 495	3692 061	9293 475	3853 693	9227 624	4014 150	9158 963	20
41	3532 027	9355 468	3694 765	9292 401	3856 377	9226 503	4016 814	9157 795	19
42	3534 748	9354 440	3697 468	9291 326	3859 060	9225 381	4019 478	9156 626	18
43	3537 469	9353 412	3700 170	9290 250	3861 744	9224 258	4022 141	9155 456	17
44	3540 190	9352 382	3702 872	9289 173	3864 427	9223 134	4024 804	9154 286	16
45	3542 910	9351 352	3705 574	9288 096	3867 110	9222 010	4027 467	9153 115	15
46	3545 630	9350 321	3708 276	9287 017	3869 792	9220 884	4030 129	9151 943	14
47	3548 350	9349 289	3710 977	9285 938	3872 474	9219 758	4032 791	9150 770	13
48	3551 070	9348 257	3713 678	9284 858	3875 156	9218 632	4030 453	9149 597	12
49	3553 789	9347 223	3716 379	9283 778	3877 837	9217 504	4038 114	9148 422	11
50	3556 508	9346 189	3719 079	9282 696	3880 518	9216 375	4040 775	9147 247	10
51	3559 226	9345 154	3721 780	9281 614	3883 199	9215 246	4043 436	9146 072	9
52	3561 944	9344 119	3724 479	9280 531	3885 880	9214 116	4046 096	9144 895	8
53	3564 662	9343 082	3727 179	9279 447	3888 560	9212 986	4048 756	9143 718	7
54	3567 380	9342 045	3729 878	9278 363	3891 240	9211 854	4051 416	9142 540	6
55	3570 097	9341 007	3732 577	9277 277	3893 919	9210 722	4054 075	9141 361	5
56	3572 814	9339 968	3735 275	9276 191	3896 598	9209 589	4056 734	9140 181	4
57	3575 531	9338 928	3737 973	9275 104	3899 277	9208 455	4059 393	9139 001	3
58	3578 248	9337 888	3740 671	9274 016	3901 955	9207 320	4062 051	9137 819	2
59	3580 964	9336 846	3743 369	9272 928	3904 633	9206 185	4064 709	6136 637	1
60	3583 679	9335 804	3746 066	9271 839	3907 311	9205 049	4067 366	9135 455	0

M	24 Deg.	65 Deg.	25 Deg.	64 Deg.	26 Deg.	63 Deg.	27 Deg.	62 Deg.	M
0	4067.366	9135.455	4226.183	9063.078	4383.711	8987.940	4539.905	8910.065	60
1	4070 024	34 271	4228 819	9061 848	4386 326	8986 665	4542 497	8908 744	59
2	4072 681	33 087	4231 455	9060 618	4388 940	8985 389	4545 088	8907 423	58
3	4075 337	31 902	4234 090	9059 386	4391 553	8984 112	4547 679	8906 100	57
4	4077 993	30 716	4236 725	9058 154	4394 166	8982 834	4550 269	8904 777	56
5	4080 649	29 529	4239 360	9056 922	4396 779	8981 555	4552 859	8903 453	55
6	4083 305	28 342	4241 994	9055 688	4399 392	8980 276	4555 449	8902 128	54
7	4085 960	27 154	4244 628	9054 454	4402 004	8978 996	4558 038	8900 803	53
8	4088 615	25 965	4247 262	9053 219	4404 615	8977 715	4560 627	8899 476	52
9	4091 269	24 775	4249 895	9051 983	4407 227	8976 433	4563 216	8898 149	51
10	4093 923	23 584	4252 528	9050 746	4409 838	8975 151	4565 804	8896 822	50
11	4096 577	22 393	4255 161	9049 509	4412 448	8973 868	4568 392	8895 493	49
12	4099 230	21 201	4257 793	9048 271	4415 059	8972 584	4570 979	8894 164	48
13	4101 883	20 008	4260 425	9047 032	4417 668	8971 299	4573 566	8892 834	47
14	4104 536	18 815	4263 056	9045 792	4420 278	8970 014	4576 153	8891 503	46
15	4107 189	17 620	4265 687	9044 551	4422 887	8968 727	4578 739	8890 171	45
16	4109 841	16 425	4268 318	9043 310	4425 496	8967 440	4581 325	8888 839	44
17	4112 492	15 229	4270 949	9042 068	4428 104	8966 153	4583 910	8887 506	43
18	4115 144	14 033	4273 579	9040 825	4430 712	8964 864	4586 496	8886 172	42
19	4117 795	12 835	4276 208	9039 582	4433 319	8963 575	4589 080	8884 838	41
20	4120 445	11 637	4278 838	9038 338	4435 927	8962 285	4591 665	8883 503	40
21	4123 096	10 438	4281 467	9037 093	4438 534	8960 994	4594 248	8882 166	39
22	4125 745	92 038	4284 095	9035 847	4441 140	8959 703	4596 832	8880 830	38
23	4128 395	08 038	4286 723	9034 600	4443 746	8958 411	4599 415	8879 492	37
24	4131 044	06 837	4289 351	9033 353	4446 352	8957 118	4601 998	8878 154	36
25	4133 693	05 635	4291 979	9032 105	4448 957	8955 824	4604 580	8876 815	35
26	4136 342	04 432	4294 606	9030 856	4451 562	8954 529	4607 162	8875 475	34
27	4138 990	03 228	4297 233	9029 606	4454 167	8953 234	4609 744	8874 134	33
28	4141 638	02 024	4299 859	9028 356	4456 771	8951 938	4612 325	8872 793	32
29	4144 285	00 819	4302 485	9027 105	4459 375	8950 641	4614 906	8871 451	31
30	4146 932	9099 613	4305 111	9025 853	4461 978	8949 344	4617 486	8870 108	30
31	4149 579	98 406	4307 736	9024 600	4464 581	8948 045	4620 066	8868 765	29
32	4152 226	97 199	4310 361	9023 347	4467 184	8946 746	4622 646	8867 420	28
33	4154 872	95 990	4312 986	9022 092	4469 786	8945 446	4625 225	8866 075	27
34	4157 517	94 781	4315 610	9020 838	4472 388	8944 146	4627 804	8864 730	26
35	4160 163	93 572	4318 234	9019 582	4474 990	8942 844	4630 382	8863 383	25
36	4162 808	92 361	4320 857	9018 325	4477 591	8941 542	4632 960	8862 036	24
37	4165 453	91 150	4323 481	9017 068	4480 192	8940 240	4635 538	8860 688	23
38	4168 097	89 938	4326 103	9015 810	4482 792	8938 936	4638 115	8859 339	22
39	4170 741	88 725	4328 726	9014 551	4485 392	8937 632	4640 692	8857 989	21
40	4173 385	87 511	4331 348	9013 292	4487 992	8936 326	4643 269	8856 639	20
41	4176 028	86 297	4333 970	9012 031	4490 591	8935 021	4645 845	8855 288	19
42	4178 671	85 082	4336 591	9010 770	4493 190	8933 714	4648 420	8853 936	18
43	4181 313	83 866	4339 212	9009 508	4495 789	8932 406	4650 996	8852 584	17
44	4183 956	82 649	4341 832	9008 246	4498 387	8931 098	4653 571	8851 230	16
45	4186 597	81 432	4344 453	9006 982	4500 984	8929 789	4656 145	8849 876	15
46	4189 239	80 214	4347 072	9005 718	4503 582	8928 480	4658 719	8848 522	14
47	4191 880	78 995	4349 692	9004 453	4506 179	8927 169	4661 293	8847 166	13
48	4194 521	77 775	4352 311	9003 188	4508 775	8925 858	4663 866	8845 810	12
49	4197 161	76 554	4354 930	9001 921	4511 372	8924 546	4666 439	8844 453	11
50	4199 801	75 333	4357 548	9000 654	4513 967	8923 234	4669 012	8843 095	10
51	4202 441	74 111	4360 166	8999 386	4516 563	8921 920	4671 584	8841 736	9
52	4205 080	72 888	4362 784	8998 117	4519 158	8920 606	4674 150	8840 377	8
53	4207 719	71 665	4365 401	8996 848	4521 753	8919 291	4676 727	8839 017	7
54	4210 358	70 440	4368 018	8995 578	4524 347	8917 975	4679 298	8837 656	6
55	4212 996	69 215	4370 634	8994 307	4526 941	8916 659	4681 869	8836 295	5
56	4215 634	67 989	4373 251	8993 035	4529 535	8915 342	4684 439	8834 933	4
57	4218 272	66 762	4375 866	8991 763	4532 128	8914 024	4687 009	8833 569	3
58	4220 909	65 535	4378 482	8990 489	4534 721	8912 705	4689 578	8832 206	2
59	4223 546	64 307	4381 097	8989 215	4537 313	8911 385	4692 147	8830 841	1
60	4226 183	63 078	4383 711	8987 940	4539 905	8910 065	4694 716	8829 476	0

A Table of Natural Sines.

M	28 Deg.	61 Deg.	29 Deg.	60 Deg.	30 Deg.	59 Deg.	31 Deg.	58 Deg.	M
0	4694.714	8829.470	4848 096	8746.197	5000.000	8660.254	5150 381	8571.673	60
1	4697 284	28 110	4850 640	8744 786	5002 519	58 799	5152 874	70 174	59
2	2699 852	26 743	4853 184	8743 375	5005 037	57 344	5155 367	68 675	58
3	4702 419	25 375	4855 727	8741 963	5007 556	55 887	5157 859	67 175	57
4	4704 986	24 007	4858 270	8740 550	5010 073	54 430	5160 351	65 674	56
5	4707 553	22 638	4860 812	8739 137	5012 591	52 973	5162 842	64 173	55
6	4710 119	21 269	4863 354	8737 722	5015 107	51 514	5165 333	62 671	54
7	4712 685	19 898	4865 895	8736 307	5017 624	50 055	5167 824	61 168	53
8	4715 250	18 527	4868 436	8734 891	5020 140	48 595	5170 314	59 664	52
9	4717 815	17 155	4870 977	8733 475	5022 655	47 134	5172 804	58 160	51
10	4720 380	15 782	4873 517	8732 058	5025 170	45 673	5175 293	56 655	50
11	4722 944	14 409	4876 057	8730 640	5027 685	44 211	5177 782	55 149	49
12	4725 508	13 035	4878 597	8729 221	5030 199	42 748	5180 270	53 643	48
13	4728 071	11 660	4881 136	8727 801	5032 713	41 284	5182 758	52 135	47
14	4730 634	10 284	4883 674	8726 381	5035 227	39 820	5185 246	50 627	46
15	4733 197	08 907	4886 212	8724 960	5037 740	38 355	5187 733	49 119	45
16	4735 759	07 530	4888 750	8723 538	5040 252	36 889	5190 219	47 609	44
17	4738 321	06 152	4891 288	8722 116	5042 765	35 423	5192 705	46 099	43
18	4740 882	04 774	4893 825	8720 693	5045 276	33 956	5195 191	44 588	42
19	4743 443	03 394	4896 361	8719 269	5047 788	32 488	5197 676	43 077	41
20	4746 004	02 014	4898 897	8717 844	5050 298	31 019	5200 161	41 564	40
21	4748 564	00 633	4901 433	8716 419	5052 809	29 549	5202 646	40 051	39
22	4751 124	8799 251	4903 968	8714 993	5055 319	28 079	5205 130	38 538	38
23	4753 683	97 869	4906 503	8713 566	5057 828	26 608	5207 613	37 023	37
24	4756 242	96 486	4909 038	8712 138	5060 338	25 137	5210 096	35 508	36
25	4758 801	95 102	4911 572	8710 710	5062 846	23 664	5212 579	33 992	35
26	4761 359	93 717	4914 105	8709 281	5065 355	22 191	5115 061	32 475	34
27	4763 917	92 332	4916 638	8707 851	5067 863	20 717	5217 543	30 958	33
28	4766 474	90 946	4919 171	8706 420	5070 370	19 243	5220 024	29 440	32
29	4769 031	89 559	4921 704	8704 989	5072 877	17 768	5222 505	27 921	31
30	4771 588	88 171	4924 236	8703 557	5075 384	16 292	5224 986	26 402	30
31	4774 144	86 783	4926 767	8702 124	5077 890	14 815	5227 466	24 881	29
32	4776 700	85 394	4929 298	8700 691	5080 396	13 337	5229 945	23 360	28
33	4779 255	84 004	4931 829	8699 256	5082 901	11 859	5232 424	21 839	27
34	4781 810	82 613	4934 359	8697 821	5085 406	10 380	5234 903	20 316	26
35	4784 364	81 222	4936 889	8696 386	5087 910	08 901	5237 381	18 793	25
36	4786 919	79 830	4939 419	8694 949	5090 414	07 420	5239 859	17 269	24
37	4789 472	78 437	4941 948	8693 512	5092 918	05 939	5242 336	15 745	23
38	4792 026	77 043	4944 476	8692 074	5095 421	04 457	5244 813	14 219	22
39	4794 579	75 649	4947 005	8690 636	5097 924	02 975	5247 290	12 693	21
40	4797 131	74 254	4949 532	8689 196	5100 426	01 491	5249 766	11 167	20
41	4799 683	72 858	4952 060	8687 756	5102 928	00 007	5252 241	09 639	19
42	4802 235	71 462	4954 587	8686 315	5105 429	8598 523	5254 717	08 111	18
43	4804 786	70 064	4957 113	8684 874	5107 930	97 037	5257 191	06 582	17
44	4807 337	68 666	4959 639	8683 431	5110 431	95 551	5259 665	05 053	16
45	4809 888	67 268	4962 165	8681 988	5112 931	94 064	5262 139	03 522	15
46	4812 438	65 868	4964 690	8680 544	5115 431	92 576	5264 613	01 991	14
47	4814 987	64 468	4967 215	8679 100	5117 930	91 088	5267 085	00 459	13
48	4817 537	63 067	4969 740	8677 655	5120 429	89 599	5269 558	8498 927	12
49	4820 086	61 665	4972 264	8676 209	5122 927	88 109	5272 030	97 394	11
50	4822 634	60 263	4974 787	8674 762	5125 425	86 619	5274 502	95 860	10
51	4825 182	58 859	4977 310	8673 314	5127 923	85 127	5276 973	94 325	9
52	4827 730	57 455	4979 833	8671 866	5130 420	83 635	5279 443	92 790	8
53	4830 277	56 051	4982 355	8670 417	5132 916	82 143	5281 914	91 254	7
54	4832 824	54 645	4984 877	8668 967	5135 413	80 640	5284 383	89 717	6
55	4835 370	53 239	4987 399	8667 517	5137 908	79 155	5286 853	88 179	5
56	4837 916	51 832	4989 920	8666 066	5140 404	77 660	5289 322	86 641	4
57	4840 462	50 425	4992 441	8664 614	5142 899	76 164	5291 790	85 102	3
58	4843 007	49 016	4994 961	8663 161	5145 393	74 668	5294 258	83 562	2
59	4845 552	47 607	4997 481	8661 708	5147 887	73 171	5296 726	82 022	1
60	4848 096	46 197	5000 000	8660 254	5150 381	71 673	5299 193	80 481	0

M	32 Deg.	57 Deg.	33 Deg.	56 Deg.	34 Deg.	55 Deg.	35 Deg.	54 Deg.	M
0	5299.193	8480.481	5446.390	8386.706	5591.929	8290 376	5735.764	8191.520	60
1	5301 659	78 939	48 830	85 121	5594 340	88 749	38 147	8189 852	59
2	5304 125	77 397	51 269	83 536	5596 751	87 121	40 529	8188 182	58
3	5306 591	75 853	53 707	81 950	5599 162	85 493	42 911	8186 512	57
4	5309 057	74 309	56 145	80 363	5601 572	83 864	45 292	8184 841	56
5	5311 521	72 765	58 583	78 775	5603 981	82 234	47 672	8183 169	55
6	5313 986	71 219	61 020	77 187	5606 390	80 603	50 053	8181 497	54
7	5316 450	69 673	63 456	75 598	5608 798	78 972	52 432	8179 824	53
8	5318 913	68 126	65 892	74 009	5611 206	77 340	54 811	8178 151	52
9	5321 376	66 579	68 328	72 418	5613 614	75 708	57 190	8176 476	51
10	5323 839	65 030	70 763	70 827	5616 021	74 074	59 568	8174 801	50
11	5326 301	63 481	73 198	69 236	5618 428	72 440	61 946	8173 125	49
12	5328 763	61 932	75 632	67 643	5620 834	70 806	64 323	8171 449	48
13	5331 224	60 381	78 066	66 050	5623 239	69 170	66 700	8169 772	47
14	5333 685	58 830	80 499	64 456	5625 645	67 534	69 076	8168 094	46
15	5336 145	57 278	82 932	62 862	5628 049	65 897	71 452	8166 416	45
16	5338 605	55 726	85 365	61 266	5630 453	64 260	73 827	8164 736	44
17	5341 065	54 172	87 797	59 670	5632 857	62 622	76 202	8163 056	43
18	5343 523	52 618	90 228	58 074	5635 260	60 983	78 576	8161 376	42
19	5345 982	51 064	92 659	56 476	5637 663	59 343	80 950	8159 695	41
20	5348 440	49 508	95 090	54 878	5640 066	57 703	83 323	8158 013	40
21	5350 898	47 952	97 520	53 279	5642 467	56 062	85 696	8156 330	39
22	5353 355	46 395	99 950	51 680	5644 869	54 420	88 069	8154 647	38
23	5355 812	44 838	5502 379	50 080	5647 270	52 778	90 440	8152 963	37
24	5358 268	43 279	04 807	48 479	5649 670	51 135	92 812	8151 278	36
25	5360 724	41 720	07 236	46 877	5652 070	49 491	95 183	8149 593	35
26	5363 179	40 161	09 663	45 275	5654 469	47 847	97 553	8147 906	34
27	5365 634	38 600	12 091	43 672	5656 868	46 202	99 923	8146 220	33
28	5368 089	37 039	14 518	42 068	5659 267	44 556	5802 292	8144 532	32
29	5370 543	35 477	16 944	40 463	5661 665	42 909	04 661	8142 844	31
30	5372 996	33 914	19 370	38 858	5664 062	41 262	07 030	8141 155	30
31	5375 449	32 351	21 795	37 252	5666 459	39 614	09 397	8139 466	29
32	5377 902	30 787	24 220	35 646	5668 856	37 965	11 765	8137 775	28
33	5380 354	29 222	26 645	34 038	5671 252	36 316	14 132	8136 084	27
34	5382 806	27 657	29 069	32 430	5673 648	34 666	16 498	8134 393	26
35	5385 257	26 091	31 492	30 822	5676 043	33 015	18 864	8132 701	25
36	5387 708	24 524	33 915	29 212	5678 437	31 364	21 230	8131 008	24
37	5390 158	22 956	36 338	27 602	5680 832	29 712	23 595	8129 314	23
38	5392 608	21 388	38 760	25 991	5683 225	28 059	25 959	8127 620	22
39	5395 058	19 819	41 182	24 380	5685 619	26 405	28 323	8125 925	21
40	5397 507	18 249	43 603	22 768	5688 011	24 751	30 687	8124 229	20
41	5399 955	16 679	46 024	21 155	5690 403	23 096	33 050	8122 532	19
42	5402 403	15 108	48 444	19 541	5692 795	21 440	35 412	8120 835	18
43	5404 851	13 536	50 864	17 927	5695 187	19 784	37 774	8119 137	17
44	5407 298	11 963	53 283	16 312	5697 577	18 127	40 136	8117 439	16
45	5409 745	10 390	55 702	14 696	5699 968	16 469	42 497	8115 740	15
46	5412 191	08 816	58 121	13 080	5702 357	14 811	44 857	8114 040	14
47	5414 637	07 241	60 539	11 463	5704 747	13 152	47 217	8112 339	13
48	5417 082	05 666	62 956	09 845	5707 136	11 492	49 577	8110 638	12
49	5419 527	04 090	65 373	08 226	5709 524	09 832	51 936	8108 936	11
50	5421 971	02 513	67 790	06 607	5711 912	08 170	54 294	8107 234	10
51	5424 415	00 936	70 206	04 987	5714 299	06 509	56 652	8105 530	9
52	5426 859	8399 357	72 621	03 366	5716 686	04 846	59 010	8103 826	8
53	5429 302	97 778	75 036	01 745	5719 073	03 183	61 367	8102 122	7
54	5431 744	96 199	77 451	00 123	5721 459	01 519	63 724	8100 416	6
55	5434 187	94 618	79 865	8298 500	5723 844	8199 854	66 080	8198 710	5
56	5436 628	93 037	82 279	96 877	5726 229	98 185	68 435	8097 004	4
57	5439 069	91 455	84 692	95 252	5728 614	96 529	70 790	8095 296	3
58	5441 510	89 873	87 105	93 628	5730 998	94 856	73 145	8093 568	2
59	5443 951	88 290	89 517	92 002	5733 381	93 189	75 499	8091 879	1
60	5446 390	86 706	91 929	90 376	5735 764	91 527	77 853	8090 170	0

M	36 Deg.	53 Deg.	37 Deg.	52 Deg.	38 Deg.	51 Deg.	39 Deg.	50 Deg.	M
0	5877.853	8090.170	6018.150	7986.355	6156.615	7880.108	6293.204	7771.460	60
1	5880 206	88 460	20 473	84 604	6158 907	78 316	6295 464	69 629	59
2	5882 558	86 749	22 795	82 853	6161 198	76 524	6297 724	67 797	58
3	5884 910	85 037	25 117	81 100	6163 489	74 732	6299 983	65 965	57
4	5887 262	83 325	27 439	79 347	6165 780	72 939	6302 242	64 132	56
5	5889 613	81 612	29 760	77 594	6168 069	71 145	04 500	62 298	55
6	5891 964	79 899	32 080	75 839	6170 359	69 350	06 758	60 464	54
7	5894 314	78 185	34 400	74 084	6172 648	67 555	09 015	58 629	53
8	5896 663	76 470	36 719	72 329	6174 936	65 759	11 272	56 794	52
9	5899 012	74 754	39 038	70 572	6177 224	63 963	13 528	54 957	51
10	5901 361	73 038	41 356	68 815	6179 511	62 165	15 784	53 121	50
11	03 709	71 321	43 674	67 058	6181 798	60 367	18 039	51 283	49
12	06 057	69 603	45 991	65 299	6184 084	58 569	20 293	49 445	48
13	08 404	67 885	48 308	63 540	6186 370	56 770	22 547	47 606	47
14	10 750	66 166	50 624	61 780	6188 655	54 970	24 800	45 767	46
15	13 096	64 446	52 940	60 020	6190 939	53 169	27 053	43 926	45
16	15 442	62 726	55 255	58 259	6193 224	51 368	29 306	42 086	44
17	17 787	61 005	57 570	56 497	6195 507	49 566	31 557	40 244	43
18	20 132	59 283	59 884	54 735	6197 790	47 764	33 809	38 402	42
19	22 476	57 560	62 198	52 972	6200 073	45 961	36 059	36 559	41
20	24 819	55 837	64 511	51 208	02 355	44 157	38 310	34 716	40
21	27 163	54 113	66 824	49 444	04 636	42 352	40 559	32 872	39
22	29 505	52 389	69 136	47 678	06 917	40 547	42 808	31 027	38
23	31 847	50 664	71 447	45 913	09 198	38 741	45 057	29 182	37
24	34 189	48 938	73 758	44 146	11 478	36 935	47 305	27 336	36
25	36 530	47 211	76 069	42 379	13 757	35 127	49 553	25 489	35
26	38 871	45 484	78 379	40 611	16 036	33 320	51 800	23 642	34
27	41 211	43 756	80 689	38 843	18 314	31 511	54 046	21 794	33
28	43 550	42 028	82 998	37 074	20 592	29 702	56 292	19 945	32
29	45 889	40 299	85 306	35 304	22 870	27 892	58 537	18 096	31
30	48 228	38 569	87 614	33 533	25 146	26 082	60 782	16 246	30
31	50 566	36 838	89 922	31 762	27 423	24 270	63 026	14 395	29
32	52 904	35 107	92 229	29 990	29 698	22 459	65 270	12 544	28
33	55 241	33 375	94 535	28 218	31 974	20 646	67 513	10 692	27
34	57 577	31 642	96 841	26 445	34 248	18 833	69 756	08 840	26
35	59 913	29 909	6199 147	24 671	36 522	17 019	71 998	06 986	25
36	62 249	28 175	01 452	22 896	38 796	15 205	74 240	05 132	24
37	64 584	26 440	03 756	21 121	41 069	13 390	76 481	03 278	23
38	66 918	24 705	06 060	19 345	43 342	11 574	78 721	01 423	22
39	69 252	22 969	08 363	17 569	45 614	09 757	80 961	7699 567	21
40	71 586	21 232	10 666	15 792	47 885	07 940	83 201	97 710	20
41	73 919	19 495	12 969	14 014	50 156	06 123	85 440	95 853	19
42	76 251	17 756	15 270	12 235	52 427	04 304	87 678	93 996	18
43	78 583	16 018	17 572	10 456	54 696	02 485	89 916	92 137	17
44	80 915	14 278	19 873	08 676	56 966	00 665	92 153	90 278	16
45	83 246	12 538	22 173	06 896	59 235	7798 845	94 390	88 418	15
46	85 577	10 797	24 473	05 115	61 503	7797 024	96 626	86 558	14
47	87 906	09 056	26 772	03 333	63 771	7795 202	98 862	84 697	13
48	90 236	07 314	29 071	01 550	66 038	7793 380	6401 097	82 835	12
49	92 565	05 571	31 369	7899 767	68 305	7791 557	6403 332	80 973	11
50	94 893	03 827	33 666	7897 983	70 571	7789 733	6405 566	79 110	10
51	97 221	02 083	35 964	7896 198	72 837	7787 909	6407 799	77 246	9
52	99 549	00 338	38 260	7894 413	75 102	7786 084	6410 032	75 382	8
53	01 876	7998 593	40 556	7892 627	77 366	7784 258	6412 264	73 517	7
54	04 202	7996 847	42 852	7890 841	79 631	7782 431	6414 496	71 652	6
55	06 528	7995 100	45 147	7889 054	81 894	7780 604	6416 728	69 785	5
56	08 854	7993 352	47 442	7887 266	84 157	7778 777	6418 958	67 918	4
57	11 179	7991 604	49 736	7885 477	86 420	7776 949	6421 189	66 051	3
58	13 503	7989 855	52 029	7883 688	88 682	7775 120	6423 418	64 183	2
59	15 827	7988 105	54 322	7881 898	90 943	7773 290	6425 647	62 314	1
60	18 150	7986 355	56 615	7880 108	93 204	7771 460	6427 876	60 444	0

M	40 Deg.	49 Deg.	41 Deg.	48 Deg.	42 Deg.	47 Deg.	43 Deg.	46 Deg.	M
0	6427.876	7660.444	6560.590	7547.096	6691.306	7431.448	6819.984	7313.537	60
1	30 104	58 574	62 785	45 187	6693 468	29 502	22 111	7311 553	59
2	32 332	56 704	64 980	43 278	6695 628	27 554	24 237	7309 568	58
3	34 559	54 832	67 174	41 368	6697 789	25 606	26 363	7307 583	57
4	36 785	52 960	69 367	39 457	6699 948	23 658	28 489	7305 597	56
5	39 011	51 087	71 560	37 546	6702 108	21 708	30 613	7303 610	55
6	41 236	49 214	73 752	35 634	04 266	19 758	32 738	7301 623	54
7	43 461	47 340	75 944	33 721	06 424	17 808	34 861	7299 635	53
8	45 685	45 465	78 135	31 808	08 582	15 857	36 984	97 646	52
9	47 909	43 590	80 326	29 894	10 739	13 905	39 107	95 657	51
10	50 132	41 714	82 516	27 980	12 895	11 953	41 229	93 668	50
11	52 355	39 838	84 706	26 065	15 051	10 000	43 350	91 677	49
12	54 577	37 960	86 895	24 149	17 206	08 046	45 471	89 686	48
13	56 798	36 082	89 083	22 233	19 361	06 092	47 591	87 695	47
14	59 019	34 204	91 271	20 316	21 515	04 137	49 711	85 703	46
15	61 240	32 325	93 458	18 398	23 668	02 181	51 830	83 710	45
16	63 460	30 445	95 645	16 480	25 821	00 225	53 948	81 716	44
17	65 679	28 564	97 831	14 561	27 973	7398 268	56 066	79 722	43
18	67 898	26 683	6600 017	12 641	30 125	96 311	58 184	77 728	42
19	70 116	24 802	02 202	10 721	32 276	94 353	60 300	75 732	41
20	72 334	22 919	04 386	08 800	34 427	92 394	62 416	73 736	40
21	74 551	21 036	06 570	06 879	36 577	90 435	64 532	71 740	39
22	76 767	19 152	08 754	04 957	38 727	88 475	66 647	69 743	38
23	78 984	17 268	10 936	03 034	40 876	86 515	68 761	67 745	37
24	81 199	15 383	13 119	01 111	43 024	84 553	70 875	65 747	36
25	83 414	13 497	15 300	7499 187	45 172	82 592	72 988	63 748	35
26	85 628	11 611	17 482	97 262	47 319	80 629	75 101	61 748	34
27	87 842	09 724	19 662	95 337	49 466	78 666	77 213	59 748	33
28	90 056	07 837	21 842	93 411	51 612	76 703	79 325	57 747	32
29	92 268	05 949	24 022	91 484	53 757	74 738	81 435	55 746	31
30	94 480	04 060	26 200	89 557	55 902	72 773	83 546	53 744	30
31	96 692	02 170	28 379	87 629	58 046	70 808	85 655	51 741	29
32	98 903	00 280	30 557	85 701	60 190	68 842	87 765	49 738	28
33	6501 114	7598 389	32 734	83 772	62 333	66 875	89 873	47 734	27
34	03 324	96 498	34 910	81 842	64 476	64 908	91 981	45 729	26
35	05 533	94 606	37 087	79 912	66 618	62 940	94 089	43 724	25
36	07 742	92 713	39 262	77 981	68 760	60 971	96 195	41 719	24
37	09 951	90 820	41 437	76 049	70 901	59 002	98 302	39 712	23
38	12 158	88 926	43 612	74 117	73 041	57 032	6900 407	37 705	22
39	14 366	87 031	45 785	72 184	75 181	55 061	02 512	35 698	21
40	16 572	85 136	47 959	70 251	77 320	53 090	04 617	33 690	20
41	18 778	83 240	50 131	68 317	79 459	51 118	06 721	31 681	19
42	20 984	81 343	52 304	66 382	81 597	49 146	08 824	29 671	18
43	23 189	79 446	54 475	64 446	83 734	47 173	10 927	27 661	17
44	25 394	77 548	56 646	62 510	85 871	45 199	13 029	25 651	16
45	27 598	75 650	58 817	60 574	88 007	43 225	15 131	23 640	15
46	29 801	73 751	60 987	58 636	90 143	41 250	17 232	21 628	14
47	32 004	71 851	63 156	56 699	92 278	39 275	19 332	19 615	13
48	34 206	69 951	65 325	54 760	94 413	37 299	21 432	17 602	12
49	36 408	68 050	67 493	52 821	96 547	35 322	23 531	15 589	11
50	38 609	66 148	69 661	50 881	98 681	33 345	25 630	13 574	10
51	40 810	64 246	71 828	48 941	6800 813	31 367	27 728	11 559	9
52	43 010	62 343	73 994	46 999	02 946	29 388	29 825	09 544	8
53	45 209	60 439	76 160	45 058	05 078	27 409	31 922	07 528	7
54	47 408	58 535	78 326	43 115	07 209	25 429	34 018	05 511	6
55	49 607	56 630	80 490	41 173	09 339	23 449	36 114	03 494	5
56	51 804	54 724	82 655	39 229	11 469	21 467	38 209	01 476	4
57	54 002	52 818	84 818	37 285	13 599	19 486	40 304	7199 457	3
58	56 198	50 911	86 981	35 340	15 728	17 503	42 398	7197 438	2
59	58 395	49 004	89 144	33 394	17 856	15 521	44 491	7195 418	1
60	60 590	47 096	91 306	31 448	19 984	13 537	46 584	7193 398	0

M	44 Deg.	45 Deg.	M
0	6946.584	7193.398	60
1	48 676	91 377	59
2	50 767	89 355	58
3	52 858	87 333	57
4	54 949	85 310	56
5	57 039	83 287	55
6	59 128	81 263	54
7	61 217	79 238	53
8	63 305	77 213	52
9	65 392	75 187	51
10	67 479	73 161	50
11	69 565	71 134	49
12	71 651	69 106	48
13	73 736	67 078	47
14	75 821	65 049	46
15	77 905	63 019	45
16	79 988	60 989	44
17	82 071	58 959	43
18	84 153	56 927	42
19	86 234	54 895	41
20	88 315	52 863	40
21	90 396	50 830	39
22	92 476	48 796	38
23	94 555	46 762	37
24	96 633	44 727	36
25	98 711	42 691	35
26	7000,789	40 655	34
27	02 866	38 618	33
28	04 942	36 581	32
29	07 018	34 543	31
30	09 093	32 504	30
31	11 167	30 465	29
32	13 241	28 426	28
33	15 314	26 385	27
34	17 387	24 344	26
35	19 459	22 303	25
36	21 531	20 260	24
37	23 601	18 218	23
38	25 672	16 174	22
39	27 741	14 130	21
40	29 811	12 086	20
41	31 879	10 041	19
42	33 947	07 995	18
43	36 014	05 948	17
44	38 081	03 901	16
45	40 147	01 854	15
46	42 213	7099 806	14
47	44 278	7097 757	13
48	46 342	7095 707	12
49	48 406	7093 637	11
50	50 469	7091 607	10
51	52 532	7089 556	9
52	54 594	7087 504	8
53	56 655	7085 451	7
54	58 716	7083 398	6
55	60 776	7081 345	5
56	62 835	7079 291	4
57	64 894	7077 230	3
58	66 953	7075 180	2
59	69 011	7073 124	1
60	71 068	7071 068	0

A

T A B L E

O F

LOGARITHMIC VERSED SINES

To every Minute of the Quadrant.

M	0 Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	5 Deg.	M
0		6.1827137	6.7847406	7.1368680	7.3866683	7.5803891	0
1	2 6264222	1970705	7919481	1416791	3902785	5832778	1
2	3 2284822	2111938	7990963	1464636	3938736	5861568	2
3	3 5806647	2250912	8061861	1512219	3974539	5890263	3
4	3 8305422	2387696	8132185	1559542	4010196	5918864	4
5	4 0243622	2522360	8201944	1606609	4045706	5947370	5
6	4 1827246	2654968	8271147	1653422	4081071	5975783	6
7	4 3166182	2785581	8339803	1699984	4116293	6004103	7
8	4 4326020	2914259	8407920	1746297	4151372	6032331	8
9	4 5349070	3041058	8475507	1792365	4186311	6060468	9
10	4 6264219	3166033	8542572	1838189	4221109	6088513	10
11	4 7092072	3289235	8609123	1883773	4255767	6116468	11
12	4 7847843	3410714	8675167	1929118	4290288	6144333	12
13	4 8543084	3530516	8740714	1974228	4324673	6172109	13
14	4 9186777	3648689	8805768	2019104	4358921	6199796	14
15	4 9786041	3765275	8870340	2063750	4393035	6227395	15
16	5 0346614	3880317	8934434	2108167	4427015	6254906	16
17	5 0873192	3993855	8998059	2152358	4460862	6282330	17
18	5 1369663	4105928	9061221	2196326	4494578	6309668	18
19	5 1839283	4216573	9123927	2240071	4528163	6336920	19
20	5 2284810	4325826	9186183	2283597	4561619	6364086	20
21	5 2708595	4433722	9247996	2326906	4594946	6391167	21
22	5 3112661	4540294	9309372	2370000	4628146	6418164	22
23	5 3498763	4645573	9370317	2412881	4661219	6445078	23
24	5 3868430	4749592	9430837	2455551	4694166	6471908	24
25	5 4223003	4852380	9490939	2498013	4726989	6498655	25
26	5 4563669	4953965	9550627	2540267	4759688	6525320	26
27	5 4891475	5054376	9609907	2582317	4792264	6551903	27
28	5 5207359	5153639	9668786	2624164	4824719	6578404	28
29	5 5512157	5251780	9727268	2665810	4857052	6604825	29
30	5 5806620	5348825	9785359	2707258	4889265	6631166	30
31	5 6091427	5444797	9843063	2748508	4921359	6657427	31
32	5 6367191	5539720	9900387	2789563	4953335	6683608	32
33	5 6634468	5633616	9957334	2830425	4985193	6709711	33
34	5 6893765	5726509	7.0013911	2871095	5016934	6735735	34
35	5 7145546	5818418	0070121	2911576	5048560	6761682	35
36	5 7390233	5909365	0125969	2951869	5080071	6787550	36
37	5 7628215	5999369	0181461	2991975	5111468	6813342	37
38	5 7859850	6088450	0236600	3031897	5142751	6839058	38
39	5 8085468	6176626	0291391	3071636	5173923	6864697	39
40	5 8305373	6263916	0345838	3111194	5204982	6890260	40
41	5 8519848	6350337	0399946	3150572	5235931	6915749	41
42	5 8729154	6435907	0453719	3189773	5266769	6941162	42
43	5 8933535	6520642	0507161	3228797	5297498	6966502	43
44	5 9133217	6604558	0560276	3267646	5328119	6991767	44
45	5 9328411	6687671	0613068	3306322	5358632	7016959	45
46	5 9519314	6769996	0665540	3344827	5389038	7042078	46
47	5 9706112	6851547	0717698	3383161	5419338	7067124	47
48	5 9888977	6932340	0769544	3421327	5449532	7092098	48
49	6 0068070	7012389	0821082	3459326	5479621	7117001	49
50	6 0243546	7091706	0872316	3497159	5509607	7141832	50
51	6 0415546	7170305	0923249	3534828	5539489	7166592	51
52	6 0584206	7248199	0973885	3572334	5569268	7191281	52
53	6 0749654	7325400	1024228	3609678	5598946	7215900	53
54	6 0912008	7401921	1074280	3646863	5628522	7240450	54
55	6 1071384	7477774	1124045	3683888	5657998	7264930	55
56	6 1227887	7552970	1173527	3720757	5687373	7289341	56
57	6 1381620	7627520	1222728	3757469	5716650	7313683	57
58	6 1532679	7701436	1271652	3794027	5745828	7337958	58
59	6 1681156	7774728	1320302	3830431	5774908	7362164	59
60	6 1827137	7847406	1368680	3866683	5803891	7386303	60

M	6 Deg.	7 Deg.	8 Deg.	9 Deg.	10 Deg.	11 Deg.	M
0	7.7386303	7.8723806	7.9881990	8.0903166	8.1816220	8.2641757	0
1	7410375	8744436	7 9900038	0919203	1830648	2654867	1
2	7434380	8765017	7 9918047	0935210	1845051	2667957	2
3	7458319	8785550	7 9936020	0951188	1859431	2681028	3
4	7482192	8806033	7 9953955	0967136	1873786	2694078	4
5	7505999	8826469	7 9971853	0983055	1888118	2707109	5
6	7529742	8846856	7 9989713	0998944	1902426	2720119	6
7	7553419	8867196	8.0007537	1014804	1916710	2733111	7
8	7577031	8887487	0025325	1030635	1930971	2746082	8
9	7600580	8907732	0043076	1046437	1945208	2759035	9
10	7624064	8927928	0060790	1062211	1959421	2771967	10
11	7647485	8948078	0078468	1077955	1973611	2784880	11
12	7670843	8968181	0096110	1093671	1987778	2797774	12
13	7694138	8988238	0113716	1109358	2001921	2810649	13
14	7717371	9008248	0131287	1125017	2016042	2823504	14
15	7740541	9028212	0148822	1140647	2030139	2836341	15
16	7763649	9048130	0166321	1156249	2044213	2849158	16
17	7786696	9068002	0183785	1171823	2058264	2861956	17
18	7809682	9087829	0201213	1187369	2072293	2874735	18
19	7832607	9107610	0218607	1202887	2086298	2887495	19
20	7855472	9127346	0235965	1218377	2100281	2900236	20
21	7878276	9147038	0253289	1233840	2114241	2912958	21
22	7901020	9166684	0270578	1249274	2128179	2925661	22
23	7923705	9186286	0287833	1264681	2142094	2938346	23
24	7946331	9205844	0305053	1280061	2155987	2951012	24
25	7968897	9225358	0322239	1295413	2169857	2963660	25
26	7991405	9244827	0339391	1310738	2183705	2976289	26
27	8013855	9264253	0356508	1326036	2197531	2988899	27
28	8036246	9283636	0373592	1341307	2211334	3001491	28
29	8058580	9302975	0390643	1356551	2225116	3014064	29
30	8080856	9322271	0407659	1371768	2238875	3026619	30
31	8103075	9341523	0424642	1386958	2252613	3039156	31
32	8125237	9360734	0441592	1402121	2266329	3051675	32
33	8147343	9379901	0458509	1417258	2280023	3064175	33
34	8169392	9399027	0475393	1432368	2293695	3076657	34
35	8191386	9418110	0492243	1447452	2307345	3089122	35
36	8213323	9437151	0509061	1462510	2320974	3101568	36
37	8235205	9456150	0525846	1477541	2334581	3113996	37
38	8257032	9475107	0542599	1492546	2348167	3126406	38
39	8278804	9494023	0559319	1507525	2361732	3138798	39
40	8300522	9512898	0576007	1522478	2375275	3151172	40
41	8322185	9531732	0592663	1537405	2388797	3163529	41
42	8343794	9550525	0609286	1552307	2402297	3175868	42
43	8365349	9569276	0625878	1567182	2415777	3188189	43
44	8386851	9587988	0642438	1582032	2429235	3200493	44
45	8408299	9606659	0658966	1596857	2442673	3212779	45
46	8429695	9625290	0675463	1611656	2456089	3225047	46
47	8451037	9643880	0691928	1626430	2469485	3237298	47
48	8472327	9662431	0708362	1641178	2482860	3249532	48
49	8493565	9680942	0724764	1655902	2496214	3261748	49
50	8514758	9699414	0741136	1670600	2509547	3273947	50
51	8535885	9717846	0757476	1685273	2522860	3286128	51
52	8556968	9736239	0773786	1699921	2536152	3298292	52
53	8577999	9754593	0700065	1714545	2549424	3310439	53
54	8598980	9772908	0806313	1729144	2562675	3322569	54
55	8619910	9791184	0822531	1743718	2575906	3334682	55
56	8640789	9809422	0838718	1758267	2589117	3346778	56
57	8661618	9827621	0854875	1772792	2602307	3358857	57
58	8682397	9845782	0871002	1787292	2615477	3370918	58
59	8703126	9863905	0887099	1801768	2628627	3382963	59
60	8723806	9881990	0903166	1816220	2641757	3394991	60

M	12 Deg.	13 Deg.	14 Deg.	15 Deg.	16 Deg.	17 Deg.	M
0	8.3394991	8.4087475	8.4728189	8.5324253	8.5881406	8.6404342	0
1	3407002	4098556	4738472	5333844	5890390	6412791	1
2	3418997	4109622	4748742	5343423	5899365	6421231	2
3	3430975	4120675	4759000	5353992	5908330	6429663	3
4	3442936	4131713	4769246	5362551	5917286	6438087	4
5	3454880	4142736	4779480	5372098	5926233	6446502	5
6	2466808	4153746	4789701	5381635	5935170	6454909	7
7	3478719	4164741	4799910	5391161	5944097	6463308	6
8	3490614	4175723	4810107	5400677	5953016	6471698	8
9	3502492	4186690	4820291	5410182	5961925	6480080	9
10	3514354	4197644	4830464	5419676	5970824	6488454	10
11	3526200	4208583	4840625	5429160	5979715	6496820	11
12	3538029	4219508	4850773	5438633	5988596	6505177	12
13	3549842	4230420	4860910	5448096	5997468	6513526	13
14	3561639	4241318	4871034	5457548	6006330	6521867	14
15	3573419	4252201	4881147	5466990	6015184	6530200	15
16	3585184	4263072	4891246	5476422	6024028	6538524	16
17	3596932	4273928	4901336	5485843	6032863	6546841	17
18	3608664	4284770	4911412	5495253	6041689	6555149	18
19	3620381	4295599	4921477	5504654	6050506	6563449	19
20	3632081	4306414	4931530	5514044	6059313	6571741	20
21	3643765	4317216	4941572	5523423	6068112	6580025	21
22	3655434	4328004	4951601	5532793	6076901	6588301	22
23	3667086	4338778	4961619	5542152	6085681	6596569	23
24	3678723	4349539	4971625	5551500	6094453	6604829	24
25	3690344	4360286	4981619	5560839	6103215	6613081	25
26	3701950	4371020	4991601	5570167	6111968	6621324	26
27	3713539	4381740	5001573	5579485	6120712	6629560	27
28	3725114	4392447	5011532	5588793	6129448	6637788	28
29	3736672	4403141	5021480	5598091	6138174	6646008	29
30	3748215	4413821	5031416	5607379	6146891	6654220	30
31	3759743	4424488	5041341	5616656	6155600	6662424	31
32	3771255	4435142	5051254	5625924	6164299	6670620	32
33	3782751	4445783	5061156	5635181	6172990	6678808	33
34	3794232	4456410	5071046	5644429	6181672	6686988	34
35	3805698	4467024	5080925	5653666	6190345	6695160	35
36	3817149	4477625	5090792	5662894	6199009	6703324	36
37	3828584	4488213	5100648	5672111	6207664	6711481	37
38	3840004	4498788	5110493	5681318	6216311	6719630	38
39	3851409	4509350	5120326	5690516	6224948	6727771	39
40	3862799	4519898	5130148	5699704	6233577	6735904	40
41	3874174	4530434	5139959	5708881	6242197	6744029	41
42	3885533	4540957	5149758	5718049	6250809	6753147	42
43	3896878	4551467	5159546	5727207	6259412	6760256	43
44	3908207	4561964	5169324	5736355	6268006	6768358	44
45	3919522	4572448	5179089	5745494	6276591	6776453	45
46	3930822	4582920	5188844	5754622	6285168	6784539	46
47	3942107	4593378	5198588	5763741	6293736	6792618	47
48	3953377	4603824	5208320	5772850	6302295	6800689	48
49	3964632	4614257	5218042	5781950	6310846	6808753	49
50	3975873	4624677	5227752	5791039	6319388	6816809	50
51	3987098	4635085	5237451	5800119	6327922	6824857	51
52	3998310	4645480	5247140	5809189	6336447	6832897	52
53	4009506	4655863	5256817	5818250	6344964	6840930	53
54	4020688	4666233	5266484	5827301	6353472	6848956	54
55	4031855	4676590	5276139	5836342	6361971	6856973	55
56	4043008	4686935	5285784	5845374	6370462	6864984	56
57	4054147	4697267	5295417	5854396	6378945	6872986	57
58	4065270	4707587	5305040	5863409	6387419	6880981	58
59	4076380	4717894	5314652	587 412	6395884	6888969	59
60	4087475	4728189	5324253	5881406	6404342	6896949	60

M	18 Deg.	19 Deg	20 Deg.	21 Deg.	22 Deg.	23 Deg.	M
0	8.6896949	8.7362485	8.7803705	8,8222961	8.8622277	8.9003406	0
1	6904921	7370030	7810866	8229774	8628774	9009613	1
2	6912886	7377570	7818022	8236582	8633265	9015816	2
3	6920844	7385102	7825171	8243385	8641752	9022013	3
4	6928794	7392628	7832314	8250182	8648233	9028207	4
5	6936736	7400147	7839452	8256973	8654710	9034395	5
6	6944672	7407659	7846583	8263759	8661181	9040579	6
7	6952599	7415165	7853708	8270539	8667648	9046759	7
8	6960520	7422664	7860827	8277314	8674109	9052934	8
9	6968432	7430156	7867940	8284083	8680566	9059104	9
10	6976338	7437642	7875047	8290848	8687018	9065270	10
11	6984236	7445121	7882149	8297606	8693464	9071431	11
12	6992127	7452593	7889244	8304360	8699906	9077588	12
13	7000010	7460059	7896333	8311107	8706342	9083740	13
14	7007886	7467518	7903416	8317850	8712774	9089887	14
15	7015755	7474971	7910494	8324587	8719201	9096030	15
16	7023617	7482417	7917565	8331318	8725623	9102169	16
17	7031471	7489857	7924630	8338044	8732040	9108303	17
18	7039318	7497290	7931690	8344765	8738452	9114432	18
19	7047158	7504716	7938743	8351480	8744859	6120557	19
20	7054990	7512136	7945791	8358190	8751261	9126678	20
21	7062815	7519549	7952833	8364895	8757658	9132794	21
22	7070633	7526956	7959869	8371594	8764051	9138905	22
23	7078444	7534357	7966899	8378288	8770438	9145012	23
24	7086247	7541751	7973923	8384976	8776821	9151115	24
25	7094044	7549138	7980941	8391660	8783198	9157213	25
26	7101833	7556519	7987953	8398337	8789571	9163306	26
27	7109615	7563894	7994960	8405010	8795939	9169396	27
28	7117390	7571262	8001961	8411677	8802303	9175480	28
29	7125157	7578623	8008956	8418339	8808661	9181561	29
30	7132918	7585979	8015945	8424996	8815014	9187636	30
31	7140671	7593327	8022926	8431647	8821363	9193708	31
32	7148418	7600670	8029906	8438294	8827707	9199775	32
33	7156157	7608006	8036877	8444934	8834046	9205837	33
34	7163889	7615336	8043843	8451570	8840380	9211895	34
35	7171614	7622659	8050803	8458200	8846710	9217949	35
36	7179332	7629976	8057758	8464826	8853034	9223999	36
37	7187044	7637286	8064707	8471445	8859354	9230043	37
38	7194748	7644591	8071649	8478000	8865669	9236084	38
39	7202445	7651889	8078587	8484670	8871980	9242120	39
40	7210135	7659180	8085518	8491274	8878285	9248152	40
41	7217818	7666466	8092444	8497873	8884586	9254179	41
42	7225494	7673745	8099364	8504467	8890882	9260202	42
43	7233163	7681018	8106278	8511055	8897173	9266221	43
44	7240825	7688284	8113187	8517639	8903460	9272235	44
45	7248480	7695544	8120090	8524217	8909742	9278245	45
46	7256129	7702798	8126988	8530790	8916019	9284251	46
47	7263770	7710046	8133879	8537358	8922291	9290252	47
48	7271404	7717288	8140765	8543921	8928559	9296249	48
49	7279032	7724523	8147646	8550479	8934822	9302242	49
50	7286653	7731752	8154521	8557032	8941080	9308231	50
51	7294267	7738975	8161390	8563579	8947334	9314215	51
52	7301874	7746192	8168253	8570121	8953583	9320194	52
53	7309474	7753403	8175111	8576659	8959827	9326170	53
54	7317067	7760607	8181964	8583191	8966066	9332141	54
55	7324654	7767805	8188810	8589718	8972301	9338108	55
56	7332233	7774997	8195652	8596240	8978532	9344070	56
57	7339806	7782183	8202487	8602757	8984757	9350029	57
58	7347373	7789363	8209317	8609268	8990978	9355983	58
59	7354932	7796537	8216142	8615775	8997194	9361933	59
60	7362485	7803705	8222961	8622277	9003406	9367878	60

M	24 Deg.	25 Deg.	26 Deg.	27 Deg.	28 Deg.	29 Deg.	M
0	8.9367878	8.9717035	9.0052061	9.0374005	9.0683803	9.0982293	0
1	9373819	9722731	0057531	0379265	0688869	0987176	1
2	9379756	9728424	0062997	0384522	0693931	0992057	2
3	9385689	9734113	0068460	0389776	0698990	0996934	3
4	9391618	9739797	0073920	0395026	0704046	1001809	4
5	9397542	9745478	0079375	0400273	0709099	1006681	5
6	9403462	9751155	0084827	0405517	0714148	1011549	6
7	9409378	9756828	0090276	0410757	0719195	1016415	7
8	9415290	9762497	0095721	0415994	0724238	1021278	8
9	9421197	9768163	0101162	0421228	0729279	1026138	9
10	9427101	9773824	0106600	0426458	0734316	1030995	10
11	9433000	9779482	0112034	0431685	0739350	1035850	11
12	9438895	9785135	0117465	0436908	0744381	1040701	12
13	9444785	9790785	0122892	0442129	0749409	1045550	13
14	9450672	9796431	0128315	0447345	0754434	1050395	14
15	9456554	9802073	0133735	0452559	0759455	1055238	15
16	9462433	9807711	0139151	0457769	0764474	1060078	16
17	9468307	9813346	0144564	0462976	0769490	1064915	17
18	9474177	9818976	0149973	0468180	0774502	1069749	18
19	9480042	9824603	0155378	0473380	0779511	1074580	19
20	9485904	9830226	0160781	0478578	0784518	1079408	20
21	9491761	9835845	0166179	0483771	0789521	1084234	21
22	9497615	9841460	0171574	0488962	0794521	1089056	22
23	9503464	9847072	0176965	0494149	0799518	1093876	23
24	9509309	9852679	0182353	0499333	0804512	1098693	24
25	9515150	9858283	0187738	0504514	0809503	1103507	25
26	9520987	9863883	0193119	0509691	0814491	1108318	26
27	9526820	9869480	0198496	0514865	0819476	1113126	27
28	9532648	9875072	0203870	0520036	0824458	1117932	28
29	9538473	9880661	0209240	0525204	0829437	1122735	29
30	9544294	9886246	0214607	0530368	0834413	1127534	30
31	9550110	9891827	0219970	0535529	0839386	1132331	31
32	9555922	9897404	0225330	0540687	0844356	1137126	32
33	9561731	9902978	0230687	0545842	0849322	1141917	33
34	9567535	9908548	0236039	0550993	0854286	1146705	34
35	9573335	9914114	0241389	0556141	0859247	1151491	35
36	9579131	9991676	0246735	0561286	0864204	1156274	36
37	9584923	9925235	0252077	0566428	0869159	1161054	37
38	9590711	9930790	0257416	0571566	0874111	1165831	38
39	9596495	9936341	0262752	0576701	0879059	1170606	39
40	9602275	9941888	0268084	0581833	0884005	1175377	40
41	9608051	9947432	0273412	0586962	0888948	1180146	41
42	9613823	9952972	0278738	0592088	0893887	1184912	42
43	9619591	9958508	0284059	0597210	0898824	1189675	43
44	9625355	9964041	0289378	0602329	0903758	1194436	44
45	9631114	9969569	0294692	0607445	0908088	1199193	45
46	9636870	9975095	0300004	0612558	0913616	1203948	46
47	9642622	9980616	0305312	0617668	0918541	1208700	47
48	9648370	9986134	0310616	0622774	0923462	1213449	48
49	9654114	9991648	0315917	0627877	0928381	1218196	49
50	9659854	9997158	0321215	0632977	0933297	1222939	50
51	9665590	9.0002665	0326509	0638074	0938210	1227680	51
52	9671322	0008168	0331800	0643168	0943120	1232419	52
53	9677050	0013667	0337088	0648258	0948027	1237154	53
54	9682774	0019163	0342372	0653346	0952931	1241887	54
55	9688494	0024655	0347652	0658430	0957832	1246617	55
56	9694210	0030144	0352930	0663511	0962730	1251344	56
57	9699922	0035628	0358204	0668589	0967625	1256068	57
58	9705630	0041109	0363474	0673663	0972517	1260790	58
59	9711335	0046587	0368741	0678735	0977406	1265568	59
60	9707035	0052061	03 7005	0683803	0982293	1270222	60

M	30 Deg.	31 Deg.	32 Deg.	33 Deg.	34 Deg.	35 Deg.	M
0	9.1270225	9.1548276	9.1817061	9 2077136	9.2329007	9.2573136	0
1	1274938	1552831	1821466	2081400	2333139	2577142	1
2	1279649	1557382	1825868	2085661	2337267	2581145	2
3	1284356	1561931	1830268	2089920	2341393	2585147	3
4	1289062	1566477	1834665	2094177	2345518	2589147	4
5	1293764	1571021	1839060	2098432	2349640	2593144	5
6	1298464	1575562	1843452	2102684	2353761	2597140	6
7	1303161	1580101	1847842	2106934	2357879	2601133	7
8	1307855	1584637	1852230	2111182	2361995	2605125	8
9	6312547	1589171	1856615	2115428	2366109	2609114	9
10	1317235	1593702	1860998	2119671	2370221	2613102	10
11	1321921	1598230	1865378	2123912	2374330	2617087	11
12	1326605	1602756	1869756	2128151	2378438	2621071	12
13	1331286	1607280	1874132	2132388	2382543	2625052	13
14	1335964	1611800	1878505	2136622	2386647	2629032	14
15	1340639	1616319	1882876	2140854	2390748	2633009	15
16	1345311	1620835	1887245	2145084	2394847	2636985	16
17	1349981	1625348	1891611	2149311	2398944	2640958	17
18	1354648	1629859	1895974	2153537	2403038	2644929	18
19	1359313	1634367	1900336	2157760	2407131	2648899	19
20	1363975	1638873	1904695	2161981	2411222	2652866	20
21	1368634	1643376	1909051	2166199	2415310	2656832	21
22	1373290	1647876	1913406	2170416	2419396	2660795	22
23	1377944	1652374	1917758	2174630	2423481	2664757	23
24	1382595	1656870	1922107	2178842	2427563	2668716	24
25	1387244	1661363	1926454	2183052	2431643	2672674	25
26	1391889	1665854	1930799	2187259	2435721	2676629	26
27	1396532	1670342	1935142	2191464	2439797	2680583	27
28	1401173	1674828	1939482	2195668	2443871	2684534	28
29	1405811	1679311	1943819	2199868	2447942	2688484	29
30	1410446	1683791	1948155	2204067	2452012	2692431	30
31	1415078	1688269	1952488	3208263	2456079	2696377	31
32	1419708	1692745	1956819	2212458	2460145	2700321	32
33	1424335	1697218	1961147	2216650	2464208	2704262	33
34	1428960	1701689	1965473	2220839	2468269	2718202	34
35	1433581	1706157	1969797	2225027	2472328	2712140	35
36	1438201	1710623	1974118	2229212	2476385	2716075	36
37	1442817	1715086	1978437	2233396	2480440	3720009	37
38	1447431	1719547	1982754	2237577	2484493	2723941	38
39	1452042	1724005	1987068	2241755	2488544	2727871	39
40	1456651	1728461	1991380	2245932	2492593	2731799	40
41	1461257	1732914	1995690	2250106	2496640	2735725	41
42	1465861	1737365	1999997	2254279	2500684	2739649	42
43	1470461	1741813	2004302	2258449	2504727	2743571	43
44	1475060	1746259	2008605	2262617	2508767	2747491	44
45	1479655	1750703	2012906	2266782	2512806	2751409	45
46	1484248	1755144	2017204	2270946	2516842	2755325	46
47	1488838	1759582	2021499	2275107	2520876	2759239	47
48	1493426	1764018	2025793	2279266	2524909	2763151	48
49	1498011	1768452	2030084	2283423	2528939	2767062	49
50	1502594	1772883	2034373	2287578	2532967	2770970	50
51	1507174	1777312	2038660	2291731	2536993	2774876	51
52	1511751	1781738	2042944	2295881	2541017	2778781	52
53	1516326	1786162	2047226	2300029	2545039	2782683	53
54	1520898	1790584	2051506	2304175	2549059	2786584	54
55	1525467	1795003	2055783	2308319	2553077	2790483	55
56	1530034	1799419	2060058	2312461	2557093	2794380	56
57	1534599	1803833	2064331	2316601	2561107	2798274	57
58	1539161	1808245	2068602	2320738	2565119	2802167	58
59	1543720	1812655	2072870	2324874	2569128	2806058	59
60	1548276	1817061	2077136	2329007	2573136	2809947	60

M	36 Deg.	37 Deg.	38 Deg.	39 Deg.	40 Deg.	41 Deg.	M
0	..2809947	9.3039829	9.3263138	9.3480205	9.3691334	9.3896806	0
1	2813834	3043604	3266806	3483772	3694804	3900184	1
2	2817720	3047376	3270473	3487337	3698272	3903561	2
3	2821603	3051148	3274137	3490900	3701739	3906936	3
4	2825484	3054917	3277800	3494462	3705205	3910309	4
5	2829364	3058684	3281461	3498022	3708669	3913682	5
6	2833241	3062450	3285121	3501580	3712131	3917052	6
7	2837117	3066214	3288778	3505137	3715592	3920421	7
8	2840990	3069976	3292434	3508692	3719051	3923789	8
9	2844862	3073736	3296089	3512246	3722508	3927155	9
10	2848732	3077494	3299741	3515798	3725965	3930520	10
11	2852600	3081251	3303392	3519348	3729419	3933883	11
12	2856466	3085006	33•7041	3522897	3732872	3937245	12
13	2860330	3088759	3310688	3526444	3736323	3940605	13
14	2864192	3092510	3314334	3529989	3739773	3943964	14
15	2868053	3096259	3317978	3533533	3743221	3947321	15
16	2871911	3100007	3321620	3537075	3746668	3950677	16
17	1875768	3103752	3325261	3540615	3750113	3954031	17
18	2879622	3107496	3328900	3544154	3753557	3957384	18
19	2883475	3111238	3332537	3547691	3756999	3960735	19
20	2887326	3114979	3336172	3551227	3760440	3964085	20
21	2891175	3118717	3339806	3554761	3763879	3967434	21
22	2895022	3122454	3343438	3558293	3767316	3970781	22
23	2898867	3126189	3347068	3561824	3770752	3974126	23
24	2902711	3129922	3350697	3565353	3774186	3977470	24
25	2906552	3133654	3354323	3568880	3777619	3980813	25
26	2910392	3137383	3357949	3572406	3781050	3984154	26
27	2914229	3141111	3361572	3575930	3784480	3987493	27
28	2918065	3144837	3365194	3579453	3787908	3990831	28
29	2921899	3148561	3368814	3582974	3791335	3994168	29
30	2925731	3152284	3372432	3586494	3794760	3997503	30
31	2929561	3156005	3376049	3590011	3798184	4000837	31
32	2933390	3159724	3379664	3593528	3801606	4004169	32
33	2937216	3163441	3383278	3597042	3805026	4007500	33
34	2941041	3167156	3386889	3600555	3808445	4010829	34
35	2944863	3170870	3390499	3604067	3811863	4014157	35
36	2948684	3174582	3394107	3607576	3815279	4017484	36
37	2952503	3178292	3397714	3611084	3818693	4020809	37
38	2956320	3182000	3401319	3614591	3822106	4024132	38
39	2960136	3185706	3404922	3618096	3825517	4027454	39
40	2963949	3189411	3408524	3621599	3828927	4030775	40
41	2967760	3193114	3412124	3625101	3832335	4034094	41
42	2971570	3196815	3415722	3628601	3835742	4037412	42
43	2975378	3200515	3419319	3632100	3839147	4040728	43
44	2979184	3204213	3422913	3635597	3842551	4044043	44
45	2982988	3207909	3426507	3639092	3845953	4047356	45
46	2986790	3211603	3430098	3642586	3849354	4050668	46
47	2990591	3215295	3433688	3646079	3852753	4053978	47
48	2994389	3218986	3437276	3649569	3856151	4057287	48
49	2998186	3222675	3440863	3653058	3859547	4060595	49
50	3001981	3226362	3444448	3656546	3862942	4063901	50
51	3005774	3230048	3448031	3660032	3866335	4067206	51
52	3009565	3233731	3451612	3663516	3869727	4070509	52
53	3013355	3237413	3455192	3666999	3873117	4073811	53
54	3017142	3241094	3458770	3670480	3876506	4077111	54
55	3020928	3244772	3462347	3673959	3879893	4080410	55
56	3024712	3248449	3465922	3677437	3883278	4083708	56
57	3028494	3252124	3469495	3680914	3886662	4087004	57
58	3032274	3255797	3473067	3684389	3890045	4090298	58
59	3036052	3259469	3476637	3687862	3893426	4093591	59
60	3039829	3263138	3480205	3691334	3896806	4096883	60

M	42 Deg.	43 Deg.	44 Deg.	45 Deg.	46 Deg.	47 Deg.	M
0	8.4096883	9.4291809	9.4481808	9.4667093	9.4847860	9.5024294	0
1	4100174	4295015	4484934	4670142	4850836	5027198	1
2	4103462	4298220	4488059	4673190	4853810	5030102	2
3	4106750	4301424	4491183	4676237	4856783	5033005	3
4	4110036	4304626	4494305	4679283	4859755	5035906	4
5	4113321	4307827	4497426	4682327	4862726	5038806	5
6	4116604	4311027	4500546	4685370	4865696	5041705	6
7	4119885	4314225	4503664	4688412	4868664	5044603	7
8	4123166	4317422	4506781	4691452	4871631	5047500	8
9	4126445	4320617	4509897	4694492	4874597	5050396	9
10	4129722	4323811	4513011	4697530	4877562	5053290	10
11	4132998	4327004	4516124	4700566	5880525	5056183	11
12	4136273	4330196	4519236	4703602	4883488	5059076	12
13	4139546	4333386	4522346	4706636	4886449	5061967	13
14	4142818	4336574	4525456	4709669	4889409	5064857	14
15	4146088	4339762	4528564	4712701	4892368	5067745	15
16	4149357	4342948	4531670	4715732	4895326	5070633	16
17	4152625	4346133	4534776	4718761	4898282	5073519	17
18	4155891	4349316	4537880	4721789	4901237	5076405	18
19	4159156	4352498	4540982	4724816	4904191	5079289	19
20	4162419	4355678	4554084	4727841	4907144	5082172	20
21	4165681	4358858	4547184	4730866	4910096	5085054	21
22	4168942	4362036	4550283	4733889	4913046	5087934	22
23	4172201	4 65212	4553380	4736911	4915996	5090814	23
24	4175459	4368387	4556477	4739932	4918944	5093693	24
25	4178715	4371561	4559572	4742951	4921891	5096570	25
26	4181970	4374734	4562665	4745969	4924836	5099446	26
27	4185223	4377905	4565758	4748986	4927781	5102321	27
28	4188475	4381075	4568849	4752002	4930724	5105195	28
29	4191726	4384243	4571939	4755016	4933667	5108068	29
30	4194975	4387411	4575027	4758030	4936608	5110940	30
31	4198223	4390576	4578115	4761042	4939547	5113810	31
32	4201470	4393741	4581201	4764052	4942486	5116679	32
33	4204715	4396904	4584286	4767062	4945424	5119548	33
34	4207959	4400066	4587369	4770070	4948360	5122415	34
35	4211201	4403227	4590451	4773078	4951295	5125281	35
36	4214442	4406386	4593532	4776083	4954229	5128146	36
37	4217681	4409544	4596612	4779088	4957162	5131009	37
38	4220920	4412700	4599690	4782092	4960093	5133872	38
39	4224156	4415855	4602767	4785094	4963024	5136734	39
40	4227392	4419009	4605843	4788095	4965953	5139594	40
41	4230626	4422162	4608918	4791095	4968881	5142453	41
42	4233858	4425313	4611991	4794093	4971808	5145311	42
43	4237089	4428463	4615063	4797091	4974734	5148168	43
44	4240319	4431611	4618134	4800087	4977658	5151024	44
45	4243548	4434758	4621203	4803082	4980582	5153879	45
46	4246775	4437904	4624271	4806075	4983504	5156733	46
47	4250000	4441049	4627338	4809068	4986425	5159585	47
48	4253225	4444192	4630404	4812059	4989345	5162436	48
49	4256447	4447334	4633468	4815049	4992264	5165287	49
50	4259669	4450475	4636531	4818038	4995182	5168136	50
51	4262889	4453614	4639593	4821026	4998098	5170984	51
52	4266108	4456752	4642654	4824012	5001013	5173831	52
53	4269325	4459889	4645713	4826997	5003927	5176677	53
54	4272541	4463024	4648771	4829981	5006840	5179521	54
55	4275756	4466158	4651828	4832964	5009752	5182365	55
56	4278969	4469291	4654884	4835946	5012663	5185207	56
57	4282181	4472422	4657938	4838926	5015572	5188049	57
58	4285392	4475553	4660991	4841905	5018480	5190888	58
59	4288601	4478681	4664043	4844883	5021388	5193728	59
60	4291809	4471808	4667093	4847860	5024294	5196566	60

M	48 Deg.	49 Deg.	50 Deg.	51 Deg.	52 Deg.	53 Deg.	M
0	9.5196566	9.5364839	9.5529265	9.5689987	9.5847139	9.6000849	0
1	5199403	5367611	5531974	5692635	5849729	6003382	1
2	5202239	5370381	5534681	5695282	5852318	6005914	2
3	5205073	5373151	5537388	5697928	5854905	6008446	3
4	5207907	5375919	5540094	5700573	5857492	6010977	4
5	5210739	5378686	5542798	5703218	5860078	6013506	5
6	5213571	5381452	5545502	5705861	5862663	6016035	6
7	5216401	5384218	5548204	5708503	5865247	6018563	7
8	5212930	5386982	5550906	5711144	5867830	6021090	8
9	5222058	5389745	5553606	5713784	5870412	6023616	9
10	5224885	5392507	5556306	5716423	5872993	6026141	10
11	5227711	5395268	5559004	5719062	5875573	6028665	11
12	5230536	5398027	5561701	5721699	5878153	6031188	12
13	5233359	5400786	5564398	5724335	5880731	6033710	13
14	5236182	5403544	5567093	5726970	5883308	6036232	14
15	5239003	5406301	5569787	5729605	5885885	6038752	15
16	5241823	5409056	5572481	5732238	5888460	6041272	16
17	5244643	5411811	5575173	5734870	5891034	6043791	17
18	5247461	5414564	5577864	5737502	5893608	6046308	18
19	5250278	5417317	5580555	5740132	5896181	6048825	19
20	5253094	5420068	5583244	5742761	5898752	6051341	20
21	5255908	5422818	5585932	5745390	5901323	6053856	21
22	5258722	5425568	5588619	5748017	5903893	6056370	22
23	5261535	5428316	5591305	5750643	5906461	6058883	23
24	5264346	5431063	5593991	5753269	5909029	6061396	24
25	5267157	5433809	5596675	5755893	5911596	6063907	25
26	5269966	5436554	5599358	5758517	5914162	6066417	26
27	5272774	5439298	5602040	5761139	5916727	6068927	27
28	5275582	5442041	5604721	5763761	5919291	6071436	28
29	5278388	5444783	5607401	5766382	5921854	6073943	29
30	5281193	5447524	5610080	5769001	5924417	6076450	30
31	5283997	5450264	5612759	5771620	5926978	6078956	31
32	5286799	5453002	5615436	5774237	5929538	6081461	32
33	5289601	5455740	5618112	5776854	5932098	6083965	33
34	5292402	5458477	5620787	5779470	5934656	6086468	34
35	5295201	5461212	5623461	5782085	5937214	6088971	35
36	5298000	5463947	5626134	5784698	5939770	6091472	36
37	5300797	5466681	5628806	5787311	5942326	6093972	37
38	5303594	5469413	5631477	5789923	5944881	6096472	38
39	5306389	5472145	5634147	5792534	5947434	6098971	39
40	5309183	5474875	5636861	5795144	5949987	6101469	40
41	5311976	5477604	5639484	5797753	5952539	6103965	41
42	5314768	5480333	5642151	5800361	5955090	6106461	42
43	5317559	5483060	5644817	5802968	5957640	6108956	43
44	5320349	5485786	5647482	5805574	5960189	6111451	44
45	5323137	5488511	5650146	5808179	5962737	6113944	45
46	5325925	5491236	5652809	5810783	5965285	6116436	46
47	5328712	5493959	5655471	5813386	5967831	6118928	47
48	5331497	5496681	5658132	5815988	5970376	6121418	48
49	5334281	5499402	5660792	5818589	5972921	6123908	49
50	5337065	5502122	5663451	5821190	5975464	6126397	50
51	5339847	5504841	5666109	5823789	5978007	6128885	51
52	5342628	5507559	5668766	5826387	5980549	6131372	52
53	5345408	5510276	5671423	5828985	5983089	6133858	53
54	5348187	5512992	5674078	5831581	5985629	6136343	54
55	5350965	5515706	5676732	5834176	5988168	6138827	55
56	5353742	5518420	5679385	5836771	5990706	6141311	56
57	5356518	5521133	5682037	5839364	5993243	6143793	57
58	5359293	5523845	5684688	5841957	5995779	6146275	58
59	5362067	5526555	5687338	5844549	5998314	6148755	59
60	5364839	5529265	5689987	5847139	6000849	6151235	60

M	54 Deg.	55 Deg.	56 Deg.	57 Deg.	58 Deg.	59 Deg.	M
0	9.6151235	9.6298412	9.6442486	9.6583558	9.6721725	9.6857076	0
1	6153714	6300838	6444861	6585884	6724003	6859309	1
2	6156192	6303264	6447236	6588210	6726281	6861541	2
3	6158669	6305688	6449610	6590535	6728558	6863772	3
4	6161146	6308112	6451983	6592858	6730835	6866002	4
5	6163621	6310535	6454355	6595182	6733110	6868231	5
6	6166096	6312957	6456726	6597504	6735385	6870460	6
7	6168569	6315378	6459097	6599825	6737659	6872688	7
8	6171042	6317799	6461467	6602146	6739932	6874915	8
9	6173514	6320218	6463836	6604466	6742205	6877144	9
10	6175985	6322637	6466204	6606785	6744476	6879368	10
11	6178455	6325055	6468571	6609103	6746747	6881593	11
12	6180924	6327472	6470937	6611421	6749017	6883817	12
13	6183392	6329888	6473303	6613737	6751287	6886040	13
14	6185860	6332303	6475667	6616053	6753555	6888263	14
15	6188326	6334717	6478031	6618368	6755823	6890485	15
16	6190792	6337131	6480394	6620683	6758090	6892706	16
17	6193256	6339543	6482757	6622996	6760356	6894926	17
18	6195720	6341955	6485118	6625309	6762622	6897146	18
19	6198183	6344366	6487479	6627621	6764886	6899365	19
20	6200645	6346776	6489839	6629932	6767150	6901583	20
21	6203107	6349185	6492197	6632242	6769413	6903801	21
22	6205567	6351594	6494556	6634552	6771676	6906017	22
23	6208026	6354001	6496913	6636860	6773937	6908233	23
24	6210485	6356408	6499269	6639168	6776198	6910449	24
25	6212943	6358814	6501625	6641475	6778458	6912663	25
26	6215400	6361219	6503980	6643781	6780717	6914877	26
27	6217855	6363623	6506334	6246087	6782976	6917090	27
28	6220311	6366026	6508687	6648392	6785234	6919302	28
29	6222765	6368429	6511039	6650696	6787491	6921513	29
30	6225218	6370830	6513391	6652999	6789747	6923724	30
31	6227670	6373231	6515741	6655301	6792002	6925934	31
32	6230122	6375631	6318092	6657603	6794257	6928143	32
33	6232573	6378030	6520441	6659903	6796511	6930352	33
34	6235022	6380428	6522789	6662203	6798764	6932559	34
35	6237471	6382825	6525136	6664502	6801016	6934766	35
36	6239919	6385222	6527483	6666801	6803268	6936973	36
37	6242367	6387618	6529829	6669098	6805519	6939178	37
38	6244813	6390012	6532174	6671395	6807769	6941383	38
39	6247258	6392406	6534518	6673691	6810018	9943587	39
40	6249703	6394800	6536861	6675986	6812266	6945790	40
41	6252147	6397192	6539204	6678281	6814514	6947993	41
42	6254589	6399583	6541546	6680574	6816761	6950194	42
43	6257031	6401974	6543887	6682867	6819007	6952396	43
44	6259473	6404364	6546227	6685159	6821253	6954596	44
45	6261913	6406753	6548566	6687450	6823498	6956795	45
46	6264352	6409141	6550904	6689741	6825741	6958994	46
47	6266791	6711528	6553243	6692030	6827985	6961192	47
48	6269228	6413914	6555579	6694319	6830227	6963390	48
49	6271665	6416300	6557915	6696607	6832469	6965586	49
50	6274101	6418685	6560250	6698895	6834710	6967782	50
51	6276536	6421068	6562585	6701181	6836950	6969977	51
52	6278970	6423452	6564918	6703467	6839189	6972172	52
53	6281403	6425834	6567251	6705752	6841428	6974365	53
54	6283836	6428215	6869583	6708036	6843665	6976558	54
55	6286267	6430596	6571914	6710319	6845902	6978750	55
56	6288698	6432975	6574245	6712602	6848139	6980942	56
57	6291128	6435354	6576574	6714884	6850374	6983132	57
58	6293557	6437732	9578903	6717165	6852609	6985322	58
59	6295985	6440109	6581231	6719445	6854843	6987512	59
60	6298412	6442486	6583558	6721725	6857076	6989700	60

M	60 Deg.	61 Deg.	62 Deg.	63 Deg.	64 Deg.	65 Deg.	M
0	9.6989700	9.7119677	9 7247087	9.7372002	9.7494494	9.7614630	0
1	6991888	7121822	7249189	7374063	7496516	7616613	1
2	6994075	7123965	7251290	7376124	7498536	7618595	2
3	6996271	7126108	7253391	7378184	7500556	7620577	3
4	6998447	7128250	7255491	7380243	7502576	7622557	4
5	7000631	7130392	7257591	7382301	7504595	7624537	5
6	7002816	7132533	7259689	7384359	7506613	7626517	6
7	7004999	7134673	7261787	7386416	7508630	7628496	7
8	7007182	7136812	7263885	7388473	7510647	7630474	8
9	7009363	7138951	7265981	7390529	7512663	7632452	9
10	7011545	7141089	7268077	7392584	7514679	7634429	10
11	7013725	7143226	7270172	7394638	7516694	7636405	11
12	7015905	7145362	7272267	7396692	7518708	7638381	12
13	7018084	7147498	7274361	7398745	7520721	7640356	13
14	7020262	7149633	7276454	7400798	7522734	7642330	14
15	7022439	7151768	7278546	7402850	7524746	7644304	15
16	7024616	7153901	7280638	7404901	7526758	7646277	16
17	7026792	7156034	7282729	7406951	7528769	7648250	17
18	7028967	7158166	7284820	7409001	7530779	7650222	18
19	7031142	7160298	7286910	7411050	7532789	7652193	19
20	7033316	7162429	2288999	7413099	7534798	7654164	20
21	7035489	7164559	7291087	7415146	7536806	7656134	21
22	7037661	7166688	7293175	7417193	7538814	7658103	22
23	7039833	7168817	7295262	7419240	7540821	7660072	23
24	7042004	7170945	7297348	7421286	7542828	7662040	24
25	7044174	7173072	7299434	7423331	7544833	7664007	25
26	7046344	7175199	7301519	7425375	7546839	7665974	26
27	7048513	7177325	7303603	7427419	7548843	7667940	27
28	7050681	7179450	7305686	7429462	7550847	7669906	28
29	7052848	7181575	7307769	7431505	7552850	7671871	29
30	7055015	7183698	7309852	7433547	7554853	7673835	30
31	7057180	7185821	7311933	7435588	7556855	7675799	31
32	7059346	7187944	7314014	7437628	7558856	7677762	32
33	7061510	7190066	7316094	7439668	7560856	7679725	33
34	7063674	7192186	7318174	7441707	7560856	7681687	34
35	7065837	7194307	7320252	7443746	7564856	7683648	35
36	7067999	7196426	7322331	7445784	7566854	7685608	36
37	7070160	7198545	7324408	7447821	7568852	7687568	37
38	7072321	7200663	7326485	7449857	7570850	7689528	38
39	7074481	7202781	7328561	7451893	7572847	7691486	39
40	7076641	7204898	7330636	7453928	7574843	7693444	40
41	7078799	7207014	7332711	7455963	7576838	7695402	41
42	7080957	7209129	7334785	7457997	7578833	7697359	42
43	7083115	7211244	7336858	7460030	7580827	7699315	43
44	7085271	7213358	7338931	7462063	7582821	7701271	44
45	7087427	7215471	7341003	7464095	7584814	7703225	45
46	7089582	7217584	7343074	7466126	7586806	7705180	46
47	7091736	7219695	7345145	7468156	7588798	7707134	47
48	7093890	7221807	7347215	7470186	7590789	7709087	48
49	7096043	7223917	7349284	7472216	7592779	7711039	49
50	7098195	7226027	7351355	7474244	7594769	7702991	50
51	7100346	7228136	7353421	7476272	7596758	7714942	51
52	7102497	7230244	7355488	7478299	7598746	7716893	52
53	7104647	7232352	7357555	7780326	7600734	7718843	53
54	7106797	7234459	7359621	7482352	7602721	7720792	54
55	7108945	7236565	7361686	7484377	7604707	7722741	55
56	7111093	7238671	7363750	7486402	7606693	7724689	56
57	7113240	7240776	7365814	7488426	7608679	7726636	57
58	7115387	7242880	7367878	7490449	7110663	7728583	58
59	7117532	7244984	7369940	7492472	7612647	7730530	59
60	7119677	7247087	7372002	7494494	7614630	7732475	60

M	66 Deg.	67 Deg.	68 Deg.	69 Deg.	70 Deg.	71 Deg.	M
0	9.7732475	9.7848090	9,7961533	9.8072860	9.8182126	9.8289381	0
1	7734420	7849998	7963406	8074698	8183930	8291152	1
2	7736365	7851906	7965278	8076536	8185733	8292922	2
3	7738308	7853813	7967149	8078372	8187536	8294692	3
4	7740252	7855720	7969020	8080208	8189338	8296461	4
5	7742194	7857626	7970890	8082044	8191140	8298229	5
6	7744136	7859531	7972760	8083879	8192941	8299997	6
7	7746077	7861436	7974629	8085713	8194742	8301765	7
8	7748018	7863340	7976498	8087547	8196542	8303532	8
9	7749958	7865243	7978366	8089380	8198341	8305299	9
10	7751898	7867146	7980233	8091213	8200140	8307064	10
11	7753836	7869049	7982100	8093045	8201938	8308830	11
12	7755775	7870950	7983966	8094877	8203736	8310595	12
13	7757712	7872852	7985832	8096708	8205533	8312359	13
14	7759649	7874752	7987697	8098538	8207330	8314123	14
15	7761586	7876652	7989561	8100368	8209126	8315886	15
16	7763521	7878551	7991425	8102197	8210922	8317649	16
17	7765457	7880450	7993288	8104026	8212717	8319411	17
18	7767391	7882348	7995151	8105854	8214511	8321172	18
19	7769325	7884246	7997013	8107682	8216305	8322933	19
20	7771258	7886143	7998875	8109509	8218099	8324694	20
21	7773191	7888039	8000735	8111335	8219891	8326454	21
22	7775123	7889935	8002596	8113161	8221684	8328213	22
23	7777055	7891830	8004456	8114986	8223475	8329972	23
24	7778985	7893725	8006315	8116811	8225266	8331731	24
25	7780916	7895618	8008173	8118635	8227057	8333488	25
26	7782845	7897512	8010031	8120459	8228847	8335246	26
27	7784774	7899405	8011889	8122282	8230636	8337002	27
28	7786703	7901297	8013746	8124104	8232425	8338759	28
29	7788630	7903188	8015602	8125926	8234214	8340514	29
30	7790558	7905079	8017458	8127748	8236002	8342269	30
31	7792484	7906970	8019313	8129569	8237789	8344024	31
32	7794410	7908859	8021167	8131389	8239576	8345778	32
33	7796335	7910749	8023021	8133208	8241362	8347532	33
34	7798260	7912673	8024875	8135027	8243147	8349285	34
35	7800184	7914525	8026728	8136846	8244932	8351037	35
36	7802108	7916413	8028580	8138664	8246717	8352789	36
37	7804031	7918300	8030432	8140481	8248501	8354540	37
38	7805953	7920186	8032283	8142298	8250284	8356291	38
39	7807875	7922071	8034133	8144114	8223067	8358041	39
40	7809796	7923956	8035983	8145930	8253849	8359791	30
41	7811716	7925841	8037832	8147745	8255631	8361540	41
42	7813636	7927725	8039681	8149560	8257412	8363289	42
43	7815555	7929608	8041529	8151374	8259193	8365037	43
44	7817474	7931491	8043377	8153187	8260973	8366785	44
45	7819392	7933373	8045224	8155000	8262753	9368532	45
46	7821309	7935254	8047070	8156812	8264532	8370278	46
47	7823226	7937135	8048916	8158624	8266310	8372024	47
48	7825143	7939015	8050762	8160435	8268088	8373770	48
49	7827058	7940895	8052606	8162246	8269866	8375515	49
50	7828973	7942774	8054451	8164056	8271642	8377259	30
51	7830888	7944653	8056294	8165866	8273419	8379003	51
52	7832801	7946531	8058137	8167675	8275194	8380746	52
53	7834715	7948408	8059980	8169483	8276970	8382489	53
54	7836627	7950285	8061821	8171291	8278744	8384231	54
55	7838539	7952161	8063663	8173098	8280518	8385973	55
56	7840450	7954037	8065503	8174905	8282292	8387714	56
57	7842361	7955912	8067344	8176711	8284005	8389455	57
58	7844271	7957786	8069183	8178516	8285837	8391195	58
59	7846181	7959660	8071022	8180322	8287609	8392935	59
60	7848090	7961533	8072860	8182126	8289381	8394674	60

M	72 Deg.	73 Deg.	74 Deg.	75 Deg.	76 Deg.	77 Deg.	M
0	9.8394674	9.8498052	9.8599560	9.8699243	9.8797140	9.8893291	0
1	8396412	8499759	8601237	8700889	8798756	8894879	1
2	8398150	8501465	8602912	8702534	8800372	8896467	2
3	8399888	8503171	8604588	8704179	8801988	8898054	3
4	8401625	8504877	8606262	8705824	8803604	8899640	4
5	8403361	8506582	8607936	8707468	8805218	8901226	5
6	8405097	8508280	8609610	8709112	8806833	8902812	6
7	8406832	8509990	8611283	8710755	8808446	8904397	7
8	8408567	8511693	8612956	8712398	8810060	8905982	8
9	8410301	8513396	8614628	8714040	8811673	8907566	9
10	8412035	8515099	8616300	8715682	8813285	8909150	10
11	8413768	8516800	8617971	8717323	8814897	8910733	11
12	8415501	8518502	8619642	8718963	8816508	8912316	12
13	8417233	8520203	8621312	8720604	8818119	8913898	13
14	8418965	8521903	8622981	8722243	8819730	8915480	14
15	8420696	8523603	8624651	8723883	8821340	8917061	15
16	8422427	8525302	8626319	8725521	8822949	8918642	16
17	8424157	8527001	8627987	8727159	8824558	8920222	17
18	8425886	8528699	8629655	8728797	8826167	8921802	18
19	8427615	8530396	8631322	8730434	8827775	8923382	19
20	8429344	8532094	8632989	8732071	8829382	8924961	20
21	8431072	8533790	8634655	8733707	8830989	8926539	21
22	8432799	8535486	8636320	8735343	8832596	8928117	22
23	8434526	8537182	8637985	8736978	8834202	8929695	23
24	8436252	8538877	8639650	8738613	8835807	8931272	24
25	8437978	8540572	8641314	8740248	8837413	8932849	25
26	8439703	8542266	8642978	4741881	8839017	8934425	26
27	8441428	8543959	8644641	8743515	8840621	8936000	27
28	8443152	8545653	8646303	8745147	8842225	8937576	28
29	8444876	8547345	8647966	8746780	8843828	8939150	29
30	8446599	8549037	8649627	8748412	8845431	8940725	30
31	8448322	8550729	8651288	8750043	8847033	8942299	31
32	8450044	8552420	8652949	8751674	8848635	8943872	32
33	8451766	8554110	8654609	8753304	8850236	8945445	33
34	8453487	8555800	8656269	8754934	8851837	8947017	34
35	8455207	8557490	8657928	8756563	8853438	8948589	35
36	8456927	8559179	8659586	8758192	8855038	8950161	36
37	8458647	8560867	8661244	8759821	8856637	8951732	37
38	8460366	8562555	8662902	8761449	8858236	8953302	38
39	8462084	8564242	8664559	8763076	8859834	8954872	39
40	8463802	8565929	8666216	8764703	8861432	8956442	40
41	8465520	8567616	8667872	8766329	8863030	8958011	41
42	8467237	8569302	8669527	8767955	8864627	8959580	42
43	8468953	8570987	8671182	8769581	8866223	8961148	43
44	8470669	8572672	8672837	8771206	8867819	8962716	44
45	8472384	8574356	8674491	8772830	8869415	8964283	45
46	8474099	8576040	8676145	8774454	8871010	8965850	46
47	8475813	8577723	8677798	8776078	8872605	8967416	47
48	8477527	8579406	8679450	8777701	8874199	8968982	48
49	8479240	8581089	8681102	8779324	8875792	8970547	49
50	8480953	8582770	8682754	8780946	8877386	8972112	50
51	8482665	8584452	8684405	8782567	8878978	8973677	51
52	8484377	8586132	8686056	8784188	8880571	8975241	52
53	8486088	8587813	8687706	8785809	8882162	8976804	53
54	8487799	8589492	8689355	8787429	8883754	8978367	54
55	8489509	8591172	8691004	8789049	8885344	8979930	55
56	8491219	8592851	8692653	8790668	8886935	8981492	56
57	8492928	8594529	8694301	8792286	8888525	8983054	57
58	8494636	8596006	8695949	8793905	8890114	8984615	58
59	8496344	8597884	8697596	8795522	8891703	8986176	59
60	8498052	8599560	8699243	8797140	8893291	8987736	60

M	78 Deg.	79 Deg.	80 Deg.	81 Deg.	82 Deg.	83 Deg.	M
0	9 8987736	9.9080510	9.9171650	9.9261188	6.9349158	9 9435591	0
1	8989296	9082043	9173155	9262667	9350611	9437019	1
2	8990855	9083575	9174660	9264146	9352064	9438446	2
3	8992414	9085106	9176165	9265624	9353516	9439873	3
4	8993973	9086637	9177669	9267101	9354968	9441300	4
5	8995531	9088167	9179172	9268579	9356419	9442726	5
6	8997088	9089697	9180675	8270055	9357870	9444151	6
7	8998645	9091227	9182178	9271532	9359321	9445577	7
8	8990202	9092756	6183680	9273008	9360771	9447001	8
9	9001758	9094285	9185182	9274483	9362220	9448426	9
10	9003313	9095813	9186683	9275958	9363670	9449850	10
11	9004869	9097341	9188184	9277433	9365119	9451273	11
12	9006423	9098868	9189685	9278907	9366567	9452696	12
13	9007978	9100395	9191185	9280380	9368015	9454119	13
14	9009531	9101921	9192684	9281854	9369462	9455541	14
15	9011085	9103447	9194183	9283327	9370909	9456963	15
16	9012638	9104973	9195682	8284799	9372356	9458385	16
17	9014190	9106498	9197180	9286271	9373802	9459806	17
18	9015742	9108022	9198678	9287743	9375248	9461226	18
19	9017294	9109547	9200175	9289214	9376694	9462646	19
20	9018845	9111070	9201672	9290684	9378139	9364066	20
21	9020395	9112593	9203169	9292155	9279583	9465486	21
22	9021945	9114116	9204665	9293624	9381027	9466904	22
23	9023495	9115639	9206160	9295094	9382471	9468323	23
24	9025044	9117161	9207656	9296563	9383914	9469741	24
25	9026593	9118682	9209150	9298031	9385357	9471159	25
26	90[illegible]8141	9120203	9210644	9299499	9386800	9472576	26
27	9029689	9121723	9212138	9300967	9388242	9473993	27
28	9031236	9123244	9213632	9302434	9389683	9475409	28
29	9032783	9124763	9215125	9303901	9391124	9476825	29
30	9034330	9126282	9216617	9305367	9392565	9478241	30
31	9035876	9127801	9218109	9306833	9394005	9479656	31
32	9037421	9129319	9219601	9308299	9395445	9481071	32
33	9038966	9130837	9221092	9309764	9396885	9482486	33
34	9040511	9132355	9222583	9311228	9398324	9483899	34
35	9042055	9133872	9224073	9312693	9399762	9485313	35
36	9043599	9135388	9225563	9314156	9401201	9486726	36
37	9045142	9136904	9227052	9315620	9402638	9488139	37
38	9046685	9138420	9228541	9317083	9404076	9489551	38
39	9048227	9139935	9230030	9318545	9405513	9490963	39
40	9049769	9141450	9231518	9320007	9406949	9492375	40
41	9051310	9142964	9233006	9321469	9408385	9493786	41
42	9052851	9144478	9234493	9322930	9409821	9495196	42
43	9054392	9145991	9235980	9324391	9411256	9496607	43
44	9055932	9147504	9237466	9325851	9412691	9498016	44
45	9057471	9149016	9238952	9327311	9414126	9499426	45
46	9059011	9150528	9240437	9328771	9415560	9500835	46
47	9060549	9152040	9241922	9330230	9416993	9502243	47
48	9062087	9153551	9243407	9331688	9418426	9503652	48
49	9063625	9155062	9244891	9333146	9419859	9505059	49
50	9065163	9156572	9246375	9334604	9421291	9506467	50
51	9066699	9158082	9247858	9336062	9422723	9507874	51
52	9068236	9159591	9249341	9337518	9424155	9509280	52
53	9069772	9161100	9240824	9338975	9425586	9510686	53
54	9071307	9162609	9252306	9340431	9427016	9512092	54
55	9072842	9164117	9253787	9341887	9428447	9513497	55
56	9074377	9165624	9255268	9343342	9429876	9514902	56
57	9075911	9167131	9256749	9344797	9431306	9516307	57
58	9077445	9168638	9258229	9346251	9432735	9517711	58
59	9078978	9170144	9259709	9347705	9434163	9519115	59
60	9080510	9171650	9261188	9349158	9435591	9520518	60